FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper B – Marking Guide

1. *(a)* $= \frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$ — M1 A2

(b) $\ln(x^2 + 7x + 12) - \ln(2x^2 + 9x + 4) = 1, \quad \ln\frac{x^2+7x+12}{2x^2+9x+4} = 1$ — M1

$\ln\frac{x+3}{2x+1} = 1, \quad \frac{x+3}{2x+1} = \mathrm{e}$ — A1

$x + 3 = \mathrm{e}(2x + 1), \quad 3 - \mathrm{e} = x(2\mathrm{e} - 1)$ — M1

$x = \frac{3-\mathrm{e}}{2\mathrm{e}-1}$ — A1 **(7)**

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2. *(a)* $x = 3,\ y = \sqrt{20} = 2\sqrt{5}$ — B1

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(3x+11)^{-\frac{1}{2}} \times 3 = \frac{3}{2}(3x+11)^{-\frac{1}{2}}$ — M1 A1

grad $= \frac{3}{4\sqrt{5}}$ — A1

$\therefore\ y - 2\sqrt{5} = \frac{3}{4\sqrt{5}}(x - 3)$ — M1

$4\sqrt{5}\,y - 40 = 3x - 9$

$3x - 4\sqrt{5}\,y + 31 = 0$ — A1

(b) normal: $y - 2\sqrt{5} = -\frac{4\sqrt{5}}{3}(x - 3)$ — M1

at Q, $x = 0\ \therefore\ y - 2\sqrt{5} = 4\sqrt{5},\quad y = 6\sqrt{5}$ — M1 A1 **(9)**

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3. *(a)* $\sin(A + B) \equiv \sin A\cos B + \cos A\sin B$

$\sin(A - B) \equiv \sin A\cos B - \cos A\sin B$

adding, $\sin(A + B) + \sin(A - B) \equiv 2\sin A\cos B$ — M1 A1

let $P = A + B,\ Q = A - B$

adding, $P + Q = 2A \Rightarrow A = \frac{P+Q}{2}$ — M1

subtracting, $P - Q = 2B \Rightarrow B = \frac{P-Q}{2}$

$\therefore\ \sin P + \sin Q \equiv 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$ — A1

(b) $2\sin 3x\cos 2x = 0$ — M1

$\sin 3x = 0$ or $\cos 2x = 0$ — A1

$3x = 0, \pi, 2\pi$ or $2x = \frac{\pi}{2}, \frac{3\pi}{2}$ — M1

$x = 0, \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}$ — A2 **(9)**

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4. *(a)* $(4, 0)$ — B1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{2}x^{\frac{3}{2}} \times \ln\frac{x}{4} + x^{\frac{5}{2}} \times \frac{1}{x} = \frac{1}{2}x^{\frac{3}{2}}(5\ln\frac{x}{4} + 2)$ — M1 A1

grad $= 8$, grad of normal $= -\frac{1}{8}$ — A1

$\therefore\ y - 0 = -\frac{1}{8}(x - 4)$ — M1

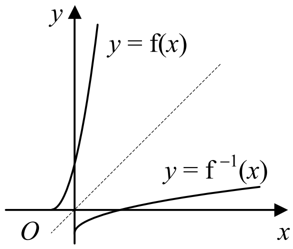
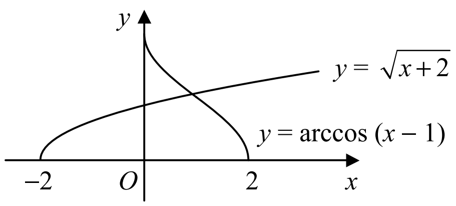
at Q, $x = 0,\ y = \frac{1}{2}$ — A1

area $= \frac{1}{2} \times \frac{1}{2} \times 4 = 1$ — A1

(c) $\frac{1}{2}x^{\frac{3}{2}}(5\ln\frac{x}{4} + 2) = 0$

$\ln\frac{x}{4} = -\frac{2}{5}$ — M1

$x = 4\mathrm{e}^{-\frac{2}{5}}$ — M1 A1 **(10)**

5.	*(a)*	$= 2[x^2 + 2x] + 2 = 2[(x+1)^2 - 1] + 2$	M1	
		$= 2(x+1)^2$	A1	
	(b)	translation by 1 unit in negative x direction stretch by scale factor of 2 in y direction (either first)	B3	
	(c)	$y = 2(x+1)^2, \quad \frac{y}{2} = (x+1)^2$	M1	
		$x + 1 = \pm\sqrt{\frac{y}{2}}$	M1	
		$x = -1 \pm \sqrt{\frac{y}{2}}$ (domain $\Rightarrow$ +), $\therefore$ $f^{-1}(x) = -1 + \sqrt{\frac{x}{2}}$, $x \in \mathbb{R}$, $x \geq 0$	A2	
	(d)		B3	
		$y = f^{-1}(x)$ is reflection of $y = f(x)$ in line $y = x$	B1	**(13)**
6.	*(a)*	$f(x) > -2$	B1	
	(b)	$x = 0,\ y = e - 2 \quad \therefore\ P(0, e - 2)$	B1	
		$y = 0,\ 0 = e^{3x+1} - 2$	M1	
		$3x + 1 = \ln 2$	M1	
		$x = \frac{1}{3}(\ln 2 - 1) \quad \therefore\ Q(\frac{1}{3}(\ln 2 - 1), 0)$	A1	
	(c)	$f'(x) = 3e^{3x+1}$	M1	
		at P, grad $= 3e$	A1	
		$\therefore\ y - (e - 2) = 3e(x - 0)$	M1	
		$y = 3ex + e - 2$	A1	
	(d)	at Q, grad $= 6$	B1	
		tangent at Q: $y - 0 = 6(x - \frac{1}{3}(\ln 2 - 1))$	M1	
		$y = 6x - 2\ln 2 + 2$		
		intersect: $3ex + e - 2 = 6x - 2\ln 2 + 2$		
		$x(3e - 6) = 4 - e - 2\ln 2$	M1	
		$x = \frac{4 - e - 2\ln 2}{3e - 6} = -0.0485$ (3sf)	A1	**(13)**
7.	*(a)*	$\arccos\theta = \frac{\pi}{3}, \quad \theta = \cos\frac{\pi}{3} = \frac{1}{2}$	M1 A1	
	(b)		B2	
			B3	
	(c)	let $f(x) = \arccos(x - 1) - \sqrt{x+2}$		
		$f(0) = 1.7,\ f(1) = -0.16$	M1 A1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(d)	$x_1 = 0.83944,\ x_2 = 0.88598,\ x_3 = 0.87233,$ $x_4 = 0.87632,\ x_5 = 0.87515,\ x_6 = 0.87549$	M1 A2	
		$\therefore\ \alpha = 0.875$ (3dp)	A1	**(14)**
			Total	**(75)**

Performance Record – C3 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	rational expressions, exponentials and logarithms	differentiation	trigonometry	differentiation	functions	exponentials and logarithms, differentiation	trigonometry, numerical methods	
Marks	7	9	9	10	13	13	14	75
Student								