

Worked Solutions

Edexcel C3 Paper C

$$1. \quad \frac{1}{x+4} - \frac{2(x-1)}{(3x+2)(x+4)}$$

$$= \frac{x+4}{(x+4)(3x+2)} - \frac{1}{3x+2}$$

$$\frac{1}{3x+2} = \frac{1}{x-5} \quad (4)$$

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$$x-5 = 3x+2 \quad x = -3\frac{1}{2}. \quad (3)$$

$$2. \quad (a) \quad 4x+1 = e^2$$

$$x = \frac{1}{4}(e^2 - 1) \quad (3)$$

$$(b) \quad 3e^{2x} - 7e^x + 2 = 0$$

$$(3e^x - 1)(e^x - 2) = 0$$

$$e^x = \frac{1}{3}; \quad e^x = 2$$

$$x = -\ln 3, \quad x = \ln 2 \quad (5)$$

$$3. \quad (a) \quad f(x) \geq 0, \quad g(x) < 8 \quad (4)$$

$$(b) \quad (x+4)^2 = 8-x$$

$$x^2 + 8x + 16 = 8-x$$

$$x^2 + 9x + 8 = 0$$

$$(x+8)(x+1) = 0$$

$$x = -8, -1, \text{ not in domain of } g. \quad (4)$$

$$4. \quad (a) \quad f'(x) = x - \frac{4}{x-3} \quad (3)$$

$$(b) \quad f'(x) < 0 \Rightarrow x < \frac{4}{x-3}$$

$$x(x-3) < 4 \quad x > 3$$

$$x^2 - 3x - 4 < 0$$

$$(x-4)(x+1) < 0$$

$$-1 < x < 4$$

$$\therefore 3 < x < 4$$

$$\left. \begin{array}{l} x^2 - 3x - 4 > 0 \text{ for } x < 3 \\ x < -1 \text{ or } x > 4 \end{array} \right\} x < -1. \quad (5)$$

$$5. \quad \tan 75^\circ = \frac{\cos(75^\circ - 15^\circ) - \cos(75^\circ + 15^\circ)}{\sin(75^\circ + 15^\circ) - \sin(75^\circ - 15^\circ)}$$

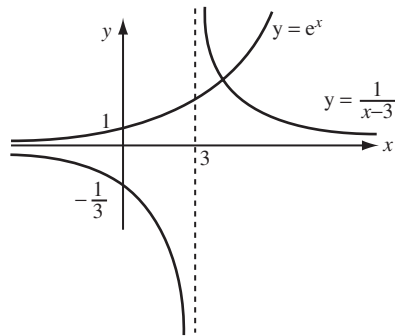
$$= \frac{\cos 60^\circ - \cos 90^\circ}{\sin 90^\circ - \sin 60^\circ}$$

$$= \frac{\frac{1}{2} - 0}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= 2 + \sqrt{3} \quad (4)$$

6. (a)



(b) curves only cross once.

(c) $x_1 = 3.04978$, $x_2 = 3.04736$, $x_3 = 3.04748$, ...
 $x = 3.047$ (3 d.p.)

7. (a) $x^2 \cdot -3e^{-3x} + e^{-3x} \cdot 2x$
 $x e^{-3x} (2 - 3x)$

(b) $2 \sec x (\sec x \tan x) = 2 \sec^2 x \tan x$

(c) $\frac{5 \sin x - 5x \cos x}{\sin^2 x}$

(d) $\frac{dx}{dy} = -\ln y \cdot \sin y + \cos y \cdot \frac{1}{y}$

$\frac{dy}{dx} = \frac{y}{\cos y - y \ln y \cdot \sin y}$

8. (a) $\frac{dy}{dx} = 12 \cos 2x - 8 \sin 2x$

$\frac{d^2y}{dx^2} = -24 \sin 2x - 16 \cos 2x$
 $= -4(6 \sin 2x + 4 \cos 2x)$
 $= -4y.$

(b) $R^2 = 6^2 + 4^2 :$

$R = \sqrt{52} = 7.211$ (3 d.p.)

$\tan \alpha = \frac{2}{3}$

$\alpha = 0.588^\circ$

(c) pt. of inflection

$\Rightarrow \frac{d^2y}{dx^2} = 0 \Rightarrow y = 0$

$\Rightarrow \sqrt{52} \sin(2x + 0.588) = 0$

$2x + 0.588 = \pi$

$x = 1.277$ (3 d.p.)