Practice Paper B

CORE THREE

crashMATHS

Name	

Duration	1 HOUR & 30 MINUTES
Total Marks Available	75 MARKS

Targets

For examiner's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
/	
/	
Total marks	

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1 Find all the values of x that satisfy the equation

$$|x+1| = x(x+1)$$

(3)

Question 1 continued

TOTAL 3 MARKS

2 (a) Solve the equation

$$4\cos 2x = 1 - 2\sin x$$

for $0 < x \leq 360^{\circ}$. **(5)**

(b) (i) Express

$$2\sin\theta + 7\cos\theta$$

in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is acute. **(3)**

(ii) Find the values of θ that satisify

$$2\sin\left(\frac{\theta}{2}\right) + 7\cos\left(\frac{\theta}{2}\right) = 6$$ **(4)**

(iii) Find the greatest possible value of k such that

$$4\sin 2\theta + 14\cos 2\theta = k$$

has real solutions. You should justify your answer. **(2)**

Question 2 continued

Question 2 continued

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Question 2 continued

TOTAL 14 MARKS

3 A curve C is defined such that

$$f(x)=e^{2x}-3e^{x}+2$$

Given that the curve C intersects the x axis at the points $(a,0)$ and $(b,0)$, where $a<b$,

(a) Find the values of a and b. **(3)**

(b) Find the coordinates of any stationary points on the curve C. **(3)**

(c) Determine the nature of any stationary points on the curve C. Justify your answer. **(3)**

(d) Hence, sketch the curve C.

On your sketch, you should clearly the coordinates of any points where the curve meets or crosses the coordinate axis and the equations of any asymptotes to C. **(3)**

Question 3 continued

Question 3 continued

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Question 3 continued

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Question 3 continued

Question 3 continued

TOTAL 12 MARKS

4 (a) Show, by sketching, that the equation

$$8-2x^2=\ln x \qquad (*)$$

has exactly one real root. **(3)**

(b) Using your sketches, explain why the root of the equation $(*)$ lies in the interval $[1,2]$. **(1)**

(c) The root of $(*)$ can be approximated using the following iterative formula

$$x_{n+1}=\sqrt{4-\frac{1}{2}\ln x_n}$$

Starting with $x_0=1$, use this iterative formula a suitable number of times to find the value of the root correct to 3 decimal places. **(5)**

Question 4 continued

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Question 4 continued

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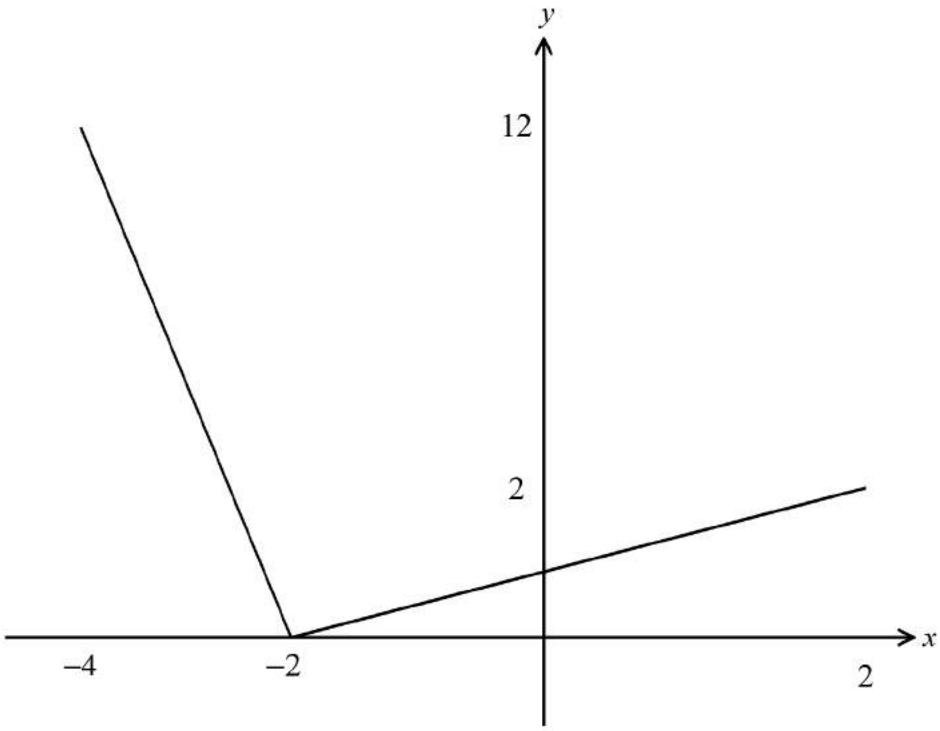
Question 4 continued

TOTAL 9 MARKS

5 A function f has domain $-4 \le x \le 2$ and is linear from $(-4,12)$ to $(-2,0)$ and from $(-2,0)$ to $(2,2)$. A sketch of the function $y = \mathrm{f}(x)$ is shown in the diagram below.



(a) State the range of the function f. **(1)**

(b) Find $\mathrm{ff}(-2)$. **(2)**

Another function g is defined such that

$$\mathrm{g} : x \rightarrow \frac{2+2x}{1-x}$$

(c) Find the largest domain and range of the function g. **(2)**

(d) Find $\mathrm{g}^{-1}(x)$. **(3)**

(e) Write down the domain and range of g^{-1}. **(1)**

(f) Solve the equation $\mathrm{gf}(x) = 6$. **(5)**

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Question 5 continued

Question 5 continued

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Question 5 continued

TOTAL 14 MARKS

6 (a) Prove that

(i) $\dfrac{\sin 2x}{\sin x + \cos x - 1} \equiv \sin x + \cos x + 1$ **(4)**

(ii) $\dfrac{\sin x - \cos x - 1}{\sin x + \cos x - 1} \equiv -\dfrac{1 + \cos x}{\sin x}$ **(5)**

(b) Hence, or otherwise, solve

$$\frac{\sin 2x}{\sin x + \cos x - 1} + \frac{\sin^2 x - \sin x \cos x - \sin x}{\sin x + \cos x - 1} = \frac{5}{12}$$

for $-\pi < x < \pi$. **(5)**

Question 6 continued

Question 6 continued

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Question 6 continued

TOTAL 14 MARKS

7 It is given without proof that

$$\frac{d^2y}{dx^2} = -\frac{d^2x}{dy^2} \times \left[\frac{dy}{dx}\right]^{-3}$$

You may use this result at any point in the question if necessary.

Given that

$$x = \cot y + \operatorname{cosec}^2 y$$

(a) Show that

$$\frac{dx}{dy} = -\operatorname{cosec}^2 y - 2\operatorname{cosec}^2 y \cot y$$ **(3)**

(b) Find $\frac{d^2x}{dy^2}$. **(3)**

(c) Hence, or otherwise, show further that

$$\frac{d^2y}{dx^2} = 2\left(\gamma^2+1\right)\left(3\gamma^2+\gamma+1\right)\left(2\gamma^3+\gamma^2+2\gamma+1\right)^3$$

where $\gamma = \cot y$. **(3)**

Question 7 continued

Question 7 continued

Question 7 continued

TOTAL 9 MARKS