Paper Reference(s) 66665/01 Edexcel GCE Core Mathematics C3 Bronze Level B1

Time: 1 hour 30 minutes

Materials required for examination	Items included with question
<u>papers</u>	
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A *	Α	В	С	D	Ε
72	66	60	54	49	43

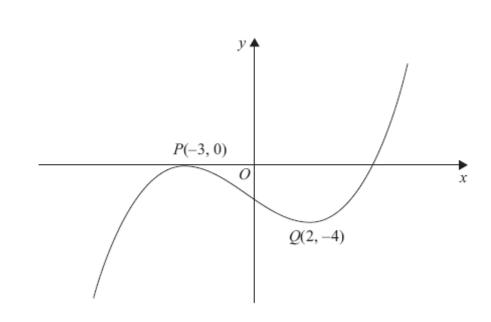
1. Express

2.

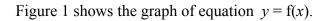
$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4) June 2013 (R)







The points P(-3, 0) and Q(2, -4) are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a)
$$y = 3f(x+2)$$
, (3)

$$(b) \quad y = \left| f(x) \right|. \tag{3}$$

On each diagram, show the coordinates of any stationary points.

January 2012

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \ge 0.$$

- (a) Write down the number of rabbits that were introduced to the island.
- (b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(c) Find
$$\frac{\mathrm{d}P}{\mathrm{d}t}$$
. (2)

(d) Find P when
$$\frac{\mathrm{d}P}{\mathrm{d}t} = 50$$
.

(3)

(1)

(2)

June 2009

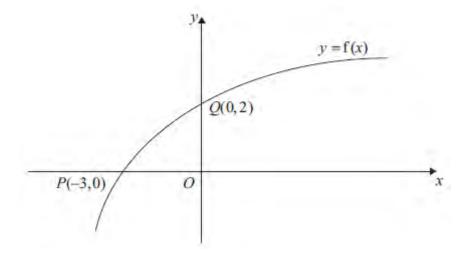


Figure 2

Figure 2 shows part of the curve with equation y = f(x), $x \in \mathbb{R}$.

The curve passes through the points Q(0, 2) and P(-3, 0) as shown.

(a) Find the value of ff
$$(-3)$$
. (2)

On separate diagrams, sketch the curve with equation

(b)
$$y = f^{-1}(x)$$
, (2)

(c)
$$y = f(|x|) - 2$$
,

(d)
$$y = 2f(\frac{1}{2}x)$$
. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

January 2013

(2)

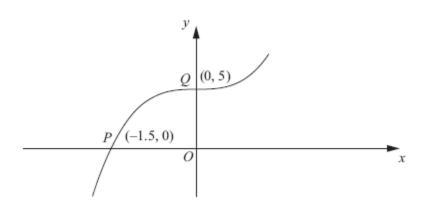


Figure 3

Figure 3 shows part of the curve with equation y = f(x). The curve passes through the points P(-1.5, 0) and Q(0, 5) as shown.

On separate diagrams, sketch the curve with equation

$(a) y = \mathbf{f}(x) $	
	(2)

(b)
$$y = f(|x|)$$
 (2)

$$(c) \quad y = 2f(3x) \tag{3}$$

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

June 2012

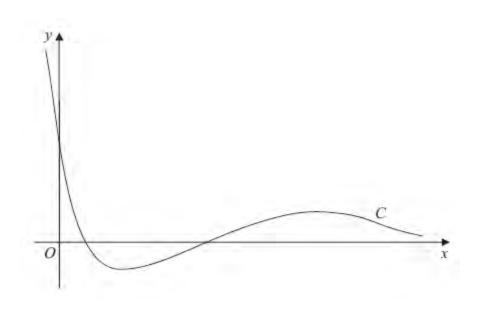


Figure 4

Figure 4 shows a sketch of the curve *C* with the equation $y = (2x^2 - 5x + 2)e^{-x}$. (*a*) Find the coordinates of the point where *C* crosses the *y*-axis.

(b) Show that C crosses the x-axis at x = 2 and find the x-coordinate of the other point where C crosses the x-axis.(3)

(c) Find
$$\frac{dy}{dx}$$
.
(3)
(d) Hence find the exact coordinates of the turning points of C.

(5)

(1)

June 2010

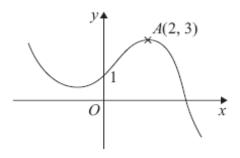


Figure 5

Figure 5 shows a sketch of the graph of y = f(x).

The graph intersects the y-axis at the point (0, 1) and the point A(2, 3) is the maximum turning point.

Sketch, on separate axes, the graphs of

- (i) y = f(-x) + 1,
- (ii) y = f(x+2) + 3,
- (iii) y = 2f(2x).

On each sketch, show the coordinates of the point at which your graph intersects the *y*-axis and the coordinates of the point to which *A* is transformed.

(9)

January 2010

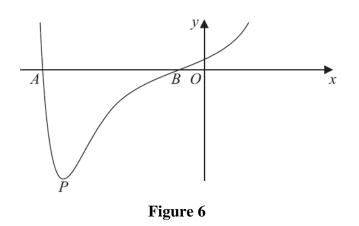


Figure 6 shows a sketch of part of the curve with equation y = f(x) where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the *x*-axis at points *A* and *B* as shown in Figure 6.

(a) Calculate the x-coordinate of A and the x-coordinate of B, giving your answers to 3 decimal places.(2)

(b) Find
$$f'(x)$$
.

The curve has a minimum turning point P as shown in Figure 6.

(c) Show that the x-coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$
(3)

(*d*) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}$$
, with $x_0 = -2.4$,

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(3)

The *x*-coordinate of *P* is α .

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

June 2013 (R)

9. (*a*) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$
(3)

Given that

$$\ln (2x^2 + 9x - 5) = 1 + \ln (x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

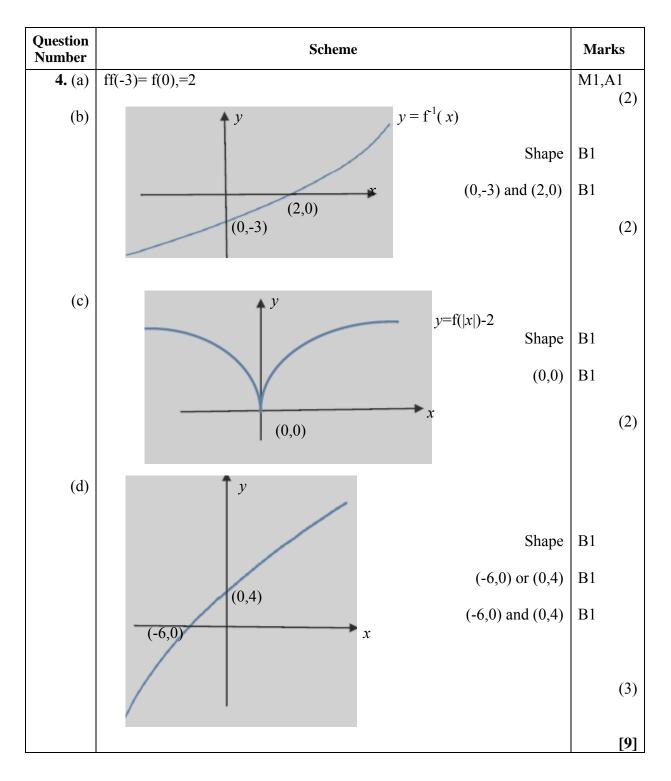
June 2010

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme						
1.	$x^{2} + x - 12 = (x + 4)(x - 3)$						
	Attempt as a single fraction						
	$(3x+5)(x-3)-2(x^2+x-12) = 3x+5-2(x+4)$						
	$\frac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)} \text{ or } \frac{3x+5-2(x+4)}{(x+4)(x-3)}$						
	$=\frac{x-3}{(x+4)(x-3)}$, $=\frac{1}{(x+4)}$ cao						
2. (a)							
(a)	y f / Sha	pe B1					
	-5.0						
	x coordinates correct 0,-12 x coordinates correct						
		(3)					
(b)							
	Shaj	pe B1					
	Max at (2,	4) B1					
	2,4 Min at (-3,0)						
	-3,0 <i>O x</i>	(3)					
		[6]					

Question Number	Scheme	Marks
3. (a)	$P = 80 e^{\frac{t}{5}}$	
	$t = 0 \implies P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
(b)	$P = 1000 \implies 1000 = 80e^{\frac{t}{5}} \implies \frac{1000}{80} = e^{\frac{t}{5}}$	M1
	$\therefore t = 5\ln\left(\frac{1000}{80}\right)$	
	t = 12.6286	A1 (2)
	$\frac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{s}}$	M1 A1 (2)
(d)	$50 = 16e^{\frac{t}{5}}$	
	:. $t = 5 \ln\left(\frac{50}{16}\right)$ {= 5.69717}	M1
	$P = 80e^{\frac{1}{5}(5\ln(\frac{50}{16}))}$ or $P = 80e^{\frac{1}{5}(5.69717)}$	M1
	$P = \frac{80(50)}{16} = \underline{250}$	A1 (3)
		[8]



Question Number	Scheme					
5. (a)	$y = \mathbf{f}(\mathbf{x}) $	Shape including cusp	B1			
	(-1.5,0) 0 × x	(-1.5, 0) and (0, 5)	B1			
	(1.5,0)		(2)			
(b)	$\mathbf{y} \qquad \mathbf{y} = \mathbf{f}(\mathbf{x})$					
		Shape	B1			
	(0,5)	(0,5)	B1			
	x 		(2)			
(c)	↑ y					
	y=2f(3x)	Shape	B1			
	(0,10)	(0,10)	B1			
	$(-0.5,0) \qquad \qquad$	(-0.5, 0)	B1			
			(3) [7]			

Question Number	Scheme	Marks
6. (a)	Either $y = 2 \operatorname{or}(0, 2)$	B1
(b)	When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$ $(2x^2 - 5x + 2) = 0 \implies (x - 2)(2x - 1) = 0$ Either $x = 2$ (for possibly B1 above) or $x = \frac{1}{2}$.	(1) B1 M1 A1
(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	(3) M1A1A 1
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$ $2x^2 - 9x + 7 = 0 \implies (2x - 7)(x - 1) = 0$ $x = \frac{7}{2}, 1$ When $x = \frac{7}{2}, y = 9e^{-\frac{7}{2}}$, when $x = 1, y = -e^{-1}$	(3) M1 M1 A1 ddM1A1 (5) [12]
7. (i)	y = f(-x) + 1	
	$A'(-2, 4)$ $(\{0\}, 2\}$ $(\{0\}, 2)$	B1 B1 B1
(;;)	y = f(x+2) + 3	(3)
(ii)	$\int y = I(x + 2) + 3$ Any translation of the original curve. Any translated maximum has either x-coordinate of 0 (can be implied) or y-coordinate of 6. The translated curve has maximum ({0}, 6) and is in the correct position on the Cartesian axes.	B1 B1 B1
		(3)

Question Number	Scheme	Marks
(iii)	$y = 2f(2x)$ Shape of with a minimum in quadrant 2 and a maximum in quadrant 1. Either ({0}, 2) or A'(1, 6) Both ({0}, 2) and A'(1, 6) $Both ({0}, 2) and A'(1, 6)$	B1 B1 B1 (3)
8. (a)	$f(x) = 0 \Longrightarrow x^2 + 3x + 1 = 0$	[9]
	$\Rightarrow x = \frac{-3 \pm \sqrt{5}}{2} = \text{awrt -0.382, -2.618}$	M1A1 (2)
(b)	Uses $vu'+uv'$ $f'(x) = e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x$	M1A1A 1 (3)
(c)	$e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x = 0$	
	$\Rightarrow e^{x^2} \left\{ 2x^3 + 6x^2 + 4x + 3 \right\} = 0$	M1
	$\Rightarrow x(2x^2+4) = -3(2x^2+1)$	M1
	$\Rightarrow x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$	A1* (3)
(d)	Sub $x_0 = -2.4$ into $x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}$	
	$x_1 = awrt - 2.420, \ x_2 = awrt - 2.427 \ x_3 = awrt - 2.430$	M1A1A 1 (3)
(e)	Sub x = - 2.425 and -2.435 into f'(x) and start to compare signs	
	f'(-2.425) = +22.4, f'(-2.435) = -15.02	M1
	Change in sign, hence $f'(x) = 0$ in between. Therefore $\alpha = -2.43$ (2dp)	A1
		(2) [13]

Question Number	Scheme	Marks	ĺ
9. (a)	$\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$	M1 B1 A1 aef (3)	
(b)	$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ $\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = e$	M1	,
	$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = e$	dM1	
	$\frac{2x-1}{x-3} = e \implies 3e-1 = x(e-2)$	M1	
	$\Rightarrow x = \frac{3e - 1}{e - 2}$	A1 aef cso	•
		(4) [7]	

Examiner reports

Question 1

The majority of the candidates found this a very straightforward question to start off the paper. The overall standard of answers was very good. The main error seen was in the expansion of -2(x + 4) = 2x + 8 or -2(x + 4) = -2x - 4. Another error was either incorrectly factorising the denominator of the first fraction or failing to factorise at the start of the question producing over complicated expressions. Only a minority of candidates failed to attempt this at all.

Question 2

Most candidates are now very good at sketching graphs in C3. The standard of presentation was also better than in similar questions from the past examination series. There were many fully correct solutions.

Part (a): When marks were lost it was usually due to slips: correctly drawn graphs mis-labelled e.g. (5, 0) rather than (-5, 0). Common errors were giving Q' as (0, -8) or (0, -4). There were also cases where candidates correctly stated that the minimum point was (0, -12) but failed to recognise Q as lying on the y-axis, indicating it to the left or right of the y-axis. A few candidates shifted the curve vertically but this was rare.

Part (b): Very well answered in general although the shape was often incorrect with 2 cusps instead of one or 2 smooth minimum points. Most candidates obtained the correct coordinates of P' and Q'. If these were incorrect it was probably carelessness, rather than a misunderstanding. e.g. Q' marked as (2, -4), P' marked (3, 0) or Q' being drawn on the y-axis. A few candidates sketched f(|x|) rather than |f(x)| but this was rare; the drawing of -f(x) was also occasionally seen.

Question 3

This question was well answered by the overwhelming majority of candidates who demonstrated their confidence in working with exponentials.

Part (a) was almost universally answered correctly, although a few candidates did try to substitute t = 1 into the equation for P in order to find the number of rabbits introduced to the island.

In part (b), most candidates were able to use natural logarithms in order to find t = 12.6 or t = 12.63. Although the expected answer was 13 years, any answer that rounded to 12.6 years was also accepted. Those candidates who continued to round their answer down to 12 or stated "in the 12th year" were not awarded the final accuracy mark as the question required candidates to find the number of years for the number of rabbits to exceed 1000. A few candidates applied a trial and error method in this part and were usually successful in gaining both marks.

In part (c), most candidates correctly stated $\frac{dP}{dt}$ as $16e^{\frac{1}{5}t}$. Common errors in this part were candidates giving answers of the form $kte^{\frac{1}{5}t}$ or $16e^{-\frac{4}{5}t}$. A few candidates tried to apply the product rule for differentiation and usually struggled to gain both marks.

In part (d), the vast majority of candidates equated their $\frac{dP}{dt}$ found in part (c) to 50 and proceeded to solve for *t*. A number of candidates failed at this point to use their value for *t* to

find *P* as required in the question. It was pleasing to see a significant minority of candidates who deduced that $P = 80e^{\frac{1}{5}t} = 5 \times 16e^{\frac{1}{5}t} = 5 \times \frac{dP}{dt} = 250.$

Question 4

Most candidates found this question accessible and a good number of completely correct solutions were seen.

Part (a) seemed to be the most challenging part of the question for many candidates. Some were confused by the fact that they were not told f(x) as a function of x was and therefore did not realise they could find the answer by using the given sketch. A proportion of these candidates attempted to find a function (usually linear) that passed through (-3, 0) and (0, 2). Some responses showed confusion over how to calculate the value of a composite function, with $ff(-3) = f(-3) \times f(-3)$ or $ff(-3) = f(-3) \times f(0)$ being common errors.

In part (b) the sketch was done well by the majority of candidates, nearly all knowing to reflect the graph of f(x) in the line y = x. A few candidates lost the mark for the shape of the graph due to an obvious minimum point drawn in the third quadrant or the curve bending back on itself in the first quadrant. However, the most common error on this part was confusion with the coordinates on the axes.

In part (c) most candidates scored both mark. Some responses did lose the first mark due to the presence of an obvious maximum point, poor symmetry or the wrong shape.

Part (d) was done well by most candidates with nearly all scoring 2 or 3 marks. It was rare for the shape of the graph to be drawn incorrectly, the most common errors made when finding the coordinates, with (-1.5, 0) instead of (-6, 0) being the mistake most often seen.

Question 5

The majority of candidates answered this question well, although the graphs were sometimes very untidy and the coordinates difficult to read. Very few candidates omitted to state the required points of intersection with the axes.

In part (a) the cusp was better drawn than in previous examinations, but there were occasional errors either with it still crossing the *x*-axis, or bending back on itself. The shape and coordinates were usually correct.

The shape of the graph in part (b) caused the most problems, with many candidates either reflecting the whole graph in the *y*-axis, or reflecting the negative *x*-values across the *y*-axis producing a K shape. A less common alternative error was to reflect in the line y = 5, leaving both upper and lower portions in (an "X" shape graph). In part c) there were some errors in the stretches but a large number of candidates answered this part accurately. There were a significant number of candidates who labelled the clearly negative intercept on the *x*-axis with a positive coordinate. The coordinates were the most problematic aspect of part (c). Labelling Q as (0, 15) and P as (-4.5, 0) were fairly common errors (e.g. candidates stretched the graph by scale factors 2 and 3 instead of 2 and $\frac{1}{2}$).

by scale factors 2 and 3 instead of 2 and $\frac{1}{3}$).

Question 6

This question was extremely well answered with 84% of candidates gaining at least 7 of the 12 marks available and about 42% gaining all 12 marks.

Nearly all candidates were successful in answering part (a). A few candidates were initially confused when attempting part (a) by believing that the curve met the *y*-axis when y = 0. These candidates quickly recovered and relabelled part (a) as their part (b) and then went onto to find in part (a) that when x = 0, y = 2. Therefore, for these candidates, part (b) was completed before part (a).

In part (b), some candidates chose to substitute x = 2 into $y = (2x^2 - 5x + 2)e^{-x}$ in order to confirm that y = 0. The majority of candidates, however, set y = 0 and solved the resulting equation to give both x = 2 and x = 0.5. Only a few candidates wrote that x = 0 is a solution of $e^{-x} = 0$.

In part (c), the product rule was applied correctly to $(2x^2 - 5x + 2)e^{-x}$ by a very high proportion of candidates with some simplifying the result to give $(-2x^2 + 9x - 7)e^{-x}$. Common errors included either e^{-x} being differentiated incorrectly to give e^{-x} or poor bracketing. The quotient rule was rarely seen, but when it was it was usually applied correctly.

In part (d), the majority of candidates set their $\frac{dy}{dx}$ in part (c) equal to 0, although a few differentiated again and set $\frac{d^2y}{dx^2} = 0$. At this stage, few candidates produced invalid

logarithmic work and lost a number of marks. Some other candidates made bracketing and/or algebraic errors in simplifying their gradient function. Most candidates realised that they needed to factorise out e^{-x} and solve the resulting quadratic with many of them correctly finding both sets of coordinates. Some candidates did not give their *y*-coordinates in terms of e, but instead wrote the decimal equivalent.

Question 7

On the whole this question was well answered, with the majority of candidates achieving at least 7 of the 9 marks available. A significant minority of candidates, however, appeared to take little care when drawing their curves with a lack of precision in labelling and in the location of turning points. The size of some candidates' sketches caused some problems – the very large whole page graphs being quite difficult for candidates to sketch well and the very small graphs were hard for candidates to label and for examiners to decipher. Some candidates, in each part, by drawing more than one set of axes, showed their working in stages. Some candidates wrote transformed coordinates of (2, 0) when they really meant (0, 2).

Part (i) seemed to seemed to cause candidates more problems than the other two parts. Often a significant number of candidates reflected the original curve through the *x*-axis and translated the resulting graph up 1 unit to give a curve which went through the origin with a minimum turning point at (2, -2). Other candidates only reflected the graph through the *y*-axis to give curve which cut the *y*-axis at (0, 1) and had a minimum turning point at (-2, 3).

Parts (ii) and (iii) were usually well done although the final mark in part (ii) was sometimes missed because of careless drawing of the maximum which appeared to be in the 1st quadrant, although labelled as being on the y axis. In part (iii), a common error was for some candidates to enlarge the original curve by a scale factor of 2.

While drawing the correct shape for (iii), with turning points in the correct quadrants, some candidates often made no attempt to indicate the stretch of scale factor $\frac{1}{2}$ parallel to the *x*-axis

in their drawing. These candidates, however, did score full marks thanks to giving correct coordinates for the turning point as well as the *y*-intercept.

Question 8

Part (a) was generally very well done, although a significant number left the answer in surd form. In some cases there was no working shown.

Part (b) was also very well answered although again failure to write down the product rule and show clear stages of working meant candidates sometimes scored zero marks. The most common mistake was not multiplying by 2x when differentiating e^{x^2} .

In part (c) the first mark was scored in most cases and good attempts were made at moving on to the given answer. Those that did not score any more marks moved on to $\dots = -3$ and then factorised the LHS by x to move onto $x = \dots$ but a sizeable number of candidates struggled with this part.

In part (d), in all but a very few cases, the first mark was scored and most went onto full marks. Main errors included having positive answers, poor rounding and obvious errors on the calculator. There did not seem to be a realisation that if the answer was radically different to -2.4 it was wrong and needed to be checked.

Part (e) was poorly done. Although most candidates were able to select an appropriate interval, many then failed to use their values correctly. Far too many answers contained substitutions into f(x) or the iteration formula. A number talked about the change in sign without calculating values.

Question 9

This question was well answered with candidates usually scoring either 3 marks (about 21%), or 5 marks (about 17%) or all 7 marks (about 46%).

The vast majority of candidates achieved all three marks in part (a). A significant minority of candidates used an alternative method of long division and were invariably successful in achieving the result of $2 + \frac{5}{(x-3)}$.

The laws of logarithms caused problems for weaker candidates in part (b). Common errors including some candidates simplifying $\ln(2x^2 + 9x - 5) - \ln(x^2 + 2x - 15)$ to $\frac{\ln(2x^2 + 9x - 5)}{\ln(x^2 + 2x - 15)}$ or other candidates manipulating the equation $\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$ into $2x^2 + 9x - 5 = e^1 + x^2 + 2x - 15$. Perhaps more disheartening was the number of candidates who were unable to make *x* the subject after correctly achieving $\frac{2x - 1}{x - 3} = e$, with some leaving a final answer of $x = \frac{1 + ex - 3e}{2}$. Those candidates who used long division in part (a) usually coped better with making *x* the subject in part (b).

			Mean score for students achieving grade:								
Qu	Max score	Modal score	Mean %	ALL	A *	Α	В	С	D	Е	U
1	4		97	3.87	4.00	3.93	3.92	3.85	3.78	3.66	3.07
2	6		90	5.40	5.93	5.75	5.52	5.18	4.82	4.52	3.02
3	8		86	6.87		7.77	7.43	6.90	5.93	4.75	2.93
4	9		81	7.26	8.82	8.32	7.62	6.88	5.70	4.98	3.24
5	7		90	6.30	6.90	6.72	6.53	6.22	5.83	5.31	4.08
6	12		80	9.56	11.83	11.28	10.42	9.08	7.36	5.55	3.32
7	9		83	7.48		8.42	7.73	7.17	6.32	5.77	3.46
8	13		71	9.27	12.32	10.51	8.68	6.99	6.07	3.61	2.41
9	7		76	5.30	6.87	6.31	5.52	4.74	4.04	3.40	2.45
	75		82	61.31		69.01	63.37	57.01	49.85	41.55	27.98

Statistics for C3 Practice Paper Bronze Level B1