

Core Mathematics C3 Advanced Level

For Edexcel

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (a) Express in the form $Ax + B$, where A and B are to be found,

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2}. \quad (4)$$

- (b) Hence solve the equation

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2} = x^2 - 9 \quad (2)$$

2. Given that $x = \sin^{-1}\left(-\frac{3}{5}\right)$, find the value of

(a) $\cos x$ (4)

(b) $\cos 2x$ (2)

3. (a) Sketch the graph of $y = e^{ax+b}$, given a and $b > 0$.

Mark the coordinates of the point where the graph meets either the x -axis or the y -axis. (2)

(b) Given that when $x = 0$, $y = 4$, find the exact value of b . (1)

(c) Given further that the gradient at the point whose x coordinate is 2 is equal to $10e^5$, find the value of a . (5)

4. Differentiate the following functions with respect to x .

(a) $\cos^2 x$ (3)

(b) $\frac{\ln x}{x}$ (3)

(c) $x^2 e^x$ (4)

5. (a) Prove that

$$\sin x + \cot x \cos x = \operatorname{cosec} x. \quad (3)$$

- (b) Hence or otherwise find the values of x , $0 < x < 180^\circ$, which satisfy the equation

$$\cot x \cos x = 3,$$

giving your answers to 1 decimal place. (4)

6. Sketch the graphs of $y = \sin x$ and $y = x^3 - 1$, for $0 < x < 2\pi$, indicating clearly the coordinates of the points where the curves meet the axes. (4)

(a) Using your graph, explain why there is only one solution to the equation

$$\sin x - x^3 + 1 = 0. \quad (1)$$

(b) If α is such that

$$\sin \alpha - \alpha^3 + 1 = 0,$$

prove that $0 < \alpha < \frac{\pi}{2}$. (2)

(c) Use the iteration

$$x_{n+1} = \sqrt[3]{\sin x_n + 1} \quad \text{and} \quad x_0 = 1$$

to obtain the values of x_1, x_2, x_3, x_4 and hence find the value of α to 4 decimal places. (3)

7. Given $a > 0$, sketch the graphs of

$$y = ax \quad \text{and} \quad y = |6a^2 - x^2|$$

on the same pair of axes. Indicate clearly the points of intersection of the graphs with the coordinate axes. (4)

(a) Show that the point $(2a, 2a^2)$ lies on both the line and the curve. (1)

(b) Write down an equation that will give the x coordinate of the second point of intersection of the line and the curve. (2)

(c) Solve the equation to find the coordinates of the second point of intersection. (4)

8. (a) Express $f(x) = \sqrt{3} \sin x + \cos x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. The values of R and α are to be given exactly. (4)

(b) Hence solve the equation

$$\sqrt{3} \sin x + \cos x = \sqrt{2}, \quad \text{where} \quad 0 < x < \pi. \quad (6)$$

(c) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. (4)

(d) You are given that $y = 2f(x) + 1$. State the maximum and minimum values of y . (3)

END

TOTAL 75 MARKS