Worked Solutions

Edexcel C3 Paper A

1. $fg(x) = g^{-1}g(x) = x.$ **(3)**

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2. (a)

(2)

(b)

(3)

(c)

(3)

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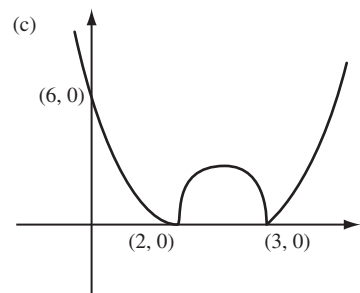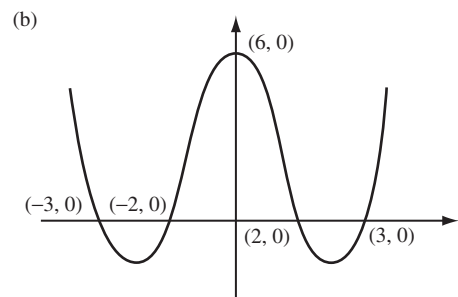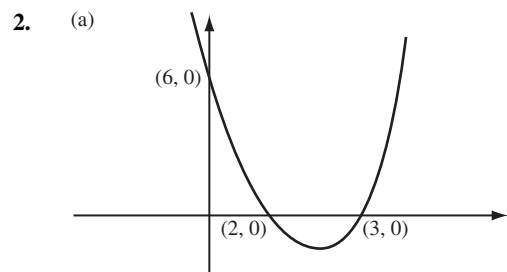
3. $e^{2x} - 7e^x + 12 = 0$

$(e^x - 3)(e^x - 4) = 0$

$e^x = 3 \Rightarrow x = \ln 3$

$e^x = 4 \Rightarrow x = \ln 4$ **(6)**

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4. $\sin A = \dfrac{3}{5} \Rightarrow \cos A = -\dfrac{4}{5}$

$\sin B = \dfrac{5}{13} \Rightarrow \cos B = -\dfrac{12}{13}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$= \frac{3}{5} \cdot \frac{-12}{13} - \frac{-4}{5} \cdot \frac{5}{13} = -\frac{16}{65}$$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{-\frac{3}{4} - \frac{5}{12}}{1 - \left(-\frac{3}{4}\right)\left(-\frac{5}{12}\right)}$$

$$= \frac{-\frac{14}{12}}{\frac{11}{16}} = \frac{-56}{33}$$ **(7)**

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5. $\dfrac{dy}{dx} = xe^x + e^x$

at turning point, $(x + 1)e^x = 0 \Rightarrow x = -1$

$y = -e^{-1}$ pt. $\left(-1, \dfrac{-1}{e}\right)$

$\dfrac{d^2y}{dx^2} = e^x + xe^x + e^x$

$x = -1$ $\dfrac{d^2y}{dx^2} = e^{-1} > 0$ $\therefore$ minimum **(8)**

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6. (a)

$$\left.\begin{array}{l} f(1) = 1 - 6 + 7 = 2 \\ f(2) = 4 - 12 + 7 = -1 \end{array}\right\} \text{change of sign}$$

(2)

(b) $7 = 6x - x^2$

$7 = x(6 - x)$

$x = \dfrac{7}{6 - x}$

(2)

(c) $x_1 = 1.75,\ x_2 = 1.6470,\ x_3 = 1.6081,\ x_4 = 1.5938,\ x_5 = 1.5886,\ x_6 = 1.5868$

Ans. $x = 1.59$ (3 s.f.)

(3)

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7. $\dfrac{dx}{dy} = -3 \sin 3y,$

$\dfrac{dy}{dx} = -\dfrac{1}{3 \sin 3y}$

$y = \dfrac{\pi}{6},\ \dfrac{dy}{dx} = -\dfrac{1}{3}$

$y = \dfrac{\pi}{6},\ x = 0.$

$y - \dfrac{\pi}{6} = -\dfrac{1}{3}(x - 0)$

$y - \dfrac{\pi}{6} = -\dfrac{1}{3}x$

$6y + 2x - \pi = 0$

(8)

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8. (a) $\dfrac{3}{(x + 2)(x + 3)} - \dfrac{2}{x + 3} = \dfrac{1}{x + 2}$

$\dfrac{3 - 2(x + 2)}{(x + 2)(x + 3)} = \dfrac{x + 3}{(x + 2)(x + 3)}$

$3 - 2x - 4 = x + 3$

$-4 = 3x$

$x = -\dfrac{4}{3}$

(5)

(b) $\dfrac{(2x - 3)(2x + 3)}{(x + 1)\left(x^2 - x + 1\right)} \times \dfrac{x + 1}{(2x + 3)(x - 5)}$

$\dfrac{2x - 3}{\left(x^2 - x + 1\right)(x - 5)}$

(4)

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9. (a) $f(1) = 1 - 1 - 3 + 3 = 0.$

$(x - 1)\left(x^2 - 3\right) = 0 \qquad x = \pm\sqrt{3}$

(4)

(b) $\tan^3 \theta - 3 \tan \theta + 4 = 1 + \tan^2 \theta$

$\tan^3 \theta - \tan^2 \theta - 3 \tan \theta + 3 = 0$

same as (i) with $x = \tan \theta$

(2)
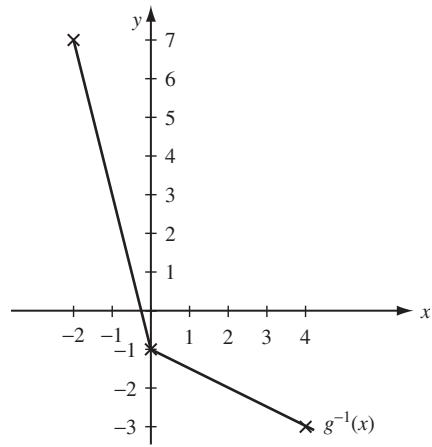
(c) $\tan \theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$\tan \theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$

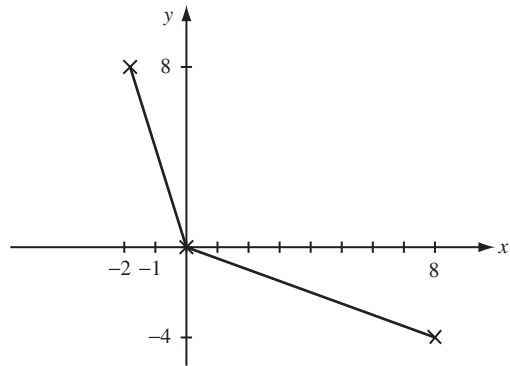$\tan \theta = -\sqrt{3} \Rightarrow \theta = \dfrac{2\pi}{3}$

(4)

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10. (*a*)

(**3**)

(*b*)

(**3**)

(*c*) g(-3) = 4

   hg (-3) = h(4)

   = -2

(**3**)