

1. (a) Using the substitution $u = 2^x$, show that the equation $4^x - 2^{(x+1)} - 15 = 0$ can be written in the form $u^2 - 2u - 15 = 0$. (2)
- (b) Hence solve the equation $4^x - 2^{(x+1)} - 15 = 0$, giving your answers to 2 decimal places. (4)
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2. **Figure 1**

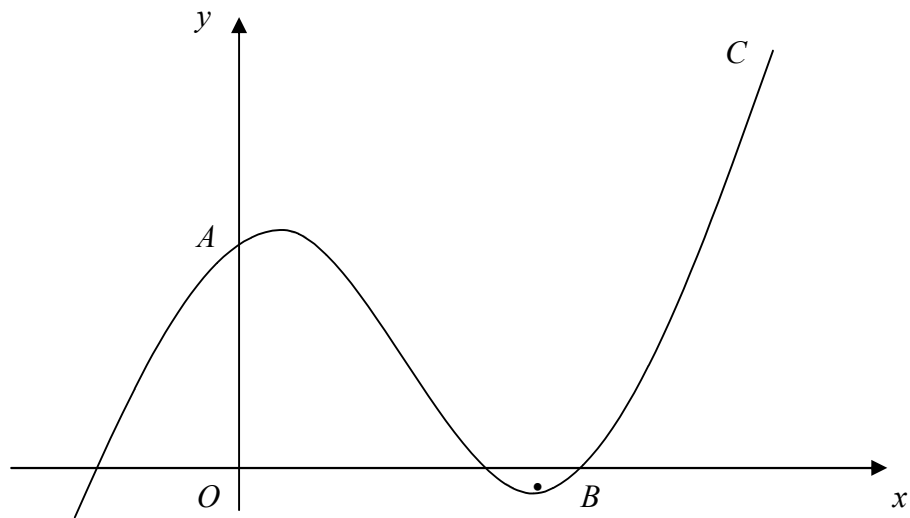


Figure 2 shows part of the curve C with equation $y = f(x)$, where

$$f(x) = 0.5e^x - x^2.$$

The curve C cuts the y -axis at A and there is a minimum at the point B .

- (a) Find an equation of the tangent to C at A . (4)

The x -coordinate of B is approximately 2.15. A more exact estimate is to be made of this coordinate using iterations $x_{n+1} = \ln g(x_n)$.

- (b) Show that a possible form for $g(x)$ is $g(x) = 4x$. (3)

- (c) Using $x_{n+1} = \ln 4x_n$, with $x_0 = 2.15$, calculate x_1 , x_2 and x_3 . Give the value of x_3 to 4 decimal places. (2)
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3. (a) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes. **(2)**
- (b) On the same axes, sketch the graph of $y = \frac{1}{x}$. **(1)**
- (c) Explain how your graphs show that there is only one solution of the equation

$$x|2x + a| - 1 = 0. \quad \mathbf{(1)}$$

- (d) Find, using algebra, the value of x for which $x|2x + 1| - 1 = 0$. **(3)**
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4. **Figure 2**

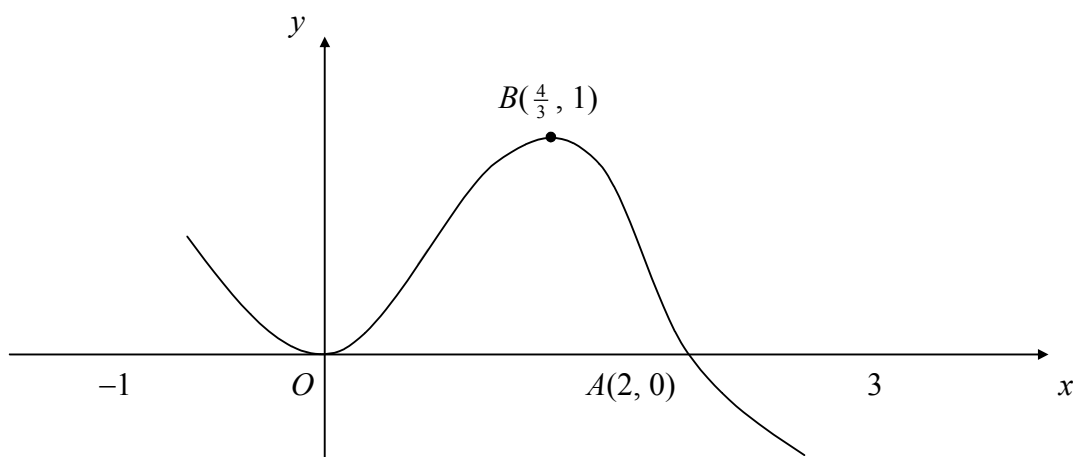


Figure 1 shows a sketch of the curve with equation $y = f(x)$, $-1 \leq x \leq 3$. The curve touches the x -axis at the origin O , crosses the x -axis at the point $A(2, 0)$ and has a maximum at the point $B(\frac{4}{3}, 1)$.

In separate diagrams, show a sketch of the curve with equation

- (a) $y = f(x + 1)$, **(3)**
- (b) $y = |f(x)|$, **(3)**
- (c) $y = f(|x|)$, **(4)**

marking on each sketch the coordinates of points at which the curve

- (i) has a turning point,
- (ii) meets the x -axis.
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5. (i) Given that $\sin x = \frac{3}{5}$, use an appropriate double angle formula to find the exact value of $\sec 2x$.

(4)

(ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \quad \left(x \neq \frac{n\pi}{2}, n \in \mathbb{Z} \right).$$

(4)

6. The function f is defined by $f: x \mapsto \frac{3x-1}{x-3}, x \in \mathbb{R}, x \neq 3$.

(a) Prove that $f^{-1}(x) = f(x)$ for all $x \in \mathbb{R}, x \neq 3$. (3)

(b) Hence find, in terms of k , $ff(k)$, where $x \neq 3$. (2)

Figure 3

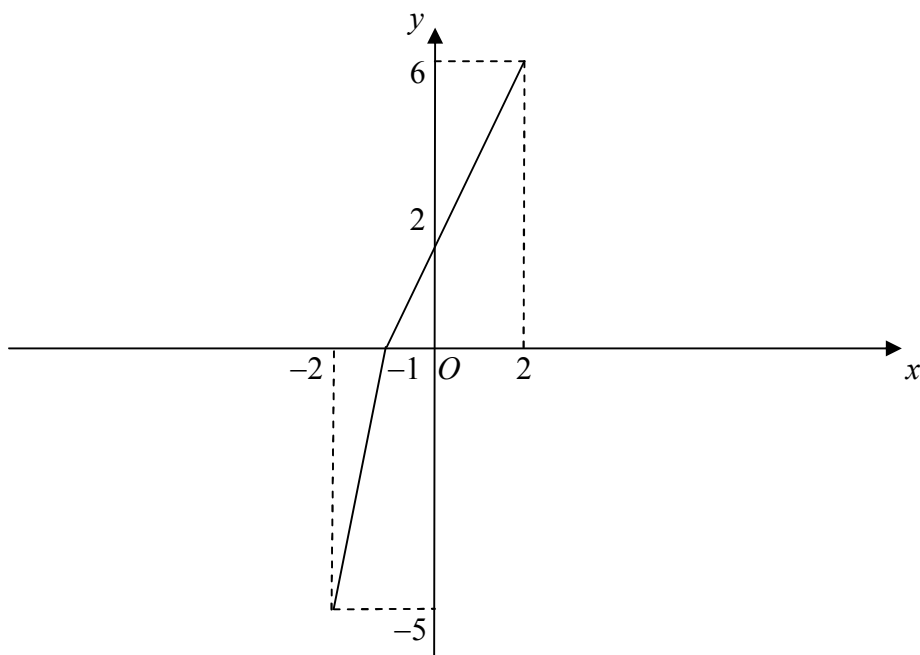


Figure 3 shows a sketch of the one-one function g , defined over the domain $-2 \leq x \leq 2$.

(c) Find the value of $fg(-2)$. (3)

(d) Sketch the graph of the inverse function g^{-1} and state its domain. (3)

The function h is defined by $h: x \mapsto 2g(x-1)$.

(e) Sketch the graph of the function h and state its range. (3)

7. (i) (a) Express $(12 \cos \theta - 5 \sin \theta)$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. (4)

(b) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 4,$$

for $0 < \theta < 90^\circ$, giving your answer to 1 decimal place. (3)

(ii) Solve

$$8 \cot \theta - 3 \tan \theta = 2,$$

for $0 < \theta < 90^\circ$, giving your answer to 1 decimal place. (5)

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8. The curve C has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, \quad x > 0.$$

The point P is a stationary point on C .

(a) Calculate the x -coordinate of P . (4)

(b) Show that the y -coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found. (2)

The point Q on C has x -coordinate 1.

(c) Find an equation for the normal to C at Q . (4)

The normal to C at Q meets C again at the point R .

(d) Show that the x -coordinate of R

(i) satisfies the equation $6 \ln x + x + \frac{2}{x} - 3 = 0$,

(ii) lies between 0.13 and 0.14. (4)

END