

Paper Reference(s)

## 6663

Edexcel GCE
Core Mathematics C3


Team Leader's use only


Time: 1 hour 30 minutes

## Materials required for examination Mathematical Formulae

Items included with question papers Nil

| Question <br> Number | Leave <br> Blank |
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| 1 |  |
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## Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has nine questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

1. The function f is given by

$$
\mathrm{f}: x \alpha \frac{x}{x^{2}-1}-\frac{1}{x+1}, x>1
$$

(a) Show that $\mathrm{f}(x)=\frac{1}{(x-1)(x+1)}$.
(b) Find the range of f .
(2)

The function g is given by

$$
\begin{equation*}
\mathrm{g}: x \alpha \frac{2}{x}, x>0 \tag{4}
\end{equation*}
$$

(c) Solve $\operatorname{gf}(x)=70$.
2. Express $\frac{y+3}{(y+1)(y+2)}-\frac{y+1}{(y+2)(y+3)}$ as a single fraction in its simplest form.
3. The function f is even and has domain $\mathbb{R}$. For $x \geq 0, \mathrm{f}(x)=x^{2}-4 a x$, where $a$ is a positive constant.
(a) In the space below, sketch the curve with equation $y=\mathrm{f}(x)$, showing the coordinates of all the points at which the curve meets the axes.
(b) Find, in terms of $a$, the value of $\mathrm{f}(2 a)$ and the value of $\mathrm{f}(-2 a)$.

Given that $a=3$,
(c) use algebra to find the values of $x$ for which $\mathrm{f}(x)=45$.
4.

$$
\mathrm{f}(x)=x^{3}+x^{2}-4 x-1
$$

The equation $\mathrm{f}(x)=0$ has only one positive root, $\alpha$.
(a) Show that $\mathrm{f}(x)=0$ can be rearranged as

$$
x=\sqrt{\left(\frac{4 x+1}{x+1}\right)}, x \neq-1
$$

The iterative formula $x_{n+1}=\sqrt{\left(\frac{4 x_{n}+1}{x_{n}+1}\right)}$ is used to find an approximation to $\alpha$.
(b) Taking $x_{1}=1$, find, to 2 decimal places, the values of $x_{2}, x_{3}$ and $x_{4}$.
(c) By choosing values of $x$ in a suitable interval, prove that $\alpha=1.70$, correct to 2 decimal places.
(d) Write down a value of $x_{1}$ for which the iteration formula $x_{n+1}=\sqrt{\left(\frac{4 x_{n}+1}{x_{n}+1}\right)}$ does not produce a valid value for $x_{2}$.

Justify your answer.
5. The functions $f$ and $g$ are defined by

$$
\begin{aligned}
& \text { f: } x \alpha|x-a|+a, x \in \mathbb{R}, \\
& \text { g: } x \alpha \quad 4 x+a, \quad x \in \mathbb{R} .
\end{aligned}
$$

where $a$ is a positive constant.
(a) On the same diagram, sketch the graphs of f and g , showing clearly the coordinates of any points at which your graphs meet the axes.
(b) Use algebra to find, in terms of $a$, the coordinates of the point at which the graphs of $f$ and $g$ intersect.
(c) Find an expression for $\mathrm{fg}(x)$.
(d) Solve, for $x$ in terms of $a$, the equation

$$
\begin{equation*}
\operatorname{fg}(x)=3 a \tag{3}
\end{equation*}
$$

6. 

Figure 1


Figure 1 shows a sketch of the curve with equation $y=\mathrm{f}(x)$, where

$$
\mathrm{f}(x)=10+\ln (3 x)-\frac{1}{2} \mathrm{e}^{x}, \quad 0.1 \leq x \leq 3.3 .
$$

Given that $\mathrm{f}(k)=0$,
(a) show, by calculation, that $3.1<k<3.2$.
(b) Find $\mathrm{f}^{\prime}(x)$.

The tangent to the graph at $x=1$ intersects the $y$-axis at the point $P$.
(c) (i) Find an equation of this tangent.
(ii) Find the exact $y$-coordinate of $P$, giving your answer in the form $a+\ln b$.
7. (a) Express $\sin x+\sqrt{ } 3 \cos x$ in the form $R \sin (x+\alpha)$, where $R>0$ and $0<\alpha<90^{\circ}$.
(b) Show that the equation $\sec x+\sqrt{3} \operatorname{cosec} x=4$ can be written in the form

$$
\begin{equation*}
\sin x+\sqrt{3} \cos x=2 \sin 2 x \tag{3}
\end{equation*}
$$

(c) Deduce from parts $(a)$ and (b) that $\sec x+\sqrt{ } 3 \operatorname{cosec} x=4$ can be written in the form

$$
\begin{equation*}
\sin 2 x-\sin \left(x+60^{\circ}\right)=0 \tag{1}
\end{equation*}
$$

## END

