GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper C
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. *(a)* Express

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1}$$

as a single fraction in its simplest form. **(3)**

(b) Hence, find the values of x such that

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1} = \frac{1}{2}.$$ **(3)**

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2. *(a)* Prove, by counter-example, that the statement

"cosec $\theta - \sin\theta > 0$ for all values of θ in the interval $0 < \theta < \pi$"

is false. **(2)**

(b) Find the values of θ in the interval $0 < \theta < \pi$ such that

$$\text{cosec}\,\theta - \sin\theta = 2,$$

giving your answers to 2 decimal places. **(5)**

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3. Solve each equation, giving your answers in exact form.

(a) $\ln(2x - 3) = 1$ **(3)**

(b) $3e^y + 5e^{-y} = 16$ **(5)**

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4. Differentiate each of the following with respect to x and simplify your answers.

(a) $\ln(3x - 2)$ **(2)**

(b) $\frac{2x+1}{1-x}$ **(3)**

(c) $x^{\frac{3}{2}} e^{2x}$ **(3)**

5.

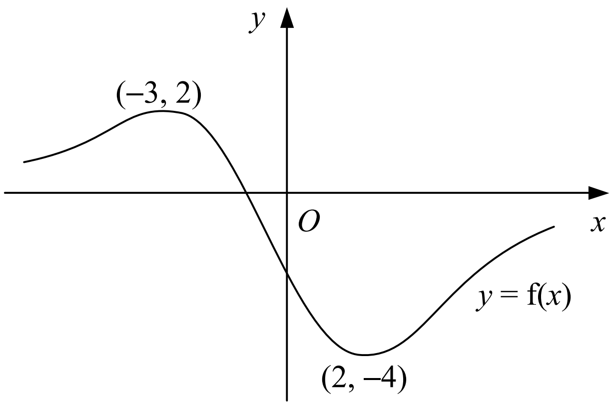

Figure 1

Figure 1 shows the curve $y = \mathrm{f}(x)$ which has a maximum point at (–3, 2) and a minimum point at (2, –4).

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}(|x|)$,

(ii) $y = 3\mathrm{f}(2x)$. **(7)**

(b) Write down the values of the constants a and b such that the curve with equation $y = a + \mathrm{f}(x + b)$ has a minimum point at the origin O. **(2)**

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6. The function f is defined by

$$\mathrm{f}(x) \equiv 4 - \ln 3x, \quad x \in \mathbb{R}, \quad x > 0.$$

(a) Solve the equation $\mathrm{f}(x) = 0$. **(2)**

(b) Sketch the curve $y = \mathrm{f}(x)$. **(2)**

(c) Find an expression for the inverse function, $\mathrm{f}^{-1}(x)$. **(3)**

The function g is defined by

$$\mathrm{g}(x) \equiv \mathrm{e}^{2-x}, \quad x \in \mathbb{R}.$$

(d) Show that

$$\mathrm{fg}(x) = x + a - \ln b,$$

where a and b are integers to be found. **(3)**

Turn over

7. *(a)* Express $4 \sin x + 3 \cos x$ in the form $R \sin (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) State the minimum value of $4 \sin x + 3 \cos x$ and the smallest positive value of x for which this minimum value occurs. **(3)**

(c) Solve the equation

$$4 \sin 2\theta + 3 \cos 2\theta = 2,$$

for θ in the interval $0 \leq \theta \leq \pi$, giving your answers to 2 decimal places. **(6)**

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8. The curve C has the equation $y = \sqrt{x} + e^{1 - 4x}$, $x \geq 0$.

(a) Find an equation for the normal to the curve at the point $(\frac{1}{4}, \frac{3}{2})$. **(4)**

The curve C has a stationary point with x-coordinate α where $0.5 < \alpha < 1$.

(b) Show that α is a solution of the equation

$$x = \tfrac{1}{4}[1 + \ln (8\sqrt{x})].$$ **(3)**

(c) Use the iteration formula

$$x_{n+1} = \tfrac{1}{4}[1 + \ln (8\sqrt{x_n})],$$

with $x_0 = 1$ to find x_1, x_2, x_3 and x_4, giving the value of x_4 to 3 decimal places. **(3)**

(d) Show that your value for x_4 is the value of α correct to 3 decimal places. **(2)**

(e) Another attempt to find α is made using the iteration formula

$$x_{n+1} = \tfrac{1}{64} e^{8x_n - 2},$$

with $x_0 = 1$. Describe the outcome of this attempt. **(2)**

END