FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper L

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. The number of people, n, in a queue at a Post Office t minutes after it opens is modelled by the differential equation

$$\frac{dn}{dt} = e^{0.5t} - 5, \quad t \geq 0.$$

(a) Find, to the nearest second, the time when the model predicts that there will be the least number of people in the queue. **(3)**

(b) Given that there are 20 people in the queue when the Post Office opens, solve the differential equation. **(4)**

(c) Explain why this model would not be appropriate for large values of t. **(1)**

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2. A curve has the equation

$$3x^2 + xy - 2y^2 + 25 = 0.$$

Find an equation for the normal to the curve at the point with coordinates (1, 4), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(8)**

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3. *(a)* Use the substitution $u = 2 - x^2$ to find

$$\int \frac{x}{2 - x^2} \, dx.$$ **(4)**

(b) Evaluate

$$\int_0^{\frac{\pi}{4}} \sin 3x \cos x \, dx.$$ **(6)**

4.

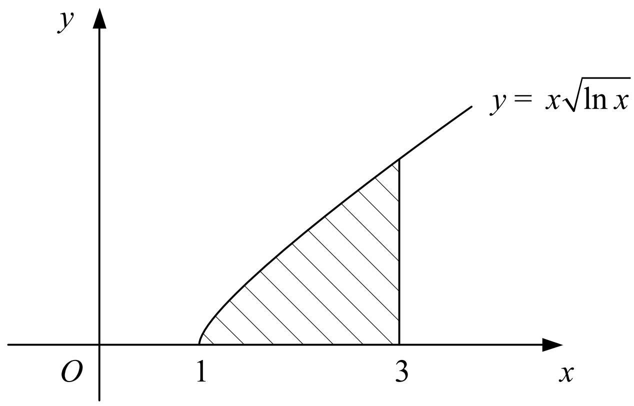

Figure 1

Figure 1 shows the curve with equation $y = x\sqrt{\ln x}$, $x \geq 1$.

The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(a) Using the trapezium rule with two intervals of equal width, estimate the area of the shaded region. **(4)**

The shaded region is rotated through 360° about the x-axis.

(b) Find the exact volume of the solid formed. **(7)**

5.

$$\mathrm{f}(x) = \frac{5 - 8x}{(1 + 2x)(1 - x)^2}.$$

(a) Express f(x) in partial fractions. **(5)**

(b) Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(6)**

(c) State the set of values of x for which your expansion is valid. **(1)**

Turn over

6.

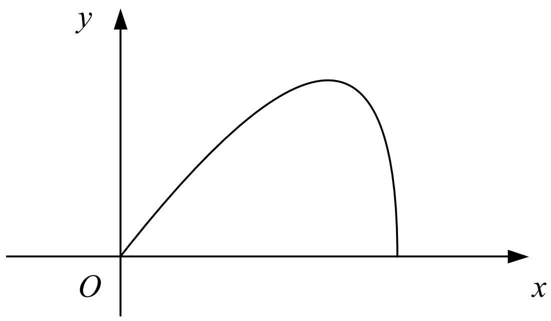

Figure 2

Figure 2 shows the curve with parametric equations

$$x = t + \sin t, \quad y = \sin t, \quad 0 \leq t \leq \pi.$$

(a) Find $\frac{dy}{dx}$ in terms of t. **(3)**

(b) Find, in exact form, the coordinates of the point where the tangent to the curve is parallel to the x-axis. **(3)**

(c) Show that the region bounded by the curve and the x-axis has area 2. **(6)**

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7. The line l_1 passes through the points A and B with position vectors $(3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ and $(8\mathbf{j} - 6\mathbf{k})$ respectively, relative to a fixed origin.

(a) Find a vector equation for l_1. **(2)**

The line l_2 has vector equation

$$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) + \mu(7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}),$$

where μ is a scalar parameter.

(b) Show that lines l_1 and l_2 intersect. **(4)**

(c) Find the coordinates of the point where l_1 and l_2 intersect. **(2)**

The point C lies on l_2 and is such that AC is perpendicular to AB.

(d) Find the position vector of C. **(6)**

END