EXAMINATION PAPER 1	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed *Where appropriate, give your answers to 3 s.f.*	
	Core Mathematics – C3
Time Allowed:-1 hour 30 minutes	

1. Express the following expression as a single fraction in its simplest form:
$$\frac{x+1}{(x-1)(x+2)}-\frac{6}{(x-1)(x+3)}$$ **[4]**

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2. $f(x) = x^4 - x - 1$
$f(x) = 0$ has a solution such that $n < x < n + 1$ where n is a positive integer.
a) i) Find a positive value of n such that the inequality is true. **[3]**
ii) Construct a simple logical argument to *prove* that such a solution exists. **[3]**
b) Using an iteration based on the equation $x = \sqrt[4]{1+x}$, find a solution to $f(x) = 0$ to 3 decimal places. **[4]**

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3. $f(x) = (x - 3)^2 + 4$
a) Calculate the equation of the function $g(x)$ where $g(x) = 1 + f(x + 1)$ **[2]**
There is a relationship between the graphs of $y = f(x)$ and $y = g(x)$.
b) i) Clearly define the transformation that takes the graph of $f(x)$ to $g(x)$. **[3]**
ii) Clearly define the transformation that takes the graph of $g(x)$ to $f(x)$. **[1]**
$h(x) = |x+2| - 3$
c) Solve the equation $h(x) = 1$ **[3]**
d) Find $fh(-3)$ **[3]**

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4. Given that $2\cos 3x \cos x = \cos 2C + \cos C$
a) Find C in terms of x. **[2]**
b) Let x be 15^o and hence, or otherwise find an *exact value* for $\cos 15^o$. Leave your answer in *surd form* and *rationalise the denominator* if necessary. **[4]**
c) Hence or otherwise solve the equation $2\cos 3x \cos x = 1$ for $0 < x \le 180^o$.
Give your answers to 1 decimal place. **[6]**

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5. $f(x) = x^3$, $g(x) = 4x - 2$
a) Find $fg(x)$, $gf(x)$ **[2]**
b) Sketch the graph of $y = g(\sin x)$ and state the coordinates of the minimum point of the graph within the range $0 < x \le 2\pi$ radians. **[4]**

$h(x) = \dfrac{x+1}{x-1}$ where x is real and $x \ne 1$

c) Find $h^{-1}(x)$ and state its domain and range. **[5]**

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6. $f(x) = \cos x + 2\sin x$
a) Express $f(x)$ in the form $R\cos(x^o - \alpha^o)$ where $0 \le \alpha < 90^o$ **[4]**
b) Solve the equation $\cos x + 2\sin x = 1$ where $0 \le x < 360^o$ **[4]**
c) For what values of x is $\dfrac{6}{6+\cos x+2\sin x}$ a maximum, where $0 < x < 360^o$? **[3]**
d) What is the value of this maximum? **[1]**

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7. a) Find $\dfrac{dy}{dx}$ when $x = 6$ and $y > 0$ and $x = y^2 - y$. **[5]**
b) i) Find the equation of the tangent to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{3}$ radians. **[5]**
ii) Find the equation of the tangent to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{6}$ radians. **[3]**
iii) Find the equation of the normal to the curve $y = \sin 3x \cos 6x$ when $x = \dfrac{\pi}{6}$ radians. **[1]**

EXAMINATION PAPER 2	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, give your answers to 3 s.f.***	
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Time Allowed:-1 hour 30 minutes	

1. Solve the simultaneous equations, $e^{3x} = ey$ and $\ln y = 6x - 2$ where e is the exponential constant. **[6]**

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2. a) Simplify the expression: $\dfrac{\tan\phi}{\tan\phi + \cot\phi}$ **[4]**

b) Hence or otherwise simplify the expression: $\dfrac{\tan^2\phi}{2 + \tan^2\phi + \cot^2\phi}$ **[2]**

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3. $y = 3e^x$

a) Sketch this curve, stating where the curve crosses the y-axis. **[2]**

b) Find the equation of the normal to the curve at the point $(\ln 3, 9)$ **[5]**

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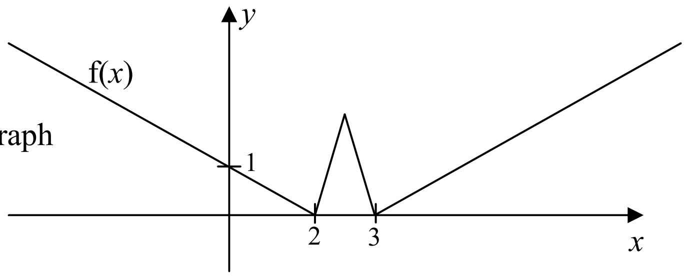
4. Sketch separately the graphs of–

a) $f(|x|)$ **[2]**

b) $2f(x + 1)$ **[3]**

In each sketch clearly show where the graph crosses or touches the x-axis and y-axis.

c) State the relationship between $f(x)$ and $|f(x)|$. **[1]**



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5. Differentiate the following expressions with respect to x:

a) $2x^4 \cos^4 x$ **[4]**

b) $\dfrac{1 + x^3}{e^{3x}}$ **[4]**

c) $\ln(x^x)$ **[4]**

---

6. $f(x) = 2 + \ln x$ for $x > 0$ with $x\varepsilon$¡ and $g(x) = 2 + e^{2x}$ with $x\varepsilon$¡ .

a) Find $fg(x)$ and $gf(x)$ simplifying your answers where possible. **[5]**

b) Find $f^{-1}(x)$ and state its range. **[4]**

c) Find $g^{-1}(x)$ and state its domain. **[4]**

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7. $f(x) = \sin 3x$ for $x\varepsilon$¡ and $g(x) = \sin x \cos x$ $0 \le x \le \pi/2$ for $x\varepsilon$¡

a) Show using trigonometric identities that $f(x + \pi/6) = -f(x - \pi/6)$ **[7]**

b) Show that $g(x)$ is an increasing function for $0 < x < \pi/4$ **[4]**

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8. a) Show that $10x^3 = \dfrac{1}{1-x}$ has 2 solutions between 0 and 0.9.

State the range that each solution must lie in. **[5]**

b) Use the iteration $x_{n+1} = \sqrt[3]{\dfrac{1}{10 - 10x_n}}$ and $x_0 = 0.7$ to find x_1, x_2, x_3, and x_4.

Give your answers to four decimal places where appropriate. **[4]**

c) Find $f(0.675)$ where $f(x) = 10x^3 - \dfrac{1}{1-x}$. Give your answer to 3 significant figures **[2]**

d) Hence using your results from b) and c) find a solution to the equation in a) to 2 decimal places and justify your answer. **[3]**

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[75]

EXAMINATION PAPER 3	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed *Where appropriate, give your answers to 3 s.f.*	
	Core Mathematics – C3
Time Allowed:-1 hour 30 minutes	

1. Solve the following equation, leaving your answer exactly:
$e^{10x} - 2e^{5x} - 3 = 0$
[5]

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2. a) Finding A and B; write $2\sin 6x \cos 5x$ in the form $\sin Ax + \sin Bx$ **[3]**
b) Show that: $\dfrac{\cot 2\phi \operatorname{cosec} 2\phi}{\tan^2\phi \sec 2\phi + \sec 2\phi} \equiv (\cos\phi \cot 2\phi)^n$ and find n. **[5]**

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3. $y = 3 - 2e^x$
a) Sketch this curve, stating where the curve crosses the x-axis and y -axis **[4]**
b) Find the equation of the normal to the curve at the point $(1, 3 - 2e)$ **[4]**

---

4. $f(x) = x^6 - x^2 - 1$
$f(x) = 0$ has a solution such that $n < x < n + 1$ where n is a positive integer.
a) Find a positive value of n such that the inequality is true. **[3]**
b) Using an iteration based on the equation $x = \sqrt[6]{1 + x^2}$, find a solution to $f(x) = 0$ to 3 decimal places. **[3]**
c) Calculate $f(-x)$ and hence find a second estimated solution of $f(x) = 0$ **[2]**

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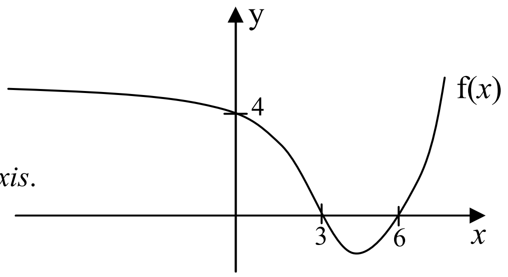
5. $f(x) = \dfrac{x+16}{x-16}$ where x is real and $x \neq 16$ and $g(x) = x^4$
a) Find $fg(x)$ and $gf(x)$ and state their domains. **[6]**
b) Find $f^{-1}(x)$ and state its domain. **[4]**

---

6. Sketch separately the following graphs:
a) $y = |f(x)|$ **[2]**
b) $y = f(|x|)$ **[2]**
c) $y = 2f(3x)$ **[4]**
Write down where each graph crosses the *x and y-axis*.



d) State the relationship between the graphs $y = 2f(3x)$ and $y = -2f(3x)$. **[1]**

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7. Differentiate the following expressions with respect to x:
a) $\sin^3 2x \cos^4 3x$ **[4]**
b) $\dfrac{e^{3x}}{x^5}$ **[4]**
c) Given that $x = \sin 5y$, prove that $\dfrac{dy}{dx} = \dfrac{1}{5\sqrt{1-x^2}}$ **[5]**

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8. a) Express $6\cos x + 8\sin x$ in the form $R\cos(x^\circ - \alpha^\circ)$ where $0 < \alpha < 90^\circ$.
Give α to two decimal places. **[3]**
b) Solve to 2 decimal places the equation $6\cos 2y + 8\sin 2y = 1$ where $0 < y < 360^\circ$. **[6]**

c) For what values of x is $\dfrac{10}{10 + 6\cos x + 8\sin x}$ a minimum, where $0 < x < 360^{\circ}$?
Give your answer to two decimal places. **[3]**

d) What is the value of this minimum? **[2]**

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[75]

EXAMINATION PAPER 4	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, give your answers to 3 s.f.***	
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Time Allowed:-1 hour 30 minutes	

1. a) Simplify the expression: $1+\dfrac{3x+2}{3x^2-x-2}$. **[4]**

   b) $f(x) = x^3 + \dfrac{23}{2}x^2 + 26x - 16$

   Show that $f(x) = 0$ has a solution between 0 and 1. **[3]**

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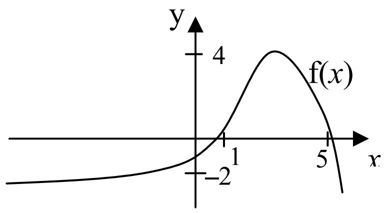
2. $f(x)$ shown, has a maximum value of 4.
   The graph cuts the x-axis at 1 and 5 and cuts the y-axis at – 2.
   Sketch separately the following graphs:



   a) $|\, f(x)\, |$ **[2]**

   b) $f(|\, x\, |)$ **[2]**

   c) $2f(x+1)$ **[3]**

---

3. a) Sketch the curve $y = 3 + 2\ln x$ and state where the curve crosses the x-axis. **[3]**

   b) Find the equation of the tangent to the curve at the point (1, 3) **[4]**

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4. The temperature of an iron ball is cooled by a 1 second blast of chilled nitrogen. The temperature of the iron ball, T°C, is given by the equation $T = 5(20 - e^t),\ for\ 0 < t \le 1$ where t is time in seconds.

   a) Find the value of T at the beginning and end of the air blast giving your answers exactly and if necessary in terms of e, the exponential constant. **[3]**

   b) i) Find $\dfrac{dT}{dt}$ **[1]**

   ii) Hence find when the iron ball is cooling at a rate of 6°C/s giving your answer exactly. **[3]**

   c) i) State the maximum rate of cooling and at what time this occurs. **[2]**

   ii) State the minimum rate of cooling and at what time this occurs. **[2]**

---

5. $f(x) = \dfrac{x^2-49}{x+7}$ where x is real and $x \ne -7$ and $g(x) = x^2 - 2$ where x is real.

   a) Show that $fg(x)$ can be written in the form $(x + A)(x - A)$ and find A. **[4]**

   b) Show that $gf(x)$ can be written in the form $\dfrac{h(x)}{(x+7)^2}$ and find $h(x)$. **[4]**

   The domain of $g(x)$ is now restricted such that $x > 5$.

   c) State the range of $g(x)$. **[1]**

   d) Find $g^{-1}(x)$ and state its domain and range. **[4]**

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6. a) Expand and simplify the expression $\left(\sqrt{11}+\sqrt{10}\right)\left(\sqrt{11}-\sqrt{10}\right)$ **[1]**

   b) Express $\cos x + 3\sin x$ in the form $R\cos(x^o - \alpha^o)$ where $0 < \alpha \le 90^o$ **[4]**

   c) Solve the equation $\cos x + 3\sin x = 1$ where $0 < x \le 360^o$ **[4]**

   d) For what values of x is $\dfrac{1}{\cos x + 3\sin x + \sqrt{11}}$ a minimum, where $0 < x \le 360^o$? **[2]**

   e) Leaving your answer exactly, calculate this minimum value. **[3]**

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7. a) Using the identity for sin (A + B), prove the identity $\sin 3x \equiv 3\sin x - 4\sin^3 x$ **[5]**

   b) Using the fact that $\dfrac{d}{dx}(\sin x) = \cos x$, prove that $\dfrac{d}{dx}(\sin ax) = a\cos ax$ **[5]**

   c) By differentiating both sides of the identity in a) find an expression equivalent to $\cos(3x)$ in terms of $\sin x$ and $\cos x$. **[3]**

d) Without a calculator (or tables) evaluate $\sin 75°$ giving your answer exactly. **[3]**

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[75]

EXAMINATION PAPER 5	Matching the syllabus written by EDEXCEL Curriculum 2004+
Calculators Allowed ***Where appropriate, give your answers to 3 s.f.***	
	Core Mathematics – C3
Time Allowed:-1 hour 30 minutes	

1. a) Simplify the expression: $1-\dfrac{1}{1+\cot^2\phi}$ **[3]**

   b) Show that : $\cos\phi+\sin\phi\tan 2\phi=\dfrac{\cos\phi}{\cos 2\phi}$ **[4]**

---

2. $f(x)=x^3-2x-3$

   The root α to the equation f(x) = 0 can be estimated using the iterative formula $x_{n+1}=\sqrt{\dfrac{3}{x_n}+2}$ with $x_0=2$.

   a) Calculate x_1, x_2, x_3 *and* x_4 giving your answers to 4 significant figures. **[3]**

   b) Prove that, to 4 significant figures, α is 1.893. **[3]**

   John found this iterative formula. He found it by first writing x^3-2x-3 in the form $x(x^2-2)-3$.

   c) Continue the likely algebraic steps that John may have taken to come across this iterative formula. **[3]**

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3. a) Solve the inequality $|2x+3|>4$ **[3]**

   b) i) Sketch a graph of $y=|(x-1)(x-3)|$ **[2]**

   The coordinates on the graph where the gradient is 1 is (a, b) where $1<a<3$.

   ii) Find the value of a. **[4]**

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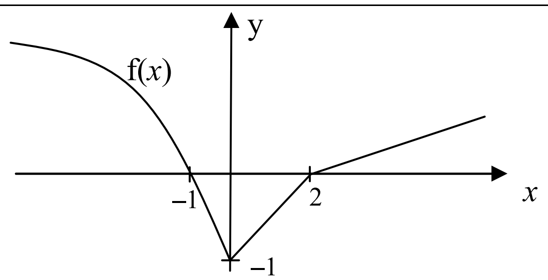
4. Sketch separately the following graphs:

   a) f(| x |) **[2]**

   b) | f(x) | **[2]**

   c) 3f(2x) **[3]**

   In each case write on where each graph crosses or touches the *x and y-axis*.



   d) Given that the curved part of the graph y = f(x) is given by f(x) = k – $3e^{x+2}$, $x\le -1$, find the value of k exactly. **[2]**

   e) Find the gradient of the steepest part of the curved part of the graph. **[3]**

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5. f(x) = x^2 – 1 with $x\varepsilon$¡ and g(x) = 1 – x^2 with $x\varepsilon$¡

   a) Find fg(x) and gf(x) and solve the equation fg(x) = gf(x) **[8]**

   For the inverse of f(x) to exist, it is necessary for the domain of f(x) to be restricted. The domain of the f(x) is now restricted such that $x\ge$ r.

   b) State the largest possible domain of f(x) such that the inverse of f(x) exists. **[2]**

   c) Assuming the domain of f(x) is appropriately restricted, then find the inverse of f(x). **[4]**

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6. f(x) = ln x and g(x) = ln 2x

   a) Find f '(x) and g '(x) **[2]**

   b) Hence find the tangent to the curve y = f(x) when x = 3. **[3]**

   c) Find the normal to the curve y = g(x) when x = 3. **[4]**

---

7. a) Using a trigonometric identity, simplify the expression: sin 2x cos 4x + cos 2x sin 4x **[2]**

   b) Using your answer to part a) and the identity sin 2x cos 4x ≡ $^1/_2$[sin 6x – sin 2x] prove that 2sin 2x cos 4x + cos 2x sin 4x ≡ $^1/_2$[3sin 6x – sin 2x] **[2]**

c) Show that the curve $y = e^{-x}\cos x$ has 2 stationary points between $0 < x < 2\pi$ and with clear working distinguish if these points are maximum or minimum points. **[11]**

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[75]