

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C3

Paper F

### MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

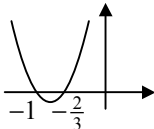
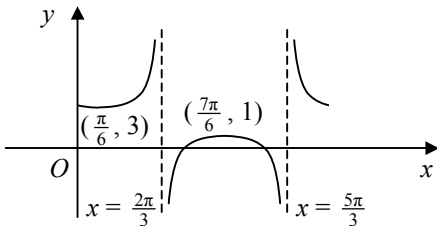


*Written by Shaun Armstrong*

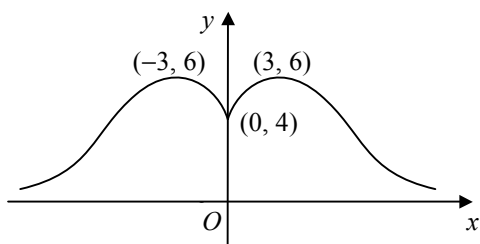
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### C3 Paper F – Marking Guide

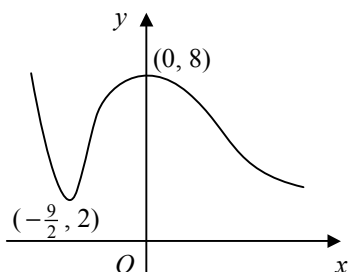
<p>1. <math>\frac{3}{\sin \theta} = -8 \cos \theta</math>  <math>3 = -8 \sin \theta \cos \theta = -4 \sin 2\theta</math>  <math>\sin 2\theta = -\frac{3}{4}</math>  <math>2\theta = 180 + 48.590, 360 - 48.590 = 228.590, 311.410</math>  <math>\theta = 114.3, 155.7</math> (1dp)</p>	<p>M1 M1 A1 M1 A2</p>	<p>(6)</p>
<p>2. (a) <math>g(x) = (x+a)^2 - a^2 + 2</math>  <math>\therefore g(x) \geq 2 - a^2</math></p> <p>(b) <math>gf(3) = g(1-3a) = (1-3a)^2 + 2a(1-3a) + 2</math>  <math>\therefore 1 - 6a + 9a^2 + 2a - 6a^2 + 2 = 7, \quad 3a^2 - 4a - 4 = 0</math>  <math>(3a+2)(a-2) = 0</math>  <math>a = -\frac{2}{3}, 2</math></p>	<p>M1 A1 A1 M1 A1 M1 A1</p>	<p>(7)</p>
<p>3. (a) <math>3x+1 = e^2</math>  <math>x = \frac{1}{3}(e^2 - 1)</math></p> <p>(b) consider <math>\ln(3x^2 + 5x + 3) \geq 0</math>  <math>\Rightarrow 3x^2 + 5x + 3 \geq 1</math>  <math>3x^2 + 5x + 2 \geq 0</math>  <math>(3x+2)(x+1) \geq 0</math>  <math>x \leq -1</math> or <math>x \geq -\frac{2}{3}</math></p> <p><math>\therefore</math> if (e.g.) <math>x = -\frac{3}{4}, \ln(3x^2 + 5x + 3) = \ln \frac{15}{16} = -0.0645\dots</math>  <math>\therefore</math> if <math>x = -\frac{3}{4}, \ln(3x^2 + 5x + 3) &lt; 0 \quad \therefore</math> statement is false</p>	<p>M1 M1 A1  M1 M1 A1 M1 A1</p> 	<p>(8)</p>
<p>4. (a) <math>\frac{dx}{dy} = 1 \times \sqrt{1-2y} + y \times \frac{1}{2}(1-2y)^{-\frac{1}{2}} \times (-2)</math>  <math>= \sqrt{1-2y} - \frac{y}{\sqrt{1-2y}} = \frac{(1-2y)-y}{\sqrt{1-2y}} = \frac{1-3y}{\sqrt{1-2y}}</math>  <math>\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\sqrt{1-2y}}{1-3y}</math></p> <p>(b) <math>y = -1, x = -\sqrt{3}, \text{grad} = \frac{1}{4}\sqrt{3}</math>  <math>\therefore y+1 = \frac{1}{4}\sqrt{3}(x+\sqrt{3})</math>  <math>4y+4 = \sqrt{3}x+3</math>  <math>\sqrt{3}x-4y-1=0 \quad [p=-4, q=-1]</math></p>	<p>M1 A1 M1 M1 A1 B1 M1 A1</p>	<p>(8)</p>
<p>5. (a) </p> <p>(b) <math>2 + \sec(x - \frac{\pi}{6}) = 0, \quad \sec(x - \frac{\pi}{6}) = -2, \quad \cos(x - \frac{\pi}{6}) = -\frac{1}{2}</math>  <math>x - \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}</math>  <math>x = \frac{5\pi}{6}, \frac{3\pi}{2}</math></p>	<p>M2 A3 M1 B1 M1 A2</p>	<p>(10)</p>

6. (a)

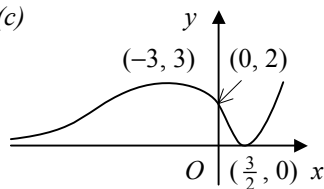


B3

(b)



(c)



M2 A2

M2 A2 (11)

7. (a)

$$\begin{aligned}
 f(x) &= 1 + \frac{4x}{2x-5} - \frac{15}{(2x-5)(x-1)} \\
 &= \frac{2x^2 - 7x + 5 + 4x(x-1) - 15}{(2x-5)(x-1)} \\
 &= \frac{6x^2 - 11x - 10}{(2x-5)(x-1)} = \frac{(3x+2)(2x-5)}{(2x-5)(x-1)} = \frac{3x+2}{x-1}
 \end{aligned}$$

B1

M1 A1

M1 A1

(b)

$$\begin{aligned}
 y &= \frac{3x+2}{x-1}, \quad y(x-1) = 3x+2 \\
 x(y-3) &= y+2 \\
 x &= \frac{y+2}{y-3} \\
 \therefore f^{-1}(x) &= \frac{x+2}{x-3}
 \end{aligned}$$

M1

M1

A1

$$f(x) = \frac{3(x-1)+5}{x-1} = 3 + \frac{5}{x-1}$$

M1

$x < 1 \therefore f(x) < 3 \therefore$  domain of  $f^{-1}(x)$  is  $x \in \mathbb{R}, x < 3$

A1

(c)

$$f(x) = 2 \Rightarrow x = f^{-1}(2) = -4$$

M1 A1

(12)

8. (a)

$$\frac{dy}{dx} = 2x - \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$$

M1 A1

$$x = 1, y = -1, \text{ grad} = \frac{7}{4}$$

A1

$$\therefore y + 1 = \frac{7}{4}(x - 1)$$

M1

$$4y + 4 = 7x - 7$$

$$7x - 4y = 11$$

A1

(b)

$$\text{SP: } 2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0$$

M1

$$\text{let } f(x) = 2x - \frac{1}{2x\sqrt{4 + \ln x}}, \quad f(0.3) = -0.40, \quad f(0.4) = 0.088$$

M1

sign change,  $f(x)$  continuous  $\therefore$  root

A1

(c)

$$2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0 \Rightarrow 2x = \frac{1}{2x\sqrt{4 + \ln x}}$$

$$x^2 = \frac{1}{4\sqrt{4 + \ln x}} = \frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}$$

M1

$$x = \sqrt{\frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4 + \ln x)^{-\frac{1}{4}}$$

A1

(d)

$$x_1 = 0.38151, x_2 = 0.37877, x_3 = 0.37900, x_4 = 0.37898 \text{ (5dp)}$$

M1 A2

(13)

Total (75)

## Performance Record – C3 Paper F

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	trigonometry	functions	exponentials and logarithms, proof	differentiation	trigonometry	functions	rational expressions, functions	differentiation, numerical methods	
Marks	6	7	8	8	10	11	12	13	75
Student									