FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper B – Marking Guide

1.	$u = x^2,\ u' = 2x,\ v' = \sin x,\ v = -\cos x$	M1	
	$I = -x^2 \cos x - \int -2x \cos x \ \mathrm{d}x = -x^2 \cos x + \int 2x \cos x \ \mathrm{d}x$	A2	
	$u = 2x,\ u' = 2,\ v' = \cos x,\ v = \sin x$	M1	
	$I = -x^2 \cos x + 2x \sin x - \int 2 \sin x \ \mathrm{d}x$	A1	
	$= -x^2 \cos x + 2x \sin x + 2 \cos x + c$	A1	**(6)**

2.	$\int \frac{1}{y^2} \ \mathrm{d}y = \int \sqrt{x} \ \mathrm{d}x$	M1	
	$-y^{-1} = \frac{2}{3}x^{\frac{3}{2}} + c$	M1 A1	
	$x = 1,\ y = -2 \Rightarrow \frac{1}{2} = \frac{2}{3} + c, \quad c = -\frac{1}{6}$	M1 A1	
	$-\frac{1}{y} = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{6}, \qquad \frac{1}{y} = \frac{1}{6} - \frac{2}{3}x^{\frac{3}{2}} = \frac{1}{6}(1 - 4x^{\frac{3}{2}})$	M1	
	$y = \dfrac{6}{1-4x^{\frac{3}{2}}}$	A1	**(7)**

3.	$8x - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A2	
	$(-1, -3) \Rightarrow -8 + 6 + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}$	M1 A1	
	grad of normal $= -4$	M1	
	$\therefore\ y + 3 = -4(x + 1) \qquad [\ y = -4x - 7\ ]$	M1 A1	**(8)**

4.	*(a)*	$= 1 + (-3)(ax) + \frac{(-3)(-4)}{2}(ax)^2 + \frac{(-3)(-4)(-5)}{3\times 2}(ax)^3 + \ldots$	M1 A1	
		$= 1 - 3ax + 6a^2x^2 - 10a^3x^3 + \ldots$	A1	
	(b)	$\frac{6-x}{(1+ax)^3} = (6 - x)(\ 1 - 3ax + 6a^2x^2 + \ldots)$		
		coeff. of $x^2 = 36a^2 + 3a = 3$	M1	
		$12a^2 + a - 1 = 0$	A1	
		$(4a - 1)(3a + 1) = 0$	M1	
		$a = -\frac{1}{3},\ \frac{1}{4}$	A1	
	(c)	$a = -\frac{1}{3} \quad \therefore \quad \frac{6-x}{(1+ax)^3} = (6 - x)(\ldots + \frac{2}{3}x^2 + \frac{10}{27}x^3 + \ldots)$	M1	
		coeff. of $x^3 = (6 \times \frac{10}{27}) + (-1 \times \frac{2}{3}) = \frac{20}{9} - \frac{2}{3} = \frac{14}{9}$	A1	**(9)**

5.	*(a)*	$= \int_1^5 \frac{1}{\sqrt{3x+1}} \ \mathrm{d}x = [\frac{2}{3}(3x+1)^{\frac{1}{2}}]_1^5$	M1 A1	
		$= \frac{2}{3}(4 - 2) = \frac{4}{3}$	M1 A1	
	(b)	$= \pi \int_1^5 \frac{1}{3x+1} \ \mathrm{d}x$	M1	
		$= \pi[\frac{1}{3}\ln\lvert 3x + 1 \rvert]_1^5$	M1 A1	
		$= \frac{1}{3}\pi(\ln 16 - \ln 4) = \frac{1}{3}\pi \ln 4 = \frac{2}{3}\pi \ln 2 \qquad [\ k = \frac{2}{3}\ ]$	M1 A1	**(9)**

6. *(a)*	$15 - 17x \equiv A(1-3x)^2 + B(2+x)(1-3x) + C(2+x)$		
	$x = -2 \quad \Rightarrow \quad 49 = 49A \quad \Rightarrow \quad A = 1$	B1	
	$x = \frac{1}{3} \quad \Rightarrow \quad \frac{28}{3} = \frac{7}{3}C \quad \Rightarrow \quad C = 4$	B1	
	coeffs $x^2 \quad \Rightarrow \quad 0 = 9A - 3B \quad \Rightarrow \quad B = 3$	M1 A1	
(b)	$= \int_{-1}^{0} \left(\frac{1}{2+x} + \frac{3}{1-3x} + \frac{4}{(1-3x)^2}\right)\,\mathrm{d}x$		
	$= [\ln\lvert 2+x\rvert - \ln\lvert 1-3x\rvert + \frac{4}{3}(1-3x)^{-1}]_{-1}^{0}$	M1 A3	
	$= (\ln 2 + 0 + \frac{4}{3}) - (0 - \ln 4 + \frac{1}{3})$	M1	
	$= 1 + \ln 8$	M1 A1	**(11)**
7. *(a)*	$x = 1 \;\therefore\; -1 + 4\cos\theta = 1,\; \cos\theta = \frac{1}{2},\; \theta = \frac{\pi}{3}, \frac{5\pi}{3}$	M1	
	$y > 0 \;\therefore\; \sin\theta > 0 \;\therefore\; \theta = \frac{\pi}{3}$	A1	
(b)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -4\sin\theta, \quad \frac{\mathrm{d}y}{\mathrm{d}\theta} = 2\sqrt{2}\cos\theta$	M1	
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{2}\cos\theta}{-4\sin\theta}$	M1 A1	
	at P, grad $= -\frac{2\sqrt{2}\times\frac{1}{2}}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{2}}{2\sqrt{3}}$	M1	
	grad of normal $= \frac{2\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{6}$	A1	
	$\therefore\; y - \sqrt{6} = \sqrt{6}(x-1)$	M1	
	$y = \sqrt{6}\,x$, when $x = 0, y = 0 \;\therefore$ passes through origin	A1	
(c)	$\cos\theta = \frac{x+1}{4},\; \sin\theta = \frac{y}{2\sqrt{2}}$	M1	
	$\therefore\; \frac{(x+1)^2}{16} + \frac{y^2}{8} = 1$	M1 A1	**(12)**
8. *(a)*	$\overrightarrow{AB} = (7\mathbf{i} - \mathbf{j} + 12\mathbf{k}) - (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = (10\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$	M1	
	$\therefore\; \mathbf{r} = (-3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \lambda(5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$	A1	
(b)	$\overrightarrow{OC} = [\mu\mathbf{i} + (5-2\mu)\mathbf{j} + (-7+7\mu)\mathbf{k}]$		
	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [(3+\mu)\mathbf{i} + (2-2\mu)\mathbf{j} + (-9+7\mu)\mathbf{k}]$	M1 A1	
	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = [(-7+\mu)\mathbf{i} + (6-2\mu)\mathbf{j} + (-19+7\mu)\mathbf{k}]$	A1	
	$\overrightarrow{AC}\cdot\overrightarrow{BC} = (3+\mu)(-7+\mu) + (2-2\mu)(6-2\mu) + (-9+7\mu)(-19+7\mu) = 0$	M1	
	$\mu^2 - 4\mu + 3 = 0$	A1	
	$(\mu-1)(\mu-3) = 0$	M1	
	$\mu = 1, 3 \quad \therefore\; \overrightarrow{OC} = (\mathbf{i} + 3\mathbf{j})$ or $(3\mathbf{i} - \mathbf{j} + 14\mathbf{k})$	A2	
(c)	$AC = \sqrt{16+0+4} = 2\sqrt{5},\; BC = \sqrt{36+16+144} = 14$	M1	
	area $= \frac{1}{2} \times 2\sqrt{5} \times 14 = 14\sqrt{5}$	M1 A1	**(13)**
		Total	**(75)**

Performance Record – C4 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	integration	differential equation	differentiation	binomial series	integration	partial fractions	parametric equations	vectors	
Marks	6	7	8	9	9	11	12	13	75
Student									