

Worked Solutions

Edexcel C3 Paper L

$$1. \quad \frac{x}{(x-3)(x+3)} - \frac{1}{(x-3)(x-1)}$$

$$\frac{x(x-1) - (x+3)}{(x-3)(x+3)(x-1)}$$

$$\frac{x^2 - 2x - 3}{(x-3)(x+3)(x-1)}$$

$$\frac{(x-3)(x+1)}{(x-3)(x+3)(x-1)} = \frac{x+1}{(x+3)(x-1)} \quad (6)$$

$$2. \quad (a) f(0) = e^3 \quad ff(0) = e^{2e^3+3} \quad (2)$$

$$(b) \text{ let } y = e^{2x+3}$$

$$\ln y = 2x + 3$$

$$\frac{1}{2}(\ln y - 3) = x \quad \therefore f^{-1}(x) = \frac{1}{2}(\ln x - 3) \quad (3)$$

$$(c) \text{ domain for } f^{-1}(x) \text{ is } x \in \mathbb{R}, x > 0 \quad (1)$$

$$3. \quad \frac{dx}{dy} = \frac{2y}{y^2+4} \Rightarrow \frac{dy}{dx} = \frac{y^2+4}{2y} = \frac{y^2}{2y} + \frac{4}{2y} \quad (6)$$

$$4. \quad (a) \left. \begin{array}{l} f(1) = \ln 1 - 3 + 5 = 2 \\ f(2) = \ln 2 - 6 + 5 = -0.306 \end{array} \right\} \text{change of sign } \therefore \text{root in interval} \quad (2)$$

$$(b) x_1 = 1.8977, x_2 = 1.8802, x_3 = 1.8771, x_4 = 1.8766 \quad (3)$$

$$(c) \left. \begin{array}{l} f(1.8765) = -9.666 \times 10^{-5} \\ f(1.8755) = 2.375 \times 10^{-3} \end{array} \right\} \text{sign change} \quad (2)$$

$$\therefore \alpha = 1.876 \text{ (4 sig. fig.)}$$

$$5. \quad (a) \frac{dy}{dx} = \sec^2 x + 2 \cos 2x$$

$$\text{at } x = \frac{\pi}{4},$$

$$\frac{dy}{dx} = (\sqrt{2})^2 + 0 = 2 \quad (4)$$

$$(b) \text{ at } x = \frac{\pi}{4},$$

$$y = 1 + 1 = 2$$

$$\text{equation of tangent is } y - 2 = 2 \left(x - \frac{\pi}{4} \right)$$

$$y = 2x + 2 - \frac{\pi}{2} \quad (3)$$

$$6. \quad (a) \text{ R.H.S} = \frac{2 \tan \theta}{\sec^2 \theta}$$

$$= 2 \cdot \frac{\sin \theta}{\cos \theta} \cdot \cos^2 \theta$$

$$= 2 \sin \theta \cos \theta$$

$$= \sin 2\theta \quad (4)$$

$$(b) 4 \tan \theta - \tan^2 \theta = 1$$

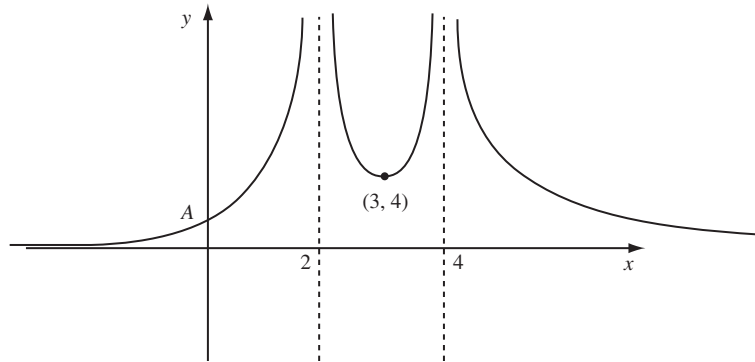
$$4 \tan \theta = 1 + \tan^2 \theta$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12} \quad (5)$$

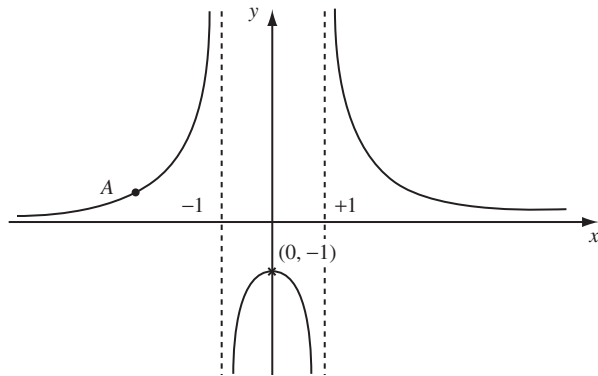
7. (a)



min. pt. at (3, 4)

pt. A $\left(0, \frac{1}{2}\right)$

(b)



asymptotes at $x = \pm 1$.

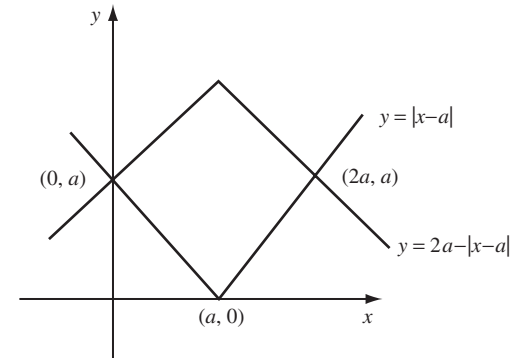
maximum at (0, -1)

A(-3, 0.125)

(5)

(4)

8. (a)



(5)

(b) points of intersection: (0, a) and (2a, a)

(2)

(c) rhombus area = $\frac{1}{2}$ product of diagonals = $\frac{1}{2}(2a)(2a) = 2a^2$

(3)

9. (a) $\cos \theta + 2 \sin \theta = \sqrt{5} \left(\frac{1}{\sqrt{5}} \cos \theta + \frac{2}{\sqrt{5}} \sin \theta \right)$;

$$R = \sqrt{5}, \quad \tan \alpha = 2$$

$$R = 2.24 \text{ (3 s.f.) } \quad \alpha = 1.11^\circ \text{ (3 s.f.)}$$

(4)

(b)

$$\left. \begin{array}{l} \text{max. } \sqrt{5} \\ \text{min. } -\sqrt{5} \end{array} \right\} \theta = 1.11$$

(2)

(c) max. depth $15 + \sqrt{5}$ (17.24 metres 2 d.p.)

$$\text{occurs at } \frac{\pi t}{12} = 1.11 \quad t = 4.24 \text{ (2 d.p.)}$$

(4)

(d) $t = 0$. $d = 15 + 1 = 16$ metres

(1)

(e) $d = 15 \Rightarrow \cos \left(\frac{\pi t}{12} - 1.11 \right) = 0$

$$\frac{\pi t}{12} - 1.11 = \frac{\pi}{2} \quad t = 10.24$$

time 2230 (nearest half hour)

(4)