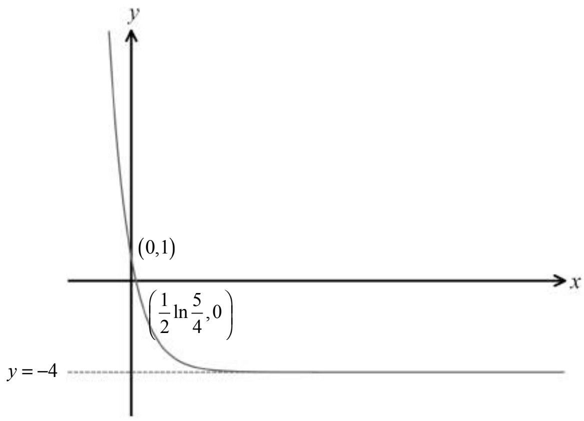
mark scheme

Practice Paper A : Core Mathematics 3

Question Number	General Scheme		Marks
1			
(a)	(Let $f(x)=x^4+x^2-8x-9$) $f(2.1)=2.1^4+2.1^2-8(2.1)-9=-1.94(19),\ (<0)$ $f(2.2)=2.2^4+2.2^2-8(2.2)-9=1.66(56),\ (>0)$	**M1** – substitutes 2.1 and 2.2 into the given equation.	M1
	Since there has been a **<u>change of sign</u>** (and $f(x)$ is continuous over the interval $[2.1,2.2]$), $x^4+x^2-8x-9=0$ has a root α in the interval $[2.1,2.2]$.	**A1** – correct substitution **and** a given conclusion	A1 **(2)**
(b)	$x^4=9+8x-x^2 \rightarrow x=\sqrt[4]{9+8x-x^2}$	**B1** – cso **AG**	B1 **(1)**
Note	This is a show that question. Candidates must thus not simply quote the final answer but show the intermediate stage above (accept terms in any order). For example, if a candidate writes $x^4+x^2-8x-9=0 \rightarrow x=\sqrt[4]{9+8x-x^2}$ This **alone** will score B0.		
(c)	$x_1=\sqrt[4]{9+8x_0-(x_0)^2}=\sqrt[4]{9+8(2.1)-(2.1)^2}$ $=2.150(56...)$	**M1** – attempts to use the iterative formula to work out x_1 (can be implied)	M1
	$x_2=2.155$ $x_3=2.156$	**A1** – x_1, x_2 or x_3 correct **A1** – x_1, x_2 and x_3 correct **to three decimal places**	A1 A1 **(3)**
(d)	$\alpha=2.2$	**B1** – cao	B1 **(1)**
		Total	**7**

2 (a)	By long division: $$\begin{array}{r} x^2+5x-5 \\ x+1\overline{)x^3+6x^2+0x-8} \\ \underbrace{x^3+x^2}_{\text{M1}} \quad \downarrow \quad \downarrow \\ 5x^2+0x \quad \downarrow \\ \underline{5x^2+5x} \quad \downarrow \\ -5x-8 \\ \underline{-5x-5} \\ -3 \end{array}$$ M1 A1	**M1** – correct attempt at long division **M1** – the *method* of long division carried out correctly **A1** – correct division	**M1** **M1** **A1**
	$\therefore \frac{x^3+6x^2+8}{x+1} \equiv x^2+5x-5-\frac{3}{x+1}$ $a=1$, $b=5$, $c=-5$, $d=-3$	**A1** – expressed in the correct form	**A1** **(4)**
(b)	$\frac{d}{dx}\left[x^2+5x-5-\frac{3}{x+1}\right] = 2x+5+...$	**B1 –** correct differentiation of *their* ax^2+bx+c	**B1**
	$\frac{d}{dx}\left[-\frac{3}{x+1}\right] = -3(-1)(x+1)^{-2} = \frac{3}{(x+1)^2}$	**M1** – correct attempt at the chain rule **A1ft** – correct differentiation	**M1** **A1**
Note	Some candidates may differentiate $(-)\frac{3}{x+1}$ by the quotient rule. If so, award **M1** for the site of $\frac{(-)3(1)}{(x+1)^2}$ and **A1ft** for correct differentiation based on *their* (b).		
	$\therefore f'(x) = 2x+5+\frac{3}{(x+1)^2}$	**A1** – cao	**A1** **(4)**
		Total	**8**

3 (a)	$\frac{5}{e^{2x}} = 4$ $e^{2x} = \frac{5}{4} \rightarrow 2x = \ln\frac{5}{4} \rightarrow x = \frac{1}{2}\ln\frac{5}{4}$ **OE**	**M1** – attempts to rearrange the equation into the form $e^{2x} = \frac{a}{b}$ **M1** – correct application of logarithms **A1** – correct value of x **oe**	**M1** **M1** **A1** **(3)**
(b)		**B1** – correct shape and in the correct quadrants **B1ft** – correct x and y intercept, ft *their* (a) for x intercept **B1** – equation of the asymptotes correctly marked on and stated	**B1** **B1** **B1** **(3)**
(c)	$x \le \frac{1}{2}\ln\frac{5}{4}$	**B1ft** – $x \le \frac{1}{2}\ln\frac{5}{4}$ ft *their* (a)	**B1** **(1)**
	Total		**7**

4 (a)	$A = \pi rl + \pi r^2$ $l = \sqrt{h^2 + r^2} = \sqrt{16^2 + x^2} = \sqrt{256 + x^2}$	**M1** – attempts to use $\pi rl + \underbrace{\pi r^2}_{\text{necessary}}$ **M1** – correct attempt to use Pythagoras to find l	**M1** **M1**
	$\therefore A = \pi x\sqrt{256 + x^2} + \pi x^2$	**A1** – cso **AG**	**A1** **(3)**
(b)	$\frac{dA}{dx} = \pi\left[\underbrace{x'\sqrt{256 + x^2} + x\left(\sqrt{256 + x^2}\right)'}_{\text{M1}}\right] + 2\pi x$	**M1** – use of the product rule to differentiate $\pi x\sqrt{256 + x^2}$ term	**M1**
	$\frac{dA}{dx} = \pi\left[\sqrt{256 + x^2} + x\left(\frac{1}{2}\right)(2x)\left(256 + x^2\right)^{-\frac{1}{2}}\right] + 2\pi x$	**M1** – use of the chain rule to differentiate $\sqrt{256 + x^2}$	**M1**
	$\frac{dA}{dx} = \pi\left[\sqrt{256 + x^2} + \frac{x^2}{\sqrt{256 + x^2}} + 2x\right]$	**A1** – correct differentiation (need not be simplified) **oe**	**A1**
	$\left.\frac{dA}{dx}\right\|_{x=12} = \pi\left[\sqrt{256 + 12^2} + \frac{12^2}{\sqrt{256 + 12^2}} + 2(12)\right]$ $\left.\frac{dA}{dx}\right\|_{x=12} = \pi\left[20 + \frac{36}{5} + 24\right] = \frac{256}{5}\pi$ **OE**	**dM1** – substitutes 12 into *their* $\frac{dA}{dx}$ **A1** – achieves the correct result, accept equivalent forms if given. If the candidate achieves this and simplifies **incorrectly,** award A0.	**M1** **A1** **(5)**
		Total	**8**

5 (a)	$4\cos\theta - 7\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $R\cos\alpha = 4$, $R\sin\alpha = 7$	**M1** – correct expression seen (or implied)	**M1**
(i)	$\tan\alpha = \frac{7}{4} \rightarrow \alpha = 1.0516... = 1.05 \ (3\text{sf})$	**M1** – correct expression for $\tan\alpha$ **A1** – $\tan\alpha$ evaluated correctly to 3 significant figures	**M1** **A1**
(ii)	$R = \sqrt{4^2 + 7^2} = \sqrt{65}$	**B1** – correct **exact** value for R. If decimal value given after, apply isw and still award **B1**	**B1** **(4)**
(b)	$2\sqrt{65}\cos(2\theta - 1.0516...) = 15\sin(2\theta - 1.0516...)$	**B1ft** – replaces $8\cos 2\theta - 14\sin 2\theta$ with twice *their* (a)	**B1**
	$\therefore \tan(2\theta - 1.0516...) = \frac{2\sqrt{65}}{15}$ $2\theta - 1.0516... = \tan^{-1}\left(\frac{2\sqrt{65}}{15}\right) = 0.8215...$, (…)	**M1** – forms equation in terms of $\tan(2\theta - 1.0516...) = ...$ **A1** – determines principal value correctly	**M1** **A1**
	<u>Other values given by:</u> $\pi + 0.8215...$, $2\pi + 0.8215...$, $3\pi + 0.8215...$	**M1** – correct attempt seen to calculate **all** other values in range	**M1**
	$\therefore 2\theta - 1.0516 = 0.8215, \ 3.9630, \ 7.1046, 10.2462$ $\therefore \theta = 0.94, \ 2.51, \ 4.08, \ 5.65$ **AWRT**	**M1** – adds α and then divides by 2 to **at least one** of their values to get θ **A1** – all values of θ given	**M1** **A1** **(6)**
(c)	$p(\theta) = 10 - 65\cos^2(\theta + 1.0516...)$	**B1ft** – seen anywhere (can be implied) ft on *their* (a)	**B1**
(i) (ii)	maximum when $\cos^2(\theta + \alpha) = 0 \Rightarrow$ maximum = 10 $\theta + 1.0516... = \frac{\pi}{2} \Rightarrow \theta = 0.52$ **AWRT**	**B1** – cao **M1** – sets $\theta + \alpha = \frac{\pi}{2}$ **A1** – 0.52	**B1** **M1** **A1** **(4)**
	Total		**14**

6	Decay will be of the form $M = 120e^{-kt}$	**B2** – decay in the correct form (need not be stated directly). (**B1** for Ae^{-kt} or e^{-kt})	**B2**
	$72.9 = 120e^{-4k}$ $e^{-4k} = \frac{72.9}{120} \Rightarrow k = -\frac{1}{4}\ln\left(\frac{72.9}{120}\right)$	**M1** – substitutes $M = 72.9$, $t = 4$ into *their* equation **A1** – correct value of k	**M1** **A1**
	$\therefore M = 120e^{\frac{1}{4}\ln\left(\frac{72.9}{120}\right)\times 10} = 34.518.. = 34.5\text{g}$ **AWRT**	**A1** – cao	**A1**
		Total	**5**

7 (i) (a)	$\frac{d}{dx}\left[e^{3x^2}\sin x\right]=e^{3x^2}\cos x+\frac{d}{dx}\left[3x^2\right]e^{3x^2}\sin x$	**M1** – differentiation in the form of the product rule **M1** – attempt to use the chain rule to differentiate e^{3x^2} **A1** – correct unsimplified differentiation	**M1** **M1** **A1**
	$\frac{d}{dx}\left[e^{3x^2}\sin x\right]=e^{3x^2}\cos x+6e^{3x^2}x\sin x$ **OE**	**A1** – correctly simplified. Accept further simplifications, but if incorrect ignore and apply isw	**A1** **(4)**
(b)	$\frac{d}{dx}\left[\cos\left(\ln\left(2x^2\right)\right)\right]=-\sin\left(\ln\left(2x^2\right)\right)\times\frac{d}{dx}\left[\ln\left(2x^2\right)\right]$	**M1** – attempts to use the chain rule **A1** – correct use of the chain rule	**M1** **A1**
	$=-\sin\left(\ln\left(2x^2\right)\right)\times\frac{4x}{2x^2}=-\frac{2}{x}\sin\left(\ln\left(2x^2\right)\right)$ **OE**	**M1** – correct method to differentiate the $\ln\left(2x^2\right)$ term **A1** – correctly simplified. Accept further simplifications, but if incorrect ignore and apply isw	**M1** **A1** **(4)**
(c)	$\frac{dy}{dx}=3(\sec x)^2\sec x\tan x=3\sec^3 x\tan x$	**M1** – application of the chain rule to find $\frac{dy}{dx}$	**M1**
	$\frac{d^2y}{dx^2}=3\sec^3 x\left(\sec^2 x\right)+\tan x\left(9\sec^3 x\tan x\right)$ $=3\sec^5 x+9\sec^3 x\tan^2 x$	**M1** – attempts to find $\frac{d^2y}{dx^2}$ using the product rule **A1** – correct differentiation	**M1** **A1**
	$=3\sec^5 x+9\sec^3 x\left(\sec^2 x-1\right)$ $=12\sec^5 x-9\sec^3 x$ $=12y^{\frac{3}{5}}-9y$	**M1** – use of $\tan^2 x=\sec^2 x-1$ **A1** – correct differentiation, all terms in the form $\sec^n x$ **A1** – cso	**M1** **A1** **(6)**
	Total		**14**

8 (a)	$\cos\left(x+\frac{\pi}{3}\right)=\sin\left(x+\frac{\pi}{4}\right)$ $\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}=\sin x\sin\frac{\pi}{4}+\cos x\cos\frac{\pi}{4}$ $\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\frac{\sqrt{2}}{2}\sin x+\frac{\sqrt{2}}{2}\cos x$ $\sqrt{2}\sin x+\sqrt{3}\sin x=\cos x-\sqrt{2}\cos x$	**M2** – applies of compound angle formula (**M1** for each side) **A1** – correct use (unsimplified) **A1** – correctly use (simplified)	**M2** **A1** **A1**
	$\tan x=\frac{1-\sqrt{2}}{\sqrt{2}+\sqrt{3}}$ $x=\tan^{-1}\left(\frac{1-\sqrt{2}}{\sqrt{2}+\sqrt{3}}\right)=-\frac{\pi}{24}$	**M1** – rearranges for $\tan x=...$ **A1** – obtains principal value of $\tan x$	**M1** **A1**
	$\therefore x=\frac{23}{24}\pi,\ \frac{25}{24}\pi$	**M1** – method to find values that are in range **A1** – correct values, ignore values outside of range, but award **A0** for additional values in range	**M1** **A1** **(8)**
(b)	If $\sin k=x \Rightarrow \tan k=\frac{x}{\sqrt{1-x^2}}$, $0<k<\frac{\pi}{2}$	**B1** – sight of $\sin k=x$ **M1** – correct method to find $\tan k$, i.e. use of triangle **A1** – correct value of $\tan k$ for $0<k<\frac{\pi}{2}$	**B1** **M1** **A1**
	Then, $\tan k=-\frac{x}{\sqrt{1-x^2}}$ for $-\frac{\pi}{2}<k<0$.	**A1ft** – cao	**A1** **(4)**
	Total		**12**
ALT	A quick alternative to (a) may be: $\cos\left(x+\frac{\pi}{3}\right)=\sin\left(x+\frac{\pi}{4}\right)$ **B1** Using $\cos x=\sin\left(\frac{\pi}{2}-x\right)$ (accept, of course, use of the reverse): $\sin\left(\frac{\pi}{2}-x-\frac{\pi}{3}\right)=\sin\left(x+\frac{\pi}{4}\right)$ **M1 M1 A1** $\frac{\pi}{6}-x=x+\frac{\pi}{4} \rightarrow 2x=-\frac{\pi}{12}$ **M1 A1**		

	Then reward such candidates a mark for attempting to find the other solutions and a mark for the final answers. Note in this scheme the third **A1** becomes **B1** here.	

Notes on alternative methods:

This mark scheme may feature some alternative solutions, but, of course, at this level, there is likely to be questions that have many others. Where alternative methods are used, you should award full marks **if the method is correct** (do **not** award full marks for methods that coincidentally lead to the right answer). If the method is *not* correct, then you should aim to mark it by being as faithful to the original scheme as you can and ensure that you award the same amount of marks for the same amount of *progress* in a question as you would award using the general scheme.