Paper Reference (complete below)								Centre No.						Surname	Initial(s)
								Candidate No.						**Signature**	

Paper Reference(s)

6663

Edexcel GCE

Pure Mathematics C1
Advanced Subsidiary
Specimen Paper

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Materials required for examination
Answer Book (AB16)
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Items included with question papers
Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Tour candidate details are printed next to the bar code above. Check that these are correct and sign your name in the signature box above.
If your candidate details are incorrect, or missing, then complete ALL the boxes above.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 10 questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

Turn over

Edexcel
Success through qualifications

Leave blank

1. Calculate $\sum_{r=1}^{20}(5+2r)$.

(3)

Leave blank

2. Find $\int (5x + 3\sqrt{x})\, dx$.

(4)

3. (a) Express $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer.

(1)

(b) Express $(4 - \sqrt{5})^2$ in the form $b + c\sqrt{5}$, where b and c are integers.

(3)

Leave blank

Leave blank

4. The points A and B have coordinates (3, 4) and (7, −6) respectively. The straight line l passes through A and is perpendicular to AB.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

5.

Figure 1

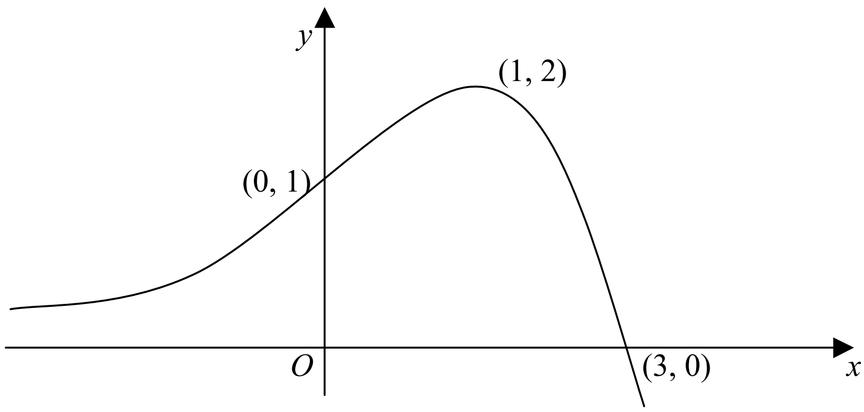

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve crosses the coordinate axes at the points (0, 1) and (3, 0). The maximum point on the curve is (1, 2).

On separate diagrams in the space opposite, sketch the curve with equation

(a) $y = \mathrm{f}(x + 1)$,

(3)

(b) $y = \mathrm{f}(2x)$.

(3)

On each diagram, show clearly the coordinates of the maximum point, and of each point at which the curve crosses the coordinate axes.

Leave blank

Leave blank

6. (a) Solve the simultaneous equations

$$y + 2x = 5,$$

$$2x^2 - 3x - y = 16.$$

(6)

(b) Hence, or otherwise, find the set of values of x for which

$$2x^2 - 3x - 16 > 5 - 2x.$$

(3)

Leave blank

Leave blank

7. Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

(a) Find the amount he plans to save in the year 2011.

(2)

(b) Calculate his total planned savings over the 20 year period from 2001 to 2020.

(3)

Ben also plans to save money over the same 20 year period. He saves £A in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben's total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

(c) calculate the value of A.

(4)

Leave blank

Leave blank

8. Given that

$$x^2 + 10x + 36 \equiv (x + a)^2 + b,$$

where a and b are constants,

(a) find the value of a and the value of b.

(3)

(b) Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots.

(2)

The equation $x^2 + 10x + k = 0$ has equal roots.

(c) Find the value of k.

(2)

(d) For this value of k, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinate axes.

(4)

Leave blank

Leave blank

9. The curve C has equation $y = f(x)$ and the point $P(3, 5)$ lies on C.

Given that

$$f'(x) = 3x^2 - 8x + 6,$$

(a) find $f(x)$.

(4)

(b) Verify that the point $(2, 0)$ lies on C.

(2)

The point Q also lies on C, and the tangent to C at Q is parallel to the tangent to C at P.

(c) Find the x-coordinate of Q.

(5)

Leave blank

Leave blank

10. The curve C has equation

$$y = x^3 - 5x + \frac{2}{x}, \qquad x \neq 0.$$

The points A and B both lie on C and have coordinates $(1, -2)$ and $(-1, 2)$ respectively.

(a) Show that the gradient of C at A is equal to the gradient of C at B.

(5)

(b) Show that an equation for the normal to C at A is

$$4y = x - 9.$$

(4)

The normal to C at A meets the y-axis at the point P. The normal to C at B meets the y-axis at the point Q.

(c) Find the length of PQ.

(4)

Leave blank

END

Paper Reference (complete below)								Centre No.					
								Candidate No.					

Surname	Initial(s)
Signature	

Paper Reference(s)

6664

Edexcel GCE

Pure Mathematics C2
Advanced Subsidiary
Specimen Paper

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
9	
Total	

Materials required for examination

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G..

Instructions to Candidates

Tour candidate details are printed next to the bar code above. Check that these are correct and sign your name in the signature box above.
If your candidate details are incorrect, or missing, then complete ALL the boxes above.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 9 questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit

Turn over

Edexcel
Success through qualifications

Leave blank

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 + 3x)^6$.

(4)

Leave blank

2. The circle C has centre (3, 4) and passes through the point (8, −8).

Find an equation for C.

(4)

Leave blank

3. The trapezium rule, with the table below, was used to estimate the area between the curve $y = \sqrt{(x^3 + 1)}$, the lines $x = 1$, $x = 3$ and the x-axis.

x	1	1.5	2	2.5	3
y	1.414	2.092	3.000		

(a) Calculate, to 3 decimal places, the values of y for $x = 2.5$ and $x = 3$.

(2)

(b) Use the values from the table and your answers to part (a) to find an estimate, to 2 decimal places, for this area.

(4)

4. Solve, for $0 \leq x < 360°$, the equation

$$3 \sin^2 x = 1 + \cos x,$$

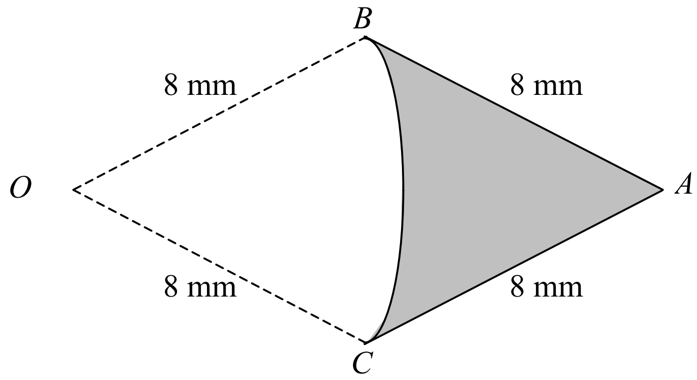
giving your answers to the nearest degree.

(7)

Leave blank

5.

Figure 1



The shaded area in Fig. 1 shows a badge ABC, where AB and AC are straight lines, with $AB = AC = 8$ mm. The curve BC is an arc of a circle, centre O, where $OB = OC = 8$ mm and O is in the same plane as ABC. The angle BAC is 0.9 radians.

(a) Find the perimeter of the badge.

(2)

(b) Find the area of the badge.

(5)

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6. At the beginning of the year 2000 a company bought a new machine for £15 000. Each year the value of the machine decreases by 20% of its value at the start of the year.

(a) Show that at the start of the year 2002, the value of the machine was £9600.

(2)

When the value of the machine falls below £500, the company will replace it.

(b) Find the year in which the machine will be replaced.

(4)

To plan for a replacement machine, the company pays £1000 at the start of each year into a savings account. The account pays interest at a fixed rate of 5% per annum. The first payment was made when the machine was first bought and the last payment will be made at the start of the year in which the machine is replaced.

(c) Using your answer to part (b), find how much the savings account will be worth immediately after the payment at the start of the year in which the machine is replaced.

(4)

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7. (a) Use the factor theorem to show that $(x + 1)$ is a factor of

$$x^3 - x^2 - 10x - 8.$$

(2)

(b) Find all the solutions of the equation $x^3 - x^2 - 10x - 8 = 0$.

(4)

(c) Prove that the value of x that satisfies

$$2 \log_2 x + \log_2 (x - 1) = 1 + \log_2 (5x + 4) \qquad \text{(I)}$$

is a solution of the equation

$$x^3 - x^2 - 10x - 8 = 0.$$

(4)

(d) State, with a reason, the value of x that satisfies equation (I).

(2)

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8. **Figure 2**

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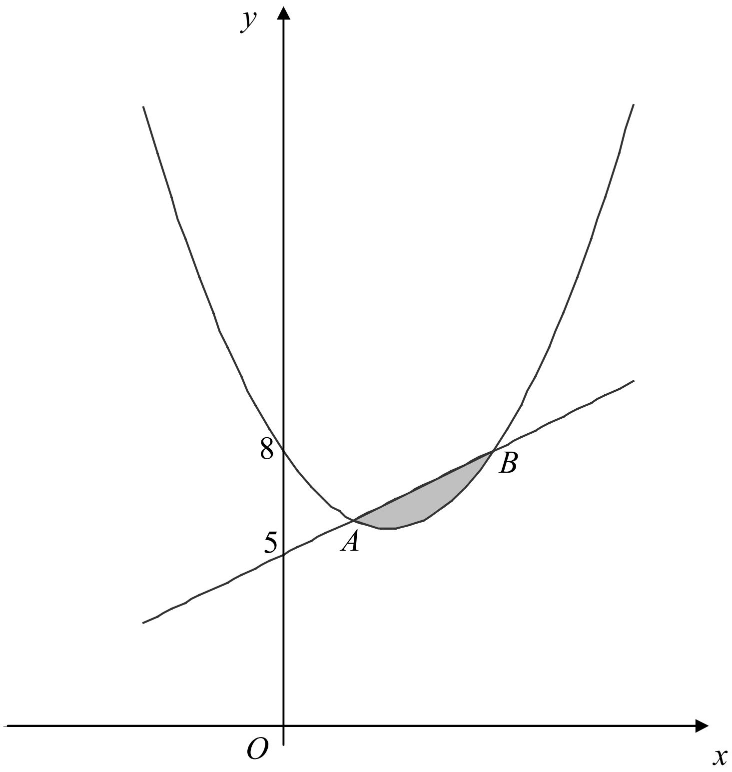

The line with equation $y = x + 5$ cuts the curve with equation $y = x^2 - 3x + 8$ at the points A and B, as shown in Fig. 2.

(a) Find the coordinates of the points A and B.

(5)

(b) Find the area of the shaded region between the curve and the line, as shown in Fig. 2.

(7)

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9. **Figure 3**

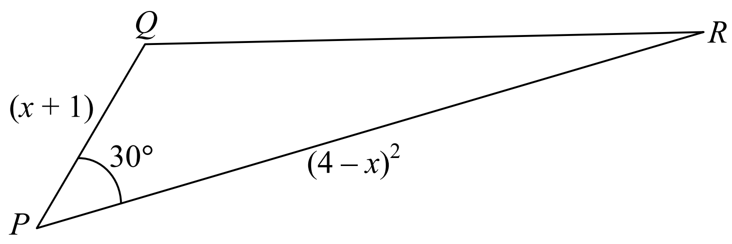

Figure 3 shows a triangle PQR. The size of angle QPR is 30°, the length of PQ is $(x + 1)$ and the length of PR is $(4 - x)^2$, where $x \in \mathbb{R}$.

(a) Show that the area A of the triangle is given by

$$A = \frac{1}{4}(x^3 - 7x^2 + 8x + 16).$$

(3)

(b) Use calculus to prove that the area of ΔPQR is a maximum when $x = \frac{2}{3}$. Explain clearly how you know that this value of x gives the maximum area.

(6)

(c) Find the maximum area of ΔPQR.

(1)

(d) Find the length of QR when the area of ΔPQR is a maximum.

(3)

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END

6665
Edexcel GCE

Pure Mathematics C3

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio *cfx* 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics C3), the paper reference (6665), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Edexcel
Success through qualifications

1. The function f is defined by

$$f: x \mapsto |x-2| - 3,\ x \in \mathbb{R}.$$

(a) Solve the equation $f(x) = 1$.

(3)

The function g is defined by

$$g: x \mapsto x^2 - 4x + 11,\ x \geq 0.$$

(b) Find the range of g.

(3)

(c) Find gf(–1).

(2)

2.

$$f(x) = x^3 - 2x - 5.$$

(a) Show that there is a root α of $f(x) = 0$ for x in the interval [2, 3].

(2)

The root α is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{\left(2 + \frac{5}{x_n}\right)},\quad x_0 = 2.$$

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures.

(3)

(c) Prove that, to 5 significant figures, α is 2.0946.

(3)

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3. (a) Using the identity for $\cos(A + B)$, prove that $\cos\theta \equiv 1 - 2\sin^2(\frac{1}{2}\theta)$.

(3)

(b) Prove that $1 + \sin\theta - \cos\theta \equiv 2\sin(\frac{1}{2}\theta)[\cos(\frac{1}{2}\theta) + \sin(\frac{1}{2}\theta)]$.

(3)

(c) Hence, or otherwise, solve the equation

$$1 + \sin\theta - \cos\theta = 0,\quad 0 \leq \theta < 2\pi.$$

(4)

4.

$$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2+2x-3}, \; x \in \mathbb{R}, x > 1.$$

(a) Show that $f(x) = \dfrac{x^2+3x+3}{x+3}$.

(5)

(b) Solve the equation $f'(x) = \frac{22}{25}$.

(5)

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5. **Figure 1**

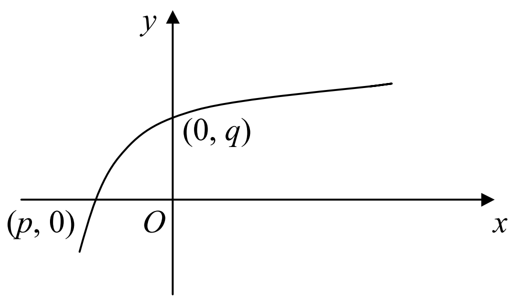

Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$. The curve meets the x-axis at $P\ (p, 0)$ and meets the y-axis at $Q\ (0, q)$.

(a) On separate diagrams, sketch the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 3f(\frac{1}{2}x)$.

In each case show, in terms of p or q, the coordinates of points at which the curve meets the axes.

(5)

Given that $f(x) = 3\ln(2x + 3)$,

(b) state the exact value of q,

(1)

(c) find the value of p,

(2)

(d) find an equation for the tangent to the curve at P.

(4)

---

6. As a substance cools its temperature, T °C, is related to the time (t minutes) for which it has been cooling. The relationship is given by the equation

$$T = 20 + 60e^{-0.1t}, \quad t \geq 0.$$

(a) Find the value of T when the substance started to cool.

(1)

(b) Explain why the temperature of the substance is always above 20°C.

(1)

(c) Sketch the graph of T against t.

(2)

(d) Find the value, to 2 significant figures, of t at the instant $T = 60$.

(4)

(e) Find $\frac{\mathrm{d}T}{\mathrm{d}t}$.

(2)

(f) Hence find the value of T at which the temperature is decreasing at a rate of 1.8 °C per minute.

(3)

---

7. (i) Given that $y = \tan x + 2\cos x$, find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \frac{\pi}{4}$.

(3)

(ii) Given that $x = \tan \frac{1}{2}y$, prove that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{1+x^2}$.

(4)

(iii) Given that $y = e^{-x}\sin 2x$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $R\,e^{-x}\cos(2x + \alpha)$. Find, to 3 significant figures, the values of R and α, where $0 < \alpha < \frac{\pi}{2}$.

(7)

END

Paper Reference(s)

6666

Edexcel GCE

Pure Mathematics C4

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI-89, TI-92, Casio CFX-9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics C4), the paper reference (6666), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has eight questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Use the binomial theorem to expand $(4-3x)^{-\frac{1}{2}}$, in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

---

2. The curve C has equation

$$13x^2 + 13y^2 - 10xy = 52.$$

Find an expression for $\frac{dy}{dx}$ as a function of x and y, simplifying your answer.

(6)

---

3. Use the substitution $x = \tan\theta$ to show that

$$\int_0^1 \frac{1}{(1+x^2)^2}\,dx = \frac{\pi}{8} + \frac{1}{4}.$$

(8)

---

4. **Figure 1**

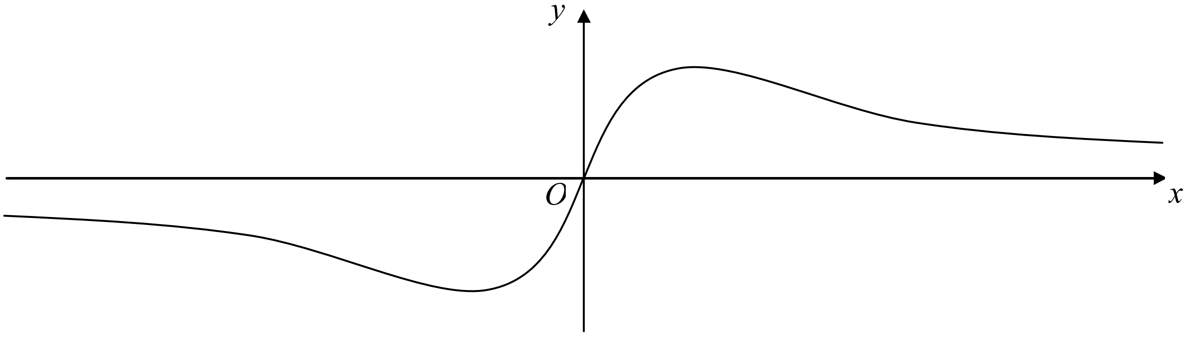

Figure 1 shows part of the curve with parametric equations

$$x = \tan t, \quad y = \sin 2t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

(a) Find the gradient of the curve at the point P where $t = \frac{\pi}{3}$.

(4)

(b) Find an equation of the normal to the curve at P.

(3)

(c) Find an equation of the normal to the curve at the point Q where $t = \frac{\pi}{4}$.

(2)

5. The vector equations of two straight lines are

$$\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \qquad \text{and}$$

$$\mathbf{r} = 2\mathbf{i} - 11\mathbf{j} + a\mathbf{k} + \mu(-3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}).$$

Given that the two lines intersect, find

(a) the coordinates of the point of intersection,

(5)

(b) the value of the constant a,

(2)

(c) the acute angle between the two lines.

(4)

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6. Given that

$$\frac{11x-1}{(1-x)^2(2+3x)} \equiv \frac{A}{(1-x)^2} + \frac{B}{(1-x)} + \frac{C}{(2+3x)},$$

(a) find the values of A, B and C.

(4)

(b) Find the exact value of $\int_0^{\frac{1}{2}} \frac{11x-1}{(1-x)^2(2+3x)}\,dx$, giving your answer in the form $k + \ln a$, where k is an integer and a is a simplified fraction.

(7)

Turn over

7. (a) Given that $u = \frac{x}{2} - \frac{1}{8}\sin 4x$, show that $\frac{du}{dx} = \sin^2 2x$.

(4)

Figure 2

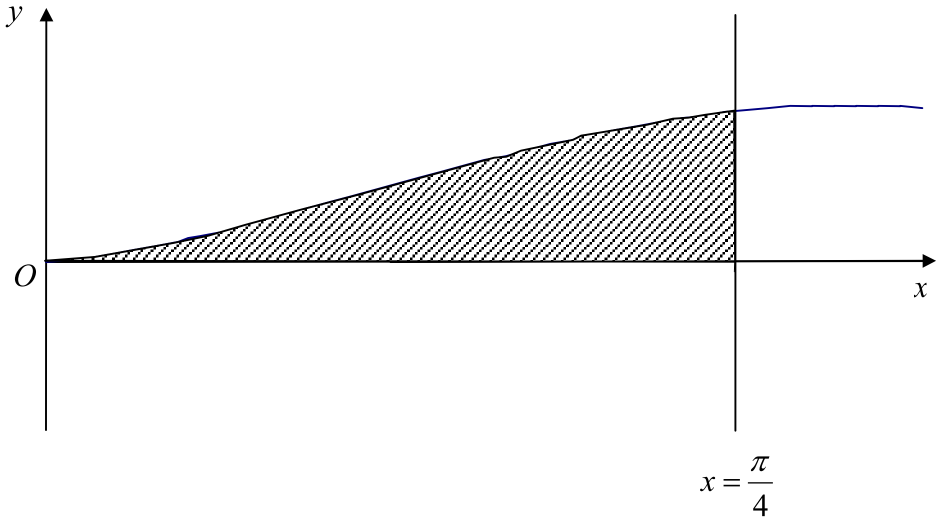


Figure 2 shows the finite region bounded by the curve $y = x^{\frac{1}{2}} \sin 2x$, the line $x = \frac{\pi}{4}$ and the x-axis. This region is rotated through 2π radians about the x-axis.

(b) Using the result in part (a), or otherwise, find the exact value of the volume generated.

(8)

8. A circular stain grows in such a way that the rate of increase of its radius is inversely proportional to the square of the radius. Given that the area of the stain at time t seconds is A cm^2,

(a) show that $\frac{dA}{dt} \propto \frac{1}{\sqrt{A}}$.

(6)

Another stain, which is growing more quickly, has area S cm² at time t seconds. It is given that

$$\frac{dS}{dt} = \frac{2e^{2t}}{\sqrt{S}}.$$

Given that, for this second stain, $S = 9$ at time $t = 0$,

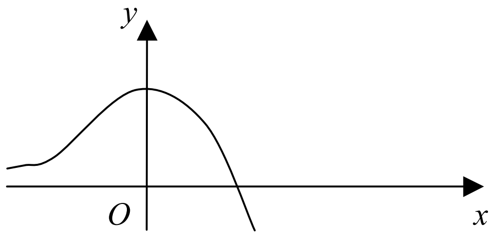
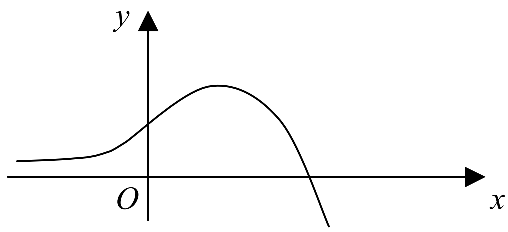
(b) solve the differential equation to find the time at which $S = 16$. Give your answer to 2 significant figures.

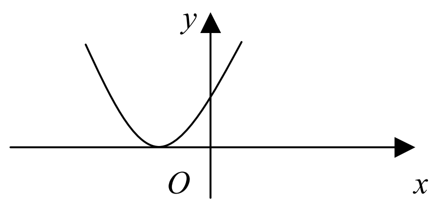
(7)

END

Turn over

EDEXCEL PURE MATHEMATICS C1 (6663) SPECIMEN PAPER MARK SCHEME

Question number	Scheme	Marks
1.	$a = 7,\ d = 2$	B1
	$S_{20} = \frac{1}{2} \times 20 \times (2 \times 7 + 19 \times 2) = 520$	M1 A1
		(3 marks)
2.	$\int (5x + 3\sqrt{x})\,\mathrm{d}x = \dfrac{5x^2}{2} + 2x^{\frac{3}{2}} + C$	M1 A1 A1 B1
		(4 marks)
3. (a)	$\sqrt{80} = 4\sqrt{5}$	B1 (1)
(b)	$(4 - \sqrt{5})^2 = 16 - 8\sqrt{5} + 5 = 21 - 8\sqrt{5}$	M1 A1 A1 (3)
		(4 marks)
4.	Gradient of $AB = \dfrac{4 - (-6)}{3 - 7}$ $\left(= -\dfrac{5}{2}\right)$	M1 A1
	Gradient of $l = \dfrac{2}{5}$	M1
	$y - 4 = \dfrac{2}{5}(x - 3)$ $2x - 5y + 14 = 0$	M1 A1 (5)
		(5 marks)
5. (a)	 Position, Shape	B1
	(0, 2), (2, 0)	B1 B1 (3)
(b)	 Position, Shape	B1
	(0, 1), $\left(\dfrac{1}{2}, 2\right)$, $\left(\dfrac{3}{2}, 0\right)$	B2 (1, 0) (3)
		(6 marks)

Question number	Scheme	Marks
6. (a)	$5 - 2x = 2x^2 - 3x - 16$ $\quad 2x^2 - x - 21 = 0$	M1 A1
	$(2x - 7)(x + 3) = 0$ $\quad x = -3, x = \frac{7}{2}$	M1 A1
	$y = 11, y = -2$	M1 A1ft (6)
(b)	Using critical values $x = -3, \quad x = \frac{7}{2}$	M1
	$x < -3, \quad x > \frac{7}{2}$	M1 A1ft (3)
		(9 marks)
7. (a)	$a + (n - 1)d = 250 + (10 \times 50) = £750$	M1 A1 (2)
(b)	$\frac{1}{2}n\left[2a + (n-1)d\right] = \frac{1}{2} \times 20 \times (500 + 19 \times 50)$, $\quad = £14500$	M1 A1, A1 (3)
(c)	B: $\frac{1}{2} \times 20 \times (2A + 19 \times 60)$ $\quad [= 10(2A + 1140)]$, = "14500"	B1, M1
	Solve for A: $A = 155$	M1 A1 (4)
		(9 marks)
8. (a)	$a = 5$, $\quad (x + 5)^2 - 25 + 36$ $\quad b = 11$	B1, M1 A1 (3)
(b)	$b^2 - 4ac = 100 - 144$, $\quad < 0$, therefore no real roots	M1 A1 (2)
(c)	Equal roots if $b^2 - 4ac = 0$ $\quad 4k = 100$ $\quad k = 25$	M1 A1 (2)
(d)	 Shape, position	B1 B1
	$(-5, 0)$ $(0, 25)$	B1 B1ft (4)
		(11 marks)

Question number	Scheme	Marks
9. (a)	$f(x) = x^3 - 4x^2 + 6x + C$	M1 A1
	$5 = 27 - 36 + 18 + C$ $\quad C = -4$	M1 A1 (4)
(b)	$x = 2$: $\quad y = 8 - 16 + 12 - 4 = 0$	M1 A1 (2)
(c)	$f'(3) = 27 - 24 + 6 = 9$, Parallel therefore equal gradient	B1, M1
	$3x^2 - 8x + 6 = 9$ $\quad 3x^2 - 8x - 3 = 0$	M1
	$(3x + 1)(x - 3) = 0$ $\quad Q$: $x = -\frac{1}{3}$	M1 A1 (5)
		(11 marks)
10. (a)	$\frac{dy}{dx} = 3x^2 - 5 - 2x^{-2}$	M1 A2(1,0)
	At both A and B, $\quad \frac{dy}{dx} = 3 \times 1 - 5 - \frac{2}{1}$ $\quad (= -4)$	M1 A1 (5)
(b)	Gradient of normal $= \frac{1}{4}$	M1 A1ft
	$y - (-2) = \frac{1}{4}(x - 1)$ $\quad 4y = x - 9$	M1 A1 (4)
(c)	Normal at A meets y-axis where $x = 0$: $\quad y = -\frac{9}{4}$	B1
	Similarly for normal at B: $\quad 4y = x + 9$ $\quad y = \frac{9}{4}$	M1 A1
	Length of $PQ = \frac{9}{4} + \frac{9}{4} = \frac{9}{2}$	A1 (4)
		(13 marks)

EDEXCEL PURE MATHEMATICS C2 (6664)SPECIMEN PAPER MARK SCHEME

Question number		Scheme		Marks
1.		$(2 + 3x)^6 = 2^6 + 6.2^5 \times 3x + \binom{6}{2} 2^4 (3x)^2$	>1 term correct	M1
		$= 64, + 576x, + 2160x^2$		B1 A1 A1
				(4 marks)
2.		$r = \sqrt{(8-3)^2 + (-8-4)^2}$, = 13	Method for r or r^2	M1 A1
		Equation: $(x - 3)^2 + (y - 4)^2 = 169$	ft their r	M1 A1ft
				(4 marks)
3.	(a)	(x = 2.5) y = 4.077 (x = 3) y = 5.292		B1 B1 (2)
	(b)	$A \approx \frac{1}{2} \times \frac{1}{2}$[1.414 + 5.292 +2(2.092 + 3.000 + 5.292)]	For $\frac{1}{2} \times \frac{1}{2}$	B1
			ft their y values	M1 A1ft
		= 6.261 = 6.26 (2 d.p.)		A1 (4)
				(6 marks)
4.		$3(1 - \cos^2 x) = 1 + \cos x$	Use of $s^2 + c^2 = 1$	M1
		$0 = 3\cos^2 x + \cos x - 2$	3TQ in cos x	M1
		$0 = (3\cos x - 2)(\cos x + 1)$	Attempt to solve	M1
		$\cos x = \frac{2}{3}$ or -1	Both	A1
		$\cos x = \frac{2}{3}$ gives $x = 48°, 312°$		B1, B1ft
		$\cos x = -1$ gives $x = 180°$		B1
				(7 marks)
5.	(a)	Arc length = $r\theta = 8 \times 0.9 = 7.2$	M1 for use of $r\theta$	M1
		Perimeter = $16 + r\theta$ = 23.2 (mm)		A1 (2)
	(b)	Area of triangle = $\frac{1}{2}.8^2.\sin(0.9) = 25.066$		M1
		Area of sector = $\frac{1}{2}.8^2.(0.9) = 28.8$		M1
		Area of segment = 28.8 – 25.066 = 3.7(33..)		A1ft
		Area of badge = triangle – segment, = 21.3 (mm^2)		M1, A1 (5)
				(7 marks)

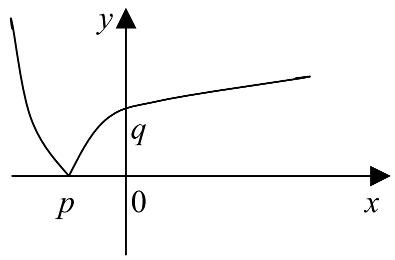
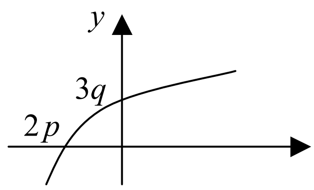
Question number		Scheme		Marks
6.	(a)	$15000 \times (0.8)^2 = 9600$ (*)	M1 for × by 0.8	M1 A1 cso (2)
	(b)	$15000 \times (0.8)^n < 500$	Suitable equation or inequality	M1
		$n \log(0.8) < \log(\frac{1}{30})$	Take logs	M1
		$n > 15.(24\ldots)$	n = is OK	A1
		So machine is replaced in 2015		A1 (4)
	(c)	$a = 1000$, $r = 1.05$, $n = 16$ (≥ 2 correct)		M1
		$S_{16} = \dfrac{1000(1.05^{16} - 1)}{1.05 - 1}$		M1 A1
		= 23 657.49 = £23 700 or £23 660 or £23657		A1 (4)
				(10 marks)
7.	(a)	$f(-1) = -1 - 1 + 10 - 8$	f(+1) or f(–1)	M1
		$= 0$ so $(x + 1)$ is a factor	= 0 and comment	A1 (2)
	(b)	$x^3 - x^2 = 2(5x + 4)$	Out of logs	M1
		i.e. $x^3 - x^2 - 10x - 8 = 0$ (*) A1 cso (4)		M1
		$x = -1, -2, 4$		A2(1, 0) (4)
	(c)	$\log_2 x^2 + \log_2 (x - 1) = 1 + \log_2 (5x + 4)$	Use of $\log x^n$	M1
		$\log_2 \left(\dfrac{x^2 (x-1)}{5x+4} \right) = 1$	Use of $\log a + \log b$	M1
	(d)	$x = 4$, since $x < 0$ is not valid in logs		B1, B1 (2)
				(12 marks)

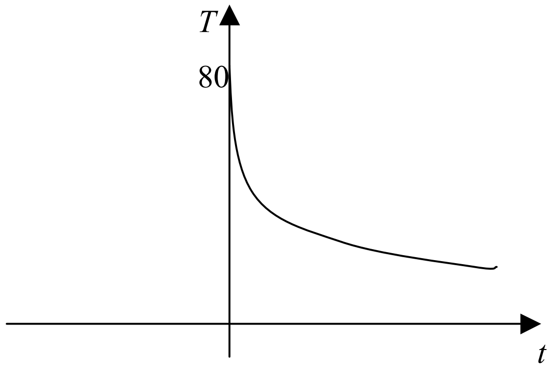
Question number		Scheme		Marks
8.	(a)	$x^2 - 3x + 8 = x + 5$	Line = curve	M1
		$x^2 - 4x + 3 = 0$	3TQ = 0	M1
		$0 = (x - 3)(x - 1)$	Solving	M1
		A is (1, 6); *B* is (3, 8)		A1; A1 (5)
	(b)	$\int (x^2 - 3x + 8)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 8x\right]$	Integration	M1 A2(1,0)
		Area below curve $= (9 - \frac{27}{2} + 24) - (\frac{1}{3} - \frac{3}{2} + 8) = 12\frac{2}{3}$	Use of Limits	M1
		Trapezium $= \frac{1}{2} \times 2 \times (6 + 8) = 14$		B1
		Area = Trapezium – Integral , = $14 - 12\frac{2}{3} = 1\frac{1}{3}$		M1, A1 (7)
				(12 marks)
ALT	(b)	$-x^2 + 4x - 3$	Line – curve	M1
		$\int (-x^2 + 4x - 3)\,dx = \left[-\frac{x^3}{3} + 2x^2 - 3x\right]$	Integration	M1 A2(1,0)
		Area = $\int_1^3 (...)\,dx = (-9 + 18 - 9) - (-\frac{1}{3} + 2 - 3)$	Use of limits	M1
		$= 1\frac{1}{3}$		A2 (7)

Question number	Scheme		Marks
9. (a)	$A = \frac{1}{2}(x+1)(4-x)^2 \sin 30°$	Use of $\frac{1}{2}ab\sin C$	M1
	$= \frac{1}{4}(x+1)(16-8x+x^2)$	Attempt to multiply out.	M1
	$= \frac{1}{4}(x^3-7x^2+8x+16)$ (*)		A1 cso (3)
(b)	$\frac{dA}{dx} = \frac{1}{4}(3x^2-14x+8)$	Ignore the $\frac{1}{4}$	M1 A1
	$\frac{dA}{dx} = 0 \Rightarrow (3x-2)(x-4) = 0$		M1
	So $x = \frac{2}{3}$ or 4	At least $x = \frac{2}{3}$ or…	A1
	e.g. $\frac{d^2A}{dx^2} = \frac{1}{4}(6x-14)$, when $x = \frac{2}{3}$ it is < 0, so maximum	Any full method	M1
	So $x = \frac{2}{3}$ gives maximum area (*)	Full accuracy	A1 (6)
(c)	Maximum area $= \frac{1}{4}(\frac{5}{3})(\frac{10}{3})^2 = 4.6$ or 4.63 or 4.630		B1 (1)
(d)	Cosine rule: $QR^2 = (\frac{5}{3})^2 + (\frac{10}{3})^4 - 2\times\frac{5}{3}\times(\frac{10}{3})^2\cos 30°$	M1 for QR or QR^2	M1 A1
	$= 94.159…$		
	$QR = 9.7$ or 9.70 or 9.704		A1 (3)
			(13 marks)

Question number	Scheme	Marks
1. (a)	$\lvert x-2 \rvert - 3 = 1$ $\qquad x = 6$	B1
	$-(x-2) - 3 = 1 \Rightarrow x = -2$	M1 A1 (3)
(b)	$g(x) = x^2 - 4x + 11 = (x-2)^2 + 7$ or $g'(x) = 2x - 4$	M1 A1
	$g'(x) = 0 \Rightarrow x = 2$	
	Range: $g(x) \geq 7$.	A1 (3)
(c)	$gf(-1) = g(0)$ correct order; $\quad = 11$	M1 A1 (2)
		(8 marks)
2. (a)	$f(2) = 8 - 4 - 5 = -1$ method shows change of sign	M1
	$f(3) = 27 - 6 - 5 = 16$ $\Rightarrow$ root with accuracy	A1 (2)
(b)	$x_1 = 2.121, \quad x_2 = 2.087, \quad x_3 = 2.097, \quad x_4 = 2.094$	M1 A2 (1, 0) (3)
(c)	Choosing suitable interval, e.g. [2.09455, 2.09465]	M1
	$f(2.09455) = -0.00001\ldots$ shows change of sign	M1
	$f(2.09465) = +0.001(099..)$ accuracy and conclusion	A1 (3)
		(8 marks)
3. (a)	$\cos(A+B) = \cos A \cos B - \sin A \sin B$ (formula sheet)	
	$\cos(\frac{1}{2}\theta + \frac{1}{2}\theta)$ $= \cos(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta) - \sin(\frac{1}{2}\theta)\sin(\frac{1}{2}\theta) = \cos^2(\frac{1}{2}\theta) - \sin^2(\frac{1}{2}\theta)$	M1
	$= \{1 - \sin^2(\frac{1}{2}\theta)\} - \sin^2(\frac{1}{2}\theta) = 1 - 2\sin^2(\frac{1}{2}\theta)$	M1 A1 (3)
(b)	$\sin\theta + 1 - \cos\theta = 2\sin(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta) + 2\sin^2(\frac{1}{2}\theta)$	M1 M1
	$= 2\sin(\frac{1}{2}\theta)[\cos(\frac{1}{2}\theta) + \sin(\frac{1}{2}\theta)]$	A1 (3)
	[M1 use of $\sin 2A = 2\sin A\cos A$; M1 use of (a)]	
(c)	$2\sin(\frac{1}{2}\theta)[\cos(\frac{1}{2}\theta) + \sin(\frac{1}{2}\theta)] = 0$	
	$\Rightarrow \sin(\frac{1}{2}\theta) = 0$ or $\cos(\frac{1}{2}\theta) + \sin(\frac{1}{2}\theta) = 0$	M1
	$\theta = 0$	B1
	$\tan\frac{1}{2}\theta = -1; \Rightarrow \theta = \frac{3}{2}\pi$	M1 A1 (4)
		(10 marks)

Question number	Scheme	Marks
4. (a)	$x^2 + 2x - 3 = (x+3)(x-1)$	B1
	$\text{f}(x) = \dfrac{x(x^2+2x-3)+3(x+3)-12}{(x+3)(x-1)}$ $\left[= \dfrac{x^3+2x^2-3}{(x+3)(x-1)}\right]$	M1A1
	$= \dfrac{(x-1)(x^2+3x+3)}{(x-1)(x+3)}$	M1
	$= \dfrac{(x^2+3x+3)}{(x+3)}$	A1 (5)
(b)	$\text{f}'(x) = \dfrac{(x+3)(2x+3)-(x^2+3x+3)}{(x+3)^2}$ $\left[= \dfrac{x^2+6x+6}{(x+3)^2}\right]$	M1 A2, 1, 0
	Setting $\text{f}'(x) = \frac{22}{25}$ and attempting to solve quadratic	M1
	$x = 2$ (only this solution)	A1 (5)
		(10 marks)
ALT (b)	ALT: $\text{f}(x) = x + \dfrac{3}{x+3}$, $\text{f}'(x) = 1 - \dfrac{3}{(x+3)^2}$	

Question number	Scheme	Marks
5. (a)	(i)  Shape correct:	B1
	Intercepts	B1 (2)
	(ii)  Shape correct	B1
	$(2p, 0)$ on x	B1
	$(0, 3q)$ on y	B1 (3)
(b)	q = 3 ln 3	B1 (1)
(c)	$\ln(2p + 3) = 0 \Rightarrow 2p + 3 = 1$; $p = -1$	M1 A1 (2)
(d)	$\frac{dy}{dx} = \frac{6}{2x+3}$; evaluated at $x = p$ (6)	M1 A1
	Equation: $y = 6(x + 1)$ any form	M1 A1ft (4)
		(12 marks)

Question number	Scheme	Marks
6. (a)	$T = 80$	B1 (1)
(b)	$e^{-0.1\,t} \geq 0$ or equivalent	B1 (1)
(c)	 Negative exponential shape	M1
	$t \geq 0$, "80"	
	clearly not $\to x$-axis	A1 (2)
(d)	$60 = 20 + 60\,e^{-0.1\,t} \Rightarrow 60\,e^{-0.1\,t} = 40$	M1
	$\Rightarrow -0.1\,t = \ln\left(\frac{2}{3}\right)$	M1A1
	$t = 4.1$	A1 (4)
(e)	$\frac{dT}{dt} = -6\,e^{-0.1\,t}$	M1A1 (2)
(f)	Using $\frac{dT}{dt} = -1.8$	B1
	Solving for t, or using value of $e^{-0.1\,t}$ (0.3)	M1
	$T = 38$	A1 (3)
		(13 marks)

Question number	Scheme	Marks
7. (i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x - 2\sin x$	B1 B1
	When $x = \frac{1}{4}\pi$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \sqrt{2}$	B1 (3)
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{2}\sec^2 \frac{1}{2}y$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sec^2\left(\frac{y}{2}\right)} = \frac{2}{1+\tan^2\left(\frac{y}{2}\right)} = \frac{2}{1+x^2}$	M1 M1 A1 (4)
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{-x}\cos 2x - \mathrm{e}^{-x}\sin 2x = \mathrm{e}^{-x}(2\cos 2x - \sin 2x)$	M1 A1 A1
	Method for R: R = 2.24 (allow $\sqrt{5}$)	M1 A1
	Method for α: $\alpha = 0.464$	M1 A1 (7)
		(14 marks)

Question number	Scheme	Marks
1.	$(4-3x)^{-\frac{1}{2}} \qquad = 4^{-\frac{1}{2}}\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	
	$= \frac{1}{2}\left(1+\frac{3}{8}x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{3}{4}x\right)^2}{2}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{3}{4}x\right)^3}{6}+.....\right)$	B1 M1
	$= \frac{1}{2} + \frac{3}{16}x, + \frac{27}{256}x^2, + \frac{135}{2048}x^3 \ldots\ldots$	A1, A1, A1
		(5 marks)
2.	$26x + 26yy'\,;\, - 10xy' - 10y = 0$	M1A1; M1A1
	$y'(26y - 10x) = 10y - 26x$	
	$y' = \frac{10y-26x}{26y-10x} = \frac{5y-13x}{13y-5x}$	M1 A1
		(6 marks)
3.	$x = \tan\theta \qquad \frac{dx}{d\theta} = \sec^2\theta \Rightarrow I = \int \frac{\sec^2\theta}{\sec^4\theta}\, d\theta$	M1 A1
	Limits $\frac{\pi}{4}$ and 0	B1
	$I = \int \cos^2\theta \;\; d\theta = \int \frac{\cos 2\theta + 1}{2}\, d\theta$	M1 A1
	$= \left[\frac{\sin 2\theta}{4} + \frac{\theta}{2}\right]_0^{\frac{\pi}{4}}$	M1 A1
	$= \frac{1}{4} + \frac{\pi}{8} \qquad (*)$	A1 cao
		(8 marks)

Question number		Scheme	Marks	
4.	(a)	$\frac{dx}{dt} = \sec^2 t \qquad \frac{dy}{dt} = 2\cos 2t, \Rightarrow \frac{dy}{dx} = \frac{2\cos 2t}{\sec^2 t}$	M1 A1, ⇒ M1	
		When $t = \frac{\pi}{3}$ gradient is $-\frac{1}{4}$	B1	(4)
	(b)	$y - \frac{\sqrt{3}}{2} = -\frac{1}{m}(x - \sqrt{3})$ P has coordinates $(\sqrt{3}, \frac{\sqrt{3}}{2})$	B1	
		$y - \frac{\sqrt{3}}{2} = 4(x - \sqrt{3})$	M1	
		$y = 4x - \frac{7}{2}\sqrt{3}$	A1	(3)
	(c)	$\frac{dy}{dx} = 0 \Rightarrow$ gradient of tan = 0, gradient of normal undefined	M1	
		$\therefore x = \tan\frac{\pi}{4}$, i.e: $x = 1$	A1	(2)
				(9 marks)
5.	(a)	$5 + \lambda = 2 - 3\mu; \qquad 3 - 2\lambda = -11 - 4\mu$	B1 B1	
		$\therefore \lambda + 3\mu + 3 = 0$		
		$2\lambda - 4\mu - 14 = 0$		
		$2\lambda + 6\mu + 6 = 0$		
		$10\mu + 20 = 0 \Rightarrow \mu = -2 \therefore \lambda = 3$	M1 A1	
		$\therefore$ point is $(8, -3, 4)$	A1	(5)
	(b)	$\therefore a - 10 = 4 \qquad \Rightarrow a = 14$	M1 A1	(2)
	(c)	$\cos\theta = \frac{-3 + 8 + 10}{\sqrt{9}\sqrt{25+25}}$	M1 A1	
		$= \frac{15}{3 \times 5\sqrt{2}} \qquad = \frac{1}{\sqrt{2}}$		
		Angle = 45°	M1 A1	(4)
				(11 marks)

Question number	Scheme	Marks
6. (a)	$11x - 1 \equiv A(2 + 3x) + B(1 - x)(2 + 3x) + C(1 - x)^2$	
	Putting $x = 1 \Rightarrow A = 2$	B1
	Putting $x = -\frac{2}{3} \Rightarrow -\frac{25}{3} = \frac{25}{9}C \quad \Rightarrow C = -3$	B1
	cf x^2 $\quad 0 = -3B + C \Rightarrow B = -1$	M1A1 (4)
(b)	$\int_0^{\frac{1}{2}} \frac{2}{(1-x)^2} - \frac{1}{(1-x)} - \frac{3}{(2+3x)} \; dx$	
	$= \left[\frac{2}{1-x} + \ln\lvert 1-x\rvert - \ln\lvert 2+3x\rvert\right]$	M1 A1ft A1ft A1ft
	$= [4 + \ln\frac{1}{2} - \ln 3\frac{1}{2} - (2 - \ln 2)]$	M1
	$= 2 + \ln \frac{\frac{1}{2}\times 2}{3\frac{1}{2}}$	M1
	$= 2 + \ln \frac{2}{7}$	A1 (7)
		(11 marks)
7. (a)	$\frac{du}{dx} = \frac{1}{2} - \frac{1}{2}\cos 4x; = \frac{1}{2} - \frac{1}{2}(1 - 2\sin^2 2x) = \sin^2 2x$	M1 A1; M1 A1 (4)
(b)	$V = \pi \int x \sin^2 2x \, dx$	M1
	$= \pi\left[x\left(\frac{x}{2} - \frac{1}{8}\sin 4x\right) - \int \frac{x}{2} - \frac{1}{8}\sin 4x \, dx\right]_0^{\frac{\pi}{4}}$	M1 A1 A1
	$= \pi\left[\frac{x^2}{2} - \frac{x}{8}\sin 4x - \left(\frac{x^2}{4} + \frac{1}{32}\cos 4x\right)\right]_0^{\frac{\pi}{4}}$	M1 A1
	$= \pi\left[\frac{\pi^2}{64} + \frac{1}{32} + \frac{1}{32}\right] = \pi\left[\frac{\pi^2}{64} + \frac{1}{16}\right]$	M1 A1 (8)
		(12 marks)

Question number	Scheme	Marks
8. (a)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^2}$	B1
	$A = \pi r^2 \qquad \therefore \frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$	M1A1
	$\therefore \frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi r \frac{k}{r^2} = \frac{(2\pi k)}{r}; = \frac{(2\pi k)}{\left(\frac{A}{\pi}\right)^{\frac{1}{2}}} = \frac{2\pi^{\frac{3}{2}}k}{\sqrt{A}}$	M1; M1
	$\therefore \frac{\mathrm{d}A}{\mathrm{d}t} \propto \frac{1}{\sqrt{A}}$ (*)	A1 (6)
(b)	$\int \sqrt{S}\,\mathrm{d}S = \int 2e^{2t}\,\mathrm{d}t$	M1
	$\frac{2}{3}S^{\frac{3}{2}} = e^{2t} + C$	M1A1
	$t = 0, S = 9 \quad \Rightarrow C = 17$	B1
	$\therefore \frac{2}{3}S^{\frac{3}{2}} = \mathrm{e}^{2t} + 17$ and use $S = 16$	M1
	$\left(\frac{128}{3} - 17\right) = \mathrm{e}^{2t} \qquad \Rightarrow t = \frac{1}{2} \ln\left[\frac{77}{3}\right]$	M1
	$= 1.6$	A1 (7)
		(13 marks)