FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper L

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. $\mathrm{f}(x) \equiv \dfrac{2x-3}{x-2}, \quad x \in \mathbb{R}, \quad x > 2.$

(a) Find the range of f. **(2)**

(b) Show that $\mathrm{ff}(x) = x$ for all $x > 2$. **(3)**

(c) Hence, write down an expression for $\mathrm{f}^{-1}(x)$. **(1)**

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2. Solve each equation, giving your answers in exact form.

(a) $\mathrm{e}^{4x-3} = 2$ **(3)**

(b) $\ln(2y-1) = 1 + \ln(3-y)$ **(4)**

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3. The curve C has the equation $y = 2\mathrm{e}^x - 6\ln x$ and passes through the point P with x-coordinate 1.

(a) Find an equation for the tangent to C at P. **(4)**

The tangent to C at P meets the coordinate axes at the points Q and R.

(b) Show that the area of triangle OQR, where O is the origin, is $\dfrac{9}{3-\mathrm{e}}$. **(4)**

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4. *(a)* Express

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)}$$

as a single fraction in its simplest form. **(5)**

(b) Hence, show that the equation

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)} = 1$$

has no real roots. **(4)**

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5. Find the values of x in the interval $-180 < x < 180$ for which

$$\tan (x + 45)^\circ - \tan x^\circ = 4,$$

giving your answers to 1 decimal place. **(9)**

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6. *(a)* Sketch on the same diagram the graphs of $y = |x| - a$ and $y = |3x + 5a|$, where a is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes. **(6)**

(b) Solve the equation

$$|x| - a = |3x + 5a|.$$ **(4)**

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7. *(a)* Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 1 - 2 \sin^2 \tfrac{x}{2}.$$ **(3)**

(b) Prove that, for $\sin x \neq 0$,

$$\frac{1 - \cos x}{\sin x} \equiv \tan \tfrac{x}{2}.$$ **(3)**

(c) Find the values of x in the interval $0 \leq x \leq 360^\circ$ for which

$$\frac{1 - \cos x}{\sin x} = 2 \sec^2 \tfrac{x}{2} - 5,$$

giving your answers to 1 decimal place where appropriate. **(6)**

Turn over

8. A curve has the equation $y = (2x + 3)\,e^{-x}$.

(a) Find the exact coordinates of the stationary point of the curve. **(4)**

The curve crosses the y-axis at the point P.

(b) Find an equation for the normal to the curve at P. **(2)**

The normal to the curve at P meets the curve again at Q.

(c) Show that the x-coordinate of Q lies in the interval $[-2, -1]$. **(3)**

(d) Use the iterative formula

$$x_{n+1} = \frac{3 - 3e^{x_n}}{e^{x_n} - 2},$$

with $x_0 = -1$, to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 2 decimal places. **(3)**

(e) Show that your value for x_4 is the x-coordinate of Q correct to 2 decimal places. **(2)**

END