Core Mathematics C3 For Edexcel
Advanced Level

Paper K
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. Express as a single fraction

$$\frac{2x^2 + x - 6}{4x^2 - 9} \times \frac{x - 1}{(x + 2)^2}. \qquad \textbf{(5)}$$

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2. The functions f and g are defined by

$$\text{f}: x \mapsto x + 2, \quad x \in \mathbb{R}$$

$$\text{g}: x \mapsto \frac{2}{x - 3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

(*a*) Express the functions gf and fg in the form : '$x \mapsto$'. In each case give the domain of the combined function. **(4)**

(*b*) Show that there are no real values of x which satisfy the equation

$$\text{fg}(x) = \text{gf}(x). \qquad \textbf{(3)}$$

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3. Differentiate with respect to x,

(*a*) $\dfrac{x}{1 - x^2}$, **(3)**

(*b*) $x^2 \ln x$, **(3)**

(*c*) $e^{\sin x}$. **(3)**

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4. (*a*) Prove that

$$\operatorname{cosec}\theta - \sin\theta \geq 0 \quad \text{for all } \theta \text{ in the range} \quad 0 < \theta < \pi. \qquad \textbf{(4)}$$

(*b*) Find the values of x, $0 < x \leq 360$, which satisfy the equation

$$\sec^2 x - 4\tan x + 2 = 0. \qquad \textbf{(5)}$$

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5. The curve

$$y = \frac{1}{4}x^4 + x^3 - 3x$$

has three turning points.

(*a*) Show that one of these turning points has an x coordinate of α where α lies in the interval [0, 1]. **(4)**

(*b*) Show that the equation

$$x^3 + 3x^2 - 3 = 0$$

can be written as

$$x = \sqrt{\frac{3}{x+3}}$$ **(2)**

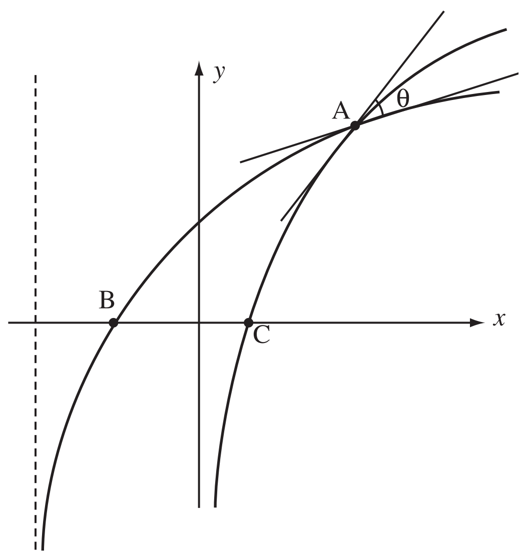
(*c*) Using the iteration

$$x_{n+1} = \sqrt{\frac{3}{x_n + 3}}, \quad \text{with } x_0 = 1,$$

find the values of x_1, x_2, x_3 and x_4. **(2)**

(*d*) Hence give the value of α to 3 decimal places. **(1)**

6.



The figure shows the curves $y = \ln 3x$ and $y = \ln(x + 2)$ intersecting at A, and crossing the x-axis at B and C.

(*a*) Write down the equations of the asymptotes to each curve. **(2)**

(*b*) Write down the coordinates of the points B and C. **(2)**

(*c*) Find the coordinates of the point A. **(1)**

The angle between the tangents to both curves at the point A is θ.

(*d*) Prove that $\theta = \arctan\left(\frac{1}{2}\right)$. **(6)**

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7. (*a*) Letting $A + B = P$, and $A - B = Q$ and using the expansions for $\sin(A \pm B)$, prove that

$$\sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right).$$ **(5)**

(*b*) Hence or otherwise solve the equation,

$\sin 4\theta - \sin 2\theta + \cos 3\theta = 0, \quad$ for $\quad 0 < \theta < 360°$. **(6)**

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8. You are given that

$$f(x) = x^2 - 3x, \quad x \in \mathbb{R}, \quad x \geq 1\frac{1}{2}.$$

(*a*) Find the range of f. **(1)**

(*b*) Write down the domain and range of f^{-1}. **(2)**

(*c*) Sketch the graph of f^{-1}, indicating clearly the coordinates of any point at which the graph intersects the coordinate axes. **(4)**

Given that $g(x) = |x - 4|, \quad x \in \mathbb{R}$,

(*d*) find an expression for $gf(x)$. **(2)**

(*e*) Solve $gf(x) = 6$. **(5)**

END **TOTAL 75 MARKS**