Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Bronze Level B3

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
73	67	61	53	46	40

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

January 2013

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2. $$\text{f}(x) = 4 \operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(a) Show that there is a root α of $\text{f}(x) = 0$ in the interval [1.2, 1.3].

(2)

(b) Show that the equation $\text{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By considering the change of sign of $\text{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

June 2010

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3. The functions f and g are defined by

$$\mathrm{f}:\ x \mapsto 2|x| + 3, \qquad x \in \mathbb{R}$$

$$\mathrm{g}:\ x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(*a*) State the range of f.

(2)

(*b*) Find fg(1).

(2)

(*c*) Find g^{-1}, the inverse function of g.

(2)

(*d*) Solve the equation

$$\mathrm{gg}(x) + [\mathrm{g}(x)]^2 = 0$$

(5)

June 2013 (R)

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4. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$.

(6)

Given that $x = \cot y$,

(ii) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{1+x^2}$.

(5)

January 2013

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5. The functions f and g are defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^x + 2, \qquad x \in \mathbb{R},$$

$$\mathrm{g}: x \mapsto \ln x, \qquad x > 0.$$

(*a*) State the range of f.

(1)

(*b*) Find fg(x), giving your answer in its simplest form.

(2)

(*c*) Find the exact value of x for which $\mathrm{f}(2x + 3) = 6$.

(4)

(*d*) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(*e*) On the same axes sketch the curves with equation $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

June 2012

6.
$$\mathrm{f}(x) = 3x^3 - 2x - 6.$$

(*a*) Show that $\mathrm{f}(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$.

(2)

(*b*) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(*c*) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

June 2008

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7. (*a*) Express $2 \cos 3x - 3 \sin 3x$ in the form $R \cos (3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x} \cos 3x.$$

(*b*) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x} \cos (3x + \alpha),$$

where R and α are the constants found in part (*a*).

(5)

(*c*) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

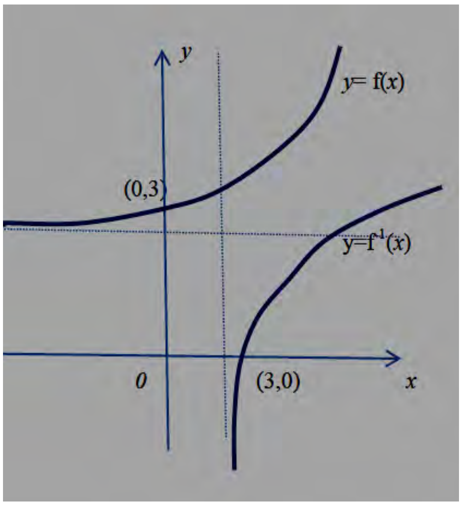
June 2011

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		[7]
2. (a)	$\mathrm{f}(1.2)=0.49166551...,\quad \mathrm{f}(1.3)=-0.048719817...$	
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha\in[1.2,1.3]$	M1A1
		(2)
(b)	$4\operatorname{cosec}x-4x+1=0 \Rightarrow 4x=4\operatorname{cosec}x+1$	M1
	$\Rightarrow x=\operatorname{cosec}x+\frac{1}{4} \Rightarrow \underline{x=\frac{1}{\sin x}+\frac{1}{4}}$	A1 *
		(2)
(c)	$x_1=\frac{1}{\sin(1.25)}+\frac{1}{4}$	M1
	$x_1=1.303757858...,\quad x_2=1.286745793...$	A1
	$x_3=1.291744613...$	A1
		(3)
(d)	$\mathrm{f}(1.2905)=0.00044566695...,\quad \mathrm{f}(1.2915)=-0.00475017278...$	M1
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha\in(1.2905,1.2915) \Rightarrow \alpha=1.291$ (3 dp)	A1
		(2)
		[9]

Question Number	Scheme	Marks
3. (a)	$f(x) \geqslant 3$	M1A1
		(2)
(b)	An attempt to find $2\|3-4x\|+3$ when $x=1$	M1
	Correct answer $fg(1)=5$	A1
		(2)
(c)	$y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\dfrac{3-y}{4}$	M1
	$g^{-1}(x)=\dfrac{3-x}{4}$	A1
		(2)
(d)	$[g(x)]^2=(3-4x)^2$	B1
	$gg(x)=3-4(3-4x)$	M1
	$gg(x)+[g(x)]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$	
	$16x^2-8x=0$	A1
	$8x(2x-1)=0 \Rightarrow x=0, 0.5$ oe	M1A1
		(5) **[11]**
4.	(i)(a) $\dfrac{dy}{dx}=3x^2\times\ln 2x+x^3\times\dfrac{1}{2x}\times 2$	M1A1A1
	$=3x^2\ln 2x+x^2$	
		(3)
	(i)(b) $\dfrac{dy}{dx}=3\boldsymbol{(x+\sin 2x)^2}\times(1+2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\dfrac{dx}{dy}=-\text{cosec}^2 y$	M1A1
	$\dfrac{dy}{dx}=-\dfrac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y=1+\cot^2 y$ and $x=\cot y$ in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ to get an expression in x	
	$\dfrac{dy}{dx}=-\dfrac{1}{\text{cosec}^2 y}=-\dfrac{1}{1+\cot^2 y}=-\dfrac{1}{1+x^2}$ cso	M1, A1*
		(5) **[11]**

Question Number	Scheme	Marks
5. (a)	f(x)>2	B1 (1)
(b)	$\text{fg}(x) = e^{\ln x} + 2, = x + 2$	M1, A1 (2)
(c)	$e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1 A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1 A1 (4)
(d)	Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$	M1
	$\text{f}^{-1}(x) = \ln(x-2), \quad x > 2.$	A1, B1ft (3)
(e)	 Shape for f(x)	B1
	(0, 3)	B1
	Shape for f^{-1}(x)	B1
	(3, 0)	B1 (4)
		[14]

Question Number	Scheme	Marks
6. (a)	$\mathrm{f}(1.4) = -0.568\ \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245\ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4, 1.45)$	A1 (2)
(b)	$3x^3 = 2x + 6$	
	$x^3 = \dfrac{2x}{3} + 2$	
	$x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ ∗	A1 cso (3)
(c)	$x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
(d)	Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01$ ...	
	$\mathrm{f}(1.4355) = 0.003\ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗ cso	A1 (3)
		[11]

Question Number	Scheme	Marks
7. (a)	$R^2 = 2^2 + 3^2$	M1
	$R = \sqrt{13}$ or 3.61…	A1
	$\tan\alpha = \dfrac{3}{2}$	M1
	$\alpha = 0.983…$	A1 (4)
(b)	$f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1A1A1
	$= e^{2x}(2\cos 3x - 3\sin 3x)$	M1
	$= e^{2x}(R\cos(3x + a))$	
	$= Re^{2x}\cos(3x + a)$	A1 * cso
		(5)
(c)	$f'(x) = 0 \Rightarrow \cos(3x + a) = 0$	M1
	$3x + a = \dfrac{\pi}{2}$	M1
	$x = 0.196…$ awrt 0.20	A1
		(3)
		[12]

Examiner reports

Question 1

This was an accessible question for candidates with many gaining all 7 marks.

Q1(a) was very well done and usually correct. Occasionally +32 was used instead of –32 giving $w = \frac{5}{2}$. Some candidates reached $w = \frac{5}{2}$ as a result of writing $(-32)^{\frac{1}{5}} = 2$.

For Q1(b) the correct expression for $\frac{dy}{dx}$ was usually achieved with nearly all candidates recognising the need to apply the chain rule. The substitution of $w = \frac{1}{2}$ occasionally produced –160 instead of +160. Some candidates mistakenly set $\frac{dy}{dx} = 0$, leading to $x = \frac{3}{2}$ and proceeded to use this as their gradient. A few used the result $m_1m_2 = -1$ and went on to find the gradient of the normal. The method mark for finding the equation of a line was gained by almost all candidates but a number of candidates lost the final accuracy mark as a result of errors made in arranging the answer in the form $y = mx + c$. An example of this was $y + 32 = 160x - 80$ followed by $y = 160x - 48$.

Question 2

All four parts of this question were well answered by the overwhelming majority of candidates who demonstrated their confidence with the topic of iteration with around 65% of candidates gaining at least 8 of the 9 marks available.

Some candidates in parts (a), (c) and (d) worked in degrees even though it was stated in the question that x was measured in radians.

In part (a), the majority of candidates evaluated both $f(1.2)$ and $f(1.3)$, although a very small number choose instead to evaluate both $f(1.15)$ and $f(1.35)$. A few candidates failed to conclude "sign change, hence root" as minimal evidence for the accuracy mark.

Most candidates found the proof relatively straightforward in part (b). A small number of candidates lost the accuracy mark by failing to explicitly write $4\text{cosec}x - 4x + 1$ as equal to 0 as part of their proof.

Part (c) was almost universally answered correctly, although a few candidates incorrectly gave x_1 as 1.3037 or x_3 as 1.2918.

The majority of candidates who attempted part (d) choose an appropriate interval for x and evaluated $f(x)$ at both ends of that interval. The majority of these candidates chose the interval $(1.2905, 1.2915)$ although incorrect intervals, such as $(1.290, 1.292)$ were seen. There were a few candidates who chose the interval $(1.2905, 1.2914)$. This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 1.291 to 3 decimal places or $\alpha = 1.291$ or even QED.

A minority of candidates who attempted part (d) by using a repeated iteration technique received no credit because the question required the candidate to consider a change of sign of $f(x)$.

Question 3

On the whole this question was well attempted.

In part (a) most candidates realised that the '3' was significant, although some stated $f(x) > 3$, or occasionally $f(x) < 3$. Many successful candidates sketched a graph. Some candidates stated $f(x) \geq 0$ or $f(x) \geq 1.5$. However the majority answered $f(x) \geq 3$ correctly. A range of appropriate notation was seen. Only a few candidates gave the range in terms of x.

In part (b), most candidates processed the functions in the correct order and realised the significance of the modulus. Occasionally the modulus was omitted, and some found values using both $x = -1$ and 1 in $f(x) = 2(3 - 4x) + 3$.

Candidates generally achieved the correct function in part (c), although a few left their answer in terms of y. A few mistakenly attempted $\frac{1}{g(x)}$, $-g(x)$ or $g'(x)$.

Part (d) was also done well. A few candidates found only one solution as a result of cancelling the x instead of factorising. There were not many numerical errors. Some candidates used as an alternative method letting $g(x) = t$ and obtaining $3 - 4t + t^2 = 0$, which they solved for t and hence found x.

Question 4

Q4(i)(a) was generally answered well, with the vast majority using the product rule. Many candidates started the question by quoting the rule. There were some errors in differentiating $\ln 2x$, obtaining either $\frac{1}{2x}$ or $\frac{2}{x}$.

In Q4(i)(b) most candidates recognised the need for the chain rule. However a considerable number obtained only $2 \cos 2x$ when differentiating $x + \sin 2x$. Of those who performed this differentiation correctly, a significant number lost marks because of incorrect bracketing in their answer. Another common error was to omit the factor 2 when differentiating $\sin 2x$. Common incorrect answers were $3(x + \sin 2x)^2 \times 2 \cos 2x$, $3(x + \sin 2x)^2 \times 1 + 2 \cos 2x$ or $3(x + \sin 2x)^2 \times (1 + \cos 2x) \times 2$.

Candidates' responses in Q4(ii) showed many concise and clear solutions. A majority expressed $\frac{dx}{dy}$ in terms of y, although many did not know, or use the formula sheet, to state that the differential of $\cot y$ is $-\text{cosec } 2y$. These candidates generally differentiated either $\frac{1}{\tan y}$ or $\frac{\cos y}{\sin y}$, often correctly. A number of able candidates proceeded more directly by using implicit differentiation. Most candidates knew that they then had to invert their result to find $\frac{dy}{dx}$, although there were many instances of negative signs appearing or disappearing without any justification. Candidates who used the identity $\text{cosec } 2y = 1 + \cot 2y$ could usually reach the correct result quite efficiently, although some failed to bracket the terms and hence obtained $-\text{cosec } 2y = -1 + \cot 2y$. However some correct use of other trig identities was also seen. At this stage some candidates confused the x and y so could not reach the stated result.

Question 5

Numerous candidates could score high marks on this question, and completely correct solutions were frequently seen. The range and domain were the least well done parts of this question.

In part (a) there was often poor use of notation, with many candidates still confusing the appropriate use of y or f(x) with that of x for the range and domain of the functions ($x > 2$ was unacceptable in part (a)). A surprising number of candidates gave the range of f(x) in part (a) as $f(x) \geq 3$ rather than $f(x) > 2$.

In part (b) most candidates applied the functions in the correct order and were able to simplify their expression correctly. Zero scoring attempts were very rare, but there was a small proportion of candidates who only got as far as $e^{\ln x} + 2$ and either did not simplify, or tried to solve $e^{\ln x} + 2 = 0$ instead.

Part (c) tested candidate's use of ln. It was answered extremely well, although with some very poor ln work was evident amongst weaker candidates. Errors in this question were essentially of three types: incorrect expressions formed by not understanding composite functions, using 2(f) + 3 instead of f($2x$ + 3); missing "+2", giving $e^{2x+3} = 6$; and incorrect ln work in solving.

Many candidates appeared to forget to state the domain in part (d). Some of those who gave a domain followed through their answer to part (a), but some gave a correct answer even if they had part (a) incorrect. Missing brackets was extremely rare.

In part (e) some candidates produced very careful and accurate graphs, but a large variety of shapes were produced by a minority, particularly for $f^{-1}(x)$ Some candidates also failed to give correct coordinates for the intersections with the axes; (0, 2) and (2, 0) were often seen, but other values did occur, or else no coordinates were given at all. Generally there was less success with the intercepts than the shapes. It was also not uncommon to find the two sketches intersecting. There were a few cases only of sketches having a max or min. Some candidates also illustrated the asymptotes, which were not required, but showed a full understanding of the functions

Question 6

In parts (a) and (d), candidates need to be aware that showing that something is true requires them to give reasons and conclusions. In this part (a), it is sufficient to say that a change of sign in the interval (1.4, 1.45) implies that there is a root in the interval (1.4, 1.45). In part(c), it would be sufficient to argue that a change of sign in the interval (1.4345, 1.4355) implies that there is a root in the interval (1.4345, 1.4355) and, hence, that $x = 1.435$ is accurate to 3 decimal places. Part (b) was very well done but candidates must put all steps in a proof and not leave it to the examiners to fill in important lines. Part (c) was very well done. Some candidates attempted part (d) using repeated iteration but the wording of the question precludes such a method and no marks could be gained this way.

Question 7

Part (a) seemed well understood and there was a lot of correct work here. R was almost always found correctly, although there were some simple arithmetic errors where candidates had not use a calculator, for example $R^2 = 2^2 + 3^3$ leading to $R = \sqrt{10}$ or $\sqrt{15}$. There were more errors in the tan alpha part; the most common mistake was giving tan θ as $\frac{2}{3}$ or $-\frac{3}{2}$. A significant number used degrees here while others gave α in terms of π. Several candidates rounded their radian answer to 0.98, or 0.982.

In part (b) the differentiation was well done and most candidates achieved the first three marks. There were some sign mistakes and $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{2x}) = 2\mathrm{e}^{x}$ also appeared. Only a few candidates omitted the 2 and the 3, possibly because they could see where they were heading. Most were then able to complete the proof, connecting their answer with part (a), although there was some unnecessary repetition of the working from part (a). As so often happens, with 'show that' questions, many lost marks by failing to show or missing out the factorising stage, candidates possibly unaware of the importance of it.

Quite surprisingly part (c) was found to be very demanding to all but the best of candidates. Many wrote $\mathrm{e}^{2x} \cos (3x + \alpha) = 0$ but did not seem to realise that $\mathrm{e}^{2x} = 0$ had no solution and hence went no further. Some confused "smallest x" with finding the minimum value so wrote $\cos (3x + \alpha) = \pm 1$ followed by $3x + \alpha = \pi$ or 2π. Some put $3x + \alpha = 0$.

Statistics for C3 Practice Paper Bronze Level B3

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	7		86	6.01	6.93	6.78	6.54	6.13	5.53	4.65	2.57
2	9		82	7.38	8.76	8.41	7.88	7.21	6.21	5.14	3.15
3	11		89	9.74	10.84	10.36	9.72	9.03	8.45	6.60	5.17
4	11		76	8.39	10.82	10.14	9.40	8.57	6.83	5.36	3.08
5	14		79	11.01	13.62	12.75	11.72	10.48	9.00	6.83	3.78
6	11		79	8.72		10.26	9.36	8.41	7.23	6.04	3.90
7	12		73	8.77	11.66	10.81	9.67	7.99	6.04	4.05	1.92
	75		**80**	**60.02**		**69.51**	**64.29**	**57.82**	**49.29**	**38.67**	**23.57**