Edexcel GCE

Core Mathematics C3
Advanced Subsidiary
Set A: Practice Paper 3

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Numb	Leave Blank
1	
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Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has nine questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

Turn over

1.

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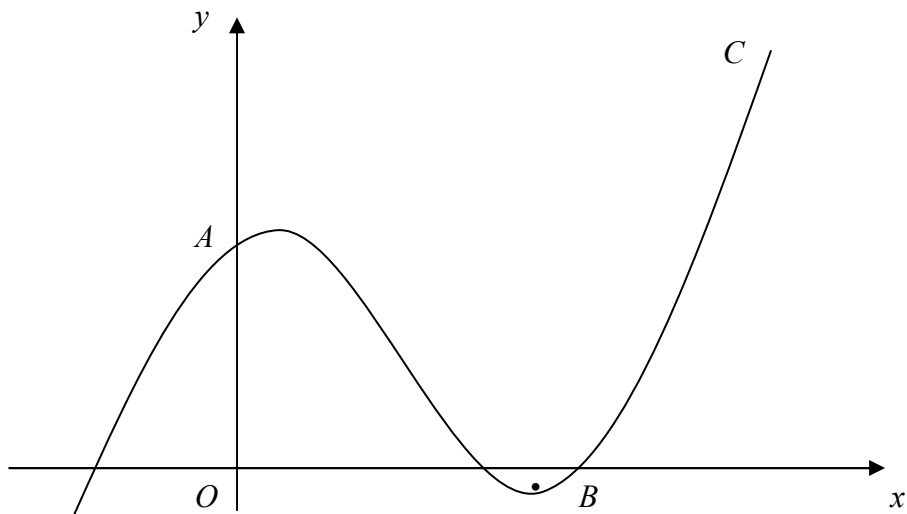**2.**   **Figure 2**

Figure 2 shows part of the curve C with equation $y = f(x)$, where

$$f(x) = 0.5e^x - x^2.$$

The curve C cuts the y-axis at A and there is a minimum at the point B.

(*a*) Find an equation of the tangent to C at A.   **(4)**

The x-coordinate of B is approximately 2.15. A more exact estimate is to be made of this coordinate using iterations $x_{n+1} = \ln g(x_n)$.

(*b*) Show that a possible form for g(x) is g(x) = 4x.   **(3)**

(*c*) Using $x_{n+1} = \ln 4x_n$, with $x_0 = 2.15$, calculate x_1 , x_2 and x_3. Give the value of x_3 to 4 decimal places.   **(2)**

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3. (*a*) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes. **(2)**

(*b*) On the same axes, sketch the graph of $y = \dfrac{1}{x}$. **(1)**

(*c*) Explain how your graphs show that there is only one solution of the equation

$$x|2x + a| - 1 = 0.$$ **(1)**

(*d*) Find, using algebra, the value of x for which $x|2x + 1| - 1 = 0$. **(3)**
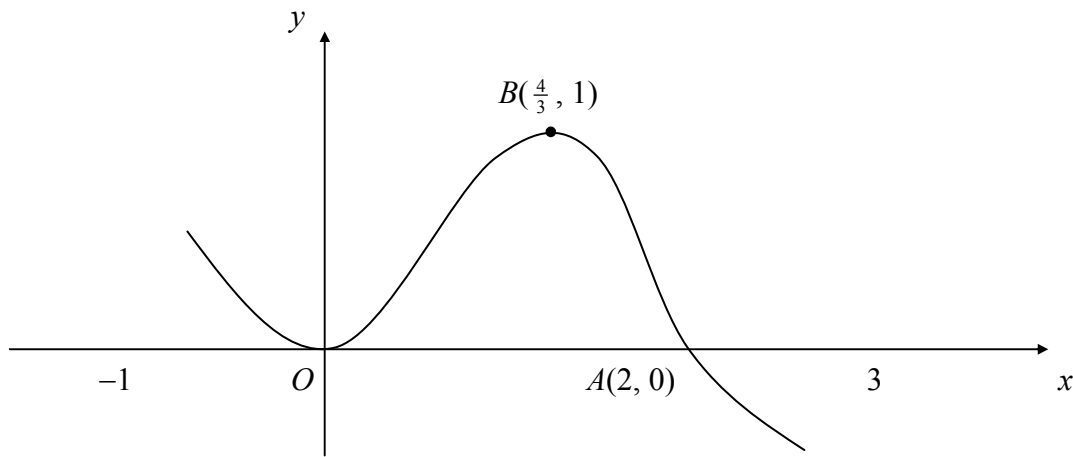
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4.   **Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $-1 \leq x \leq 3$. The curve touches the x-axis at the origin O, crosses the x-axis at the point $A(2, 0)$ and has a maximum at the point $B(\frac{4}{3}, 1)$.

In separate diagrams, show a sketch of the curve with equation

(*a*) $y = f(x + 1)$, **(3)**

(*b*) $y = |f(x)|$, **(3)**

(*c*) $y = f(|x|)$, **(4)**

marking on each sketch the coordinates of points at which the curve

(i) has a turning point,

(ii) meets the x-axis.

3

5. (i) Given that $\sin x = \dfrac{3}{5}$, use an appropriate double angle formula to find the exact value of sec $2x$.

(4)

(ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \qquad \left(x \neq \frac{n\pi}{2}, n \in Z \right).$$

(4)

6. The function f is defined by f: $x \rightarrow \dfrac{3x-1}{x-3}$, $x \in \mathbb{j}$, $x \neq 3$.

(a) Prove that $f^{-1}(x) = f(x)$ for all $x \in \mathbb{j}$, $x \neq 3$. **(3)**

$\mathbb{j}$

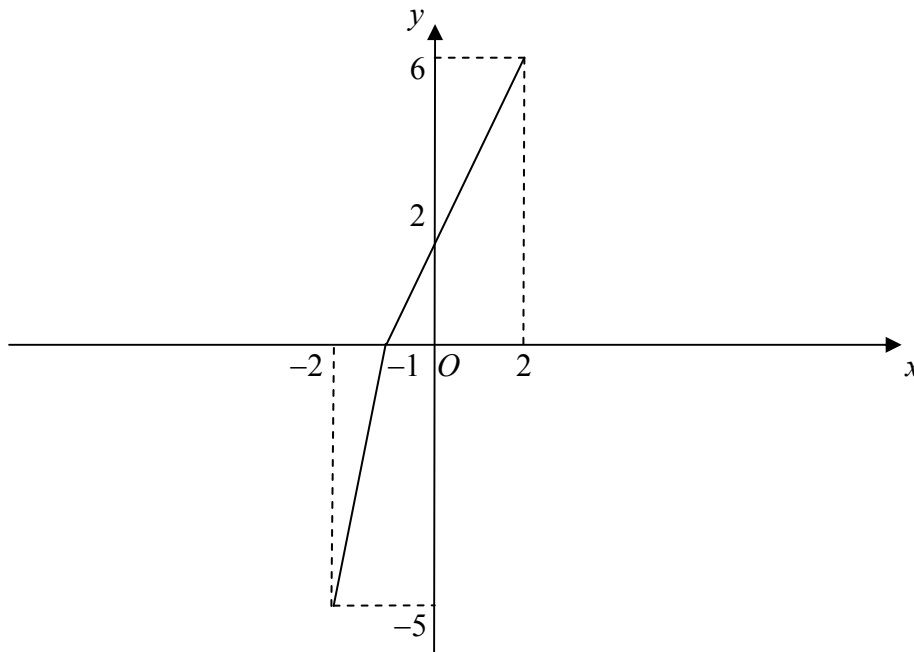(b) Hence find, in terms of k, ff(k), where $x \neq 3$. **(2)**

Figure 3

Figure 3 shows a sketch of the one-one function g, defined over the domain $-2 \le x \le 2$.

(c) Find the value of fg(-2). **(3)**

(d) Sketch the graph of the inverse function g^{-1} and state its domain. **(3)**

The function h is defined by h: $x \mapsto 2g(x-1)$.

(e) Sketch the graph of the function h and state its range. **(3)**

7. (i) (*a*) Express (12 cos θ − 5 sin θ) in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(*b*) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 4,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place. **(3)**

(ii) Solve

$$8 \cot \theta - 3 \tan \theta = 2,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place. **(5)**

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8. The curve C has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, \quad x > 0.$$

The point P is a stationary point on C.

(*a*) Calculate the x-coordinate of P. **(4)**

(*b*) Show that the y-coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found. **(2)**

The point Q on C has x-coordinate 1.

(*c*) Find an equation for the normal to C at Q. **(4)**

The normal to C at Q meets C again at the point R.

(*d*) Show that the x-coordinate of R

(i) satisfies the equation $6 \ln x + x + \dfrac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14. **(4)**

END