

Worked Solutions

Edexcel C3 Paper I

1. (a) $y = \frac{1-2x}{2-x} \Rightarrow$

$$2y - xy = 1 - 2x$$

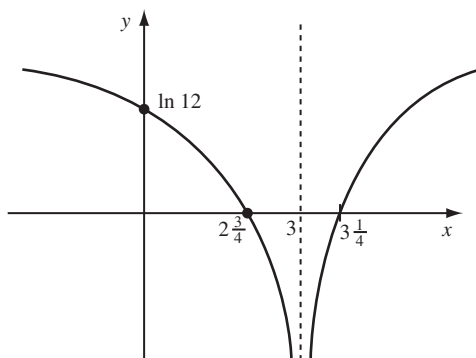
$$x(2-y) = 1 - 2y$$

$$x = \frac{1-2y}{2-y} \quad (3)$$

So $f'(x) = \frac{1-2x}{2-x}$

(b) $ff(k^2) = ff^{-1}(k^2)$
 $= k^2 \quad (2)$

2. (a)



(b) $(0, \ln 12); (2\frac{3}{4}, 0); (3\frac{1}{4}, 0) \quad (3)$

3. (a) $\frac{x(x-2)(x+3) - (x+3) + 5}{(x-2)(x+3)}$
 $\frac{x^3 + x^2 - 7x + 2}{(x-2)(x+3)}$
 $\frac{(x-2)(x^2 + 3x - 1)}{(x-2)(x+3)} \quad (5)$

(b) $f'(x) = \frac{(x+3)(2x+3) - (x^2 + 3x - 1)}{(x+3)^2}$
 $= \frac{x^2 + 6x + 10}{x^2 + 6x + 9}$
 $= 1 + \frac{1}{(x+3)^2}$

$$\frac{1}{(x+3)^2} = \frac{1}{25} \Rightarrow x+3 = \pm 5$$

$$x = 2 \text{ or } -8$$

but $x > 2$, so there are no solutions. (5)

4. (a) $\frac{dy}{dx} = \frac{x \cdot 5e^{5x} - e^{5x}}{x^2} = 0$
 $\frac{dy}{dx} = 0, \quad \frac{(5x-1)e^{5x}}{x^2} = 0 \Rightarrow x = \frac{1}{5} \quad (4)$

(b) (i) $\frac{dy}{dx} = 2 \cdot \sin 3y \cdot \cos 3y \cdot 3 \quad (3)$

(ii) $y = \frac{\pi}{12}, 3y = \frac{\pi}{4} : \frac{dy}{dx} = \frac{1}{2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 3} = \frac{1}{3} \quad (3)$

5. (a) $f'(x) = \frac{1}{2x} - \frac{2}{x^3}, \quad x > 0$

$\frac{1}{2x} = \frac{2}{x^3} \Rightarrow x^2 = 4, \quad x = 2$

(b) $f(x) = \frac{1}{2} \ln 2 + \frac{1}{4}$

$= 2^{-1} \ln 2 + 2^{-2} \quad k = 2$

(c) $f(x) = \frac{1}{2} \ln 1 + \frac{1}{1^2} = 1 \quad Q(1, 1)$

$x = 1 \quad f'(x) = \frac{1}{2} - 2 = -1\frac{1}{2}$

gradient of normal is $\frac{2}{3}$

$y - 1 = \frac{2}{3}(x - 1)$

$3y - 3 = 2x - 2$

$3y - 2x - 1 = 0$

6. (a) $R^2 = 2.5^2 + 6^2$

$R = 6.5$

$\tan \alpha = \frac{6}{2.5}$

$\alpha = 1.176^\circ$

(b) $5 \sin x \cos x - 12 \sin^2 x$

$= 2.5(\sin 2x) + 6(-2 \sin^2 x + 1) - 6$

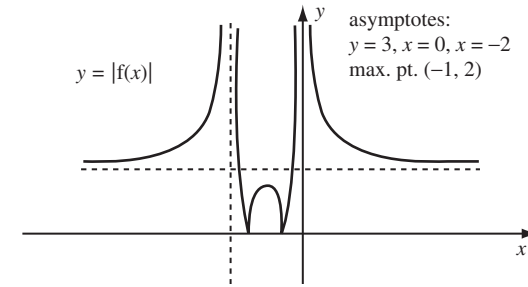
$= 2.5 \sin 2x + 6 \cos 2x - 6$

(c) $= 6.5 \sin(2x + 1.176) - 6$

max. value $6.5 - 6 = 0.5$

7. (a)

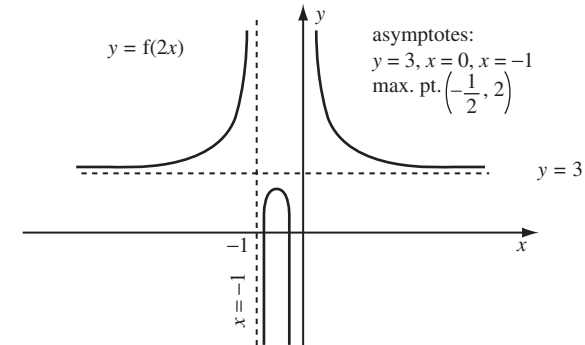
$y = |f(x)|$
 asymptotes:
 $y = 3, x = 0, x = -2$
 max. pt. $(-1, 2)$



(4)

(b)

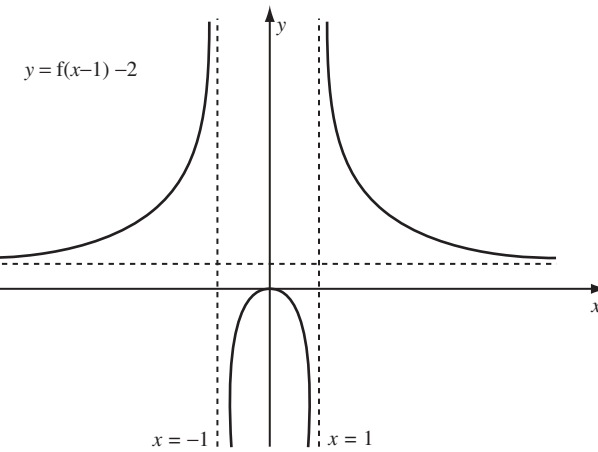
$y = f(2x)$
 asymptotes:
 $y = 3, x = 0, x = -1$
 max. pt. $(-\frac{1}{2}, 2)$



(2)

(3)

(c)



(4)

(3)

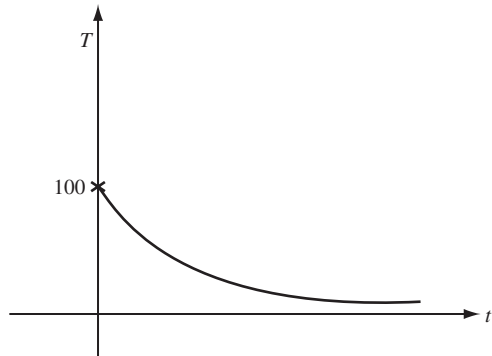
asymptotes $y = 1 \quad x = 1 \quad x = -1 \quad$ max. pt. $(0, 0)$

(d) $x = 0$

(2)

(2)

8. (a)



(3)

(b) $T = 15 + 85e^{-\frac{1}{2}} = 15 + 85 \times 0.606 = 66.6^\circ$ (1 d.p.)

(2)

(c) $40 = 15 + 85e^{-\frac{t}{8}} \quad \therefore e^{-\frac{t}{8}} = \frac{25}{85}$

$$e^{\frac{t}{8}} = 3.4$$

$$\therefore t = 8 \ln 3.4$$

$$t = 9.79 \text{ mins.}$$

$$= 9 \text{ mins. } 47 \text{ secs.}$$

(3)

(d) $\frac{dT}{dt} = -\frac{85}{8} \cdot e^{-\frac{t}{8}} ; -\frac{85}{8} \cdot e^{-\frac{t}{8}} = -1.7$

$$t = 8 \ln 6.25 = 14.7 \text{ mins. (1 d.p.)}$$

(4)

$$T = 15 + 85 \times 0.16$$

$$T = 28.6$$

(e) $15^\circ C$

(1)