mark scheme

Practice Paper C : Core Mathematics 2

Question Number	General Scheme		Marks
1	$\int_1^3 \left(\frac{2+3\sqrt{x}}{x^2}\right) dx$	**B1:** sets up a correct integral to evaluate (seen anywhere).	**B1**
	$\int_1^3 \left(\frac{2+3\sqrt{x}}{x^2}\right) dx = \left[-\frac{2}{x}-\frac{6}{\sqrt{x}}\right]_1^3$	**M2:** correct method to find the general integral **(see note)** **A1:** correct general integral. **Constant <u>not</u> needed**	**M2** **A1**
Note	**Award M1 for one error in working and M0 for more than one error.**		
	$\left[-\frac{2}{x}-\frac{6}{\sqrt{x}}\right]_1^3 = \left(-\frac{2}{3}-\frac{6}{\sqrt{3}}\right)-\left(-\frac{2}{1}-\frac{6}{\sqrt{1}}\right)$ $= -\frac{2}{3}-\frac{6}{\sqrt{3}}+8 = \frac{22}{3}-2\sqrt{3}$	**M1:** correct substitution of limits **A1:** cao	**M1** **A1**
		Total	**6**

Question Number	General Scheme		Marks
2	$\overbrace{\underbrace{{}^5C_0(1)^5(x)^0}_{\text{M1}} + {}^5C_1(1)^4 x + {}^5C_2(1)^3(x)^2 + {}^5C_3(1)^2(x)^3 + {}^5C_4(1)^1(x)^4 + {}^5C_5(1)^0(x)^5}^{\text{M1 A1}}$ **M1:** one term correctly expressed (need not be the first term) **M1**: a complete expansion with all terms containing coefficients and the powers on each term adding to 5. **Accept** $\binom{n}{r}$ in replacement for ${}^nC_r$. **A1**: a correct unsimplified expansion. $\therefore (1+x)^5 = 1+5x+10x^2+10x^3+5x^4+x^5$ **A1**: a correct simplified expansion.		M1 M1 A1 A1
	$1+x=0.0172 \ \rightarrow \ x=-0.9828$	**M1:** attempts to find the value of x that will compute $(0.0172)^5$.	M1
	$(0.0172)^5 = 1+5(-0.9828)+10(-0.9828)^2+...+(-0.9828)^5$ $\therefore (0.0172)^5 \simeq 1.51\times10^{-9}$ **M1** – correctly substitutes *their* x into *their* binomial expansion **A1** – cso		M1 A1
	Total		7

Question Number	General Scheme		Marks
4	$\log_3(x^2-5x+6)-\log_3(2x^2-26x+60)=2$ $\log_3\left(\dfrac{x^2-5x+6}{2x^2-26x+60}\right)=2$	**M1:** use of the rule that $\log a-\log b=\log\left(\frac{a}{b}\right)$	**M1**
	$\log_3\left(\dfrac{x^2-5x+6}{2x^2-26x+60}\right)=2$ $\dfrac{x^2-5x+6}{2x^2-26x+60}=9$	**M1:** correct attempt to obtain an equation independent of logs **A1:** correct working	**M1** **A1**
	$x^2-5x+6=9(2x^2-26x+60)$ $(x-2)(x-3)-18(x-3)(x-10)=0$ $(x-3)(x-2-18x+180)=0$ $(x-3)(178-17x)=0$ $\therefore x=3,\ x=\dfrac{178}{17}$	**M1:** a valid attempt to solve the resultant quadratic **A1: both** solutions	**M1** **A1**
	Total		**5**

Question Number	General Scheme		Marks
5 (a)	$\frac{dy}{dx} = 2x^3 - 128x$	**B1:** correct derivative	**B1**
	$2x^3 - 128x = 0$	**M1:** sets *their* $\frac{dy}{dx} = 0$	**M1**
	$2x\left(x^2 - 64\right) = 0$ $2x(x-8)(x+8) = 0$ $x = 0,\ x = 8,\ x = -8$	**M1:** correct method to find x **A1:** obtains the correct values of x	**M1** **A1** **(4)**
(b)	$\frac{d^2y}{dx^2} = 6x^2 - 128$	**B1ft:** correct second derivative	**B1ft**
	$\left.\frac{d^2y}{dx^2}\right\|_{x=\ldots} = \ldots$	**M1:** evaluates the second derivative at *their* stationary points	**M1**
	Maximum at $x = 0$, as $\left.\frac{d^2y}{dx^2}\right\|_{x=0} < 0$ Minimum at $x = \pm 8$, as $\left.\frac{d^2y}{dx^2}\right\|_{x=\pm 8} > 0$	**A1ft:** award when the nature of the stationary points is correctly determined **and** an explicit justification. Ft of *their* (a)	**A1ft** **(3)**
		Total	**7**

Question Number	General Scheme		Marks
6 (a)	$S_n = \frac{a(1-r^n)}{1-r}$	**B1:** cao	**B1** **(1)**
(b)	$S_\infty = \frac{2a}{1-\frac{1}{s}}$ $= \frac{2a}{\frac{s-1}{s}} = \frac{2as}{s-1}$	**B1:** use of $\frac{1}{s}$ as common ratio **M1:** correct substitution **A1:** correct expression for S_∞	**B1** **M1** **A1** **(3)**
(c)	$\frac{a(1-r^n)}{1-r} = 4\left(\frac{2as}{s-1}\right)$	**M1:** correct expression formed	**M1**
	$\frac{1-r^n}{1-r} = \frac{8s}{s-1}$ $(s-1)(1-r^n) = 8s(1-r)$ $s - sr^n - 1 + r^n - 8s + 8rs = 0$ $s - sr^n - 8s + 8rs = 1 - r^n$	**M1:** attempts to make s the subject	**M1**
	$s(1-r^n-8+8r) = 1-r^n$ $\therefore s = \frac{1-r^n}{1-r^n-8+8r}$	**M1:** factorises s out **A1:** correct expression of s in terms of r.	**M1** **A1** **(4)**
		Total	**8**

Question Number	General Scheme		Marks
7 (a)	Either $x+3$ or $x-1$ or $2x-1$	**M1:** attempts to substitute values into the given expression **A1:** one correct factor	**M1** **A1** **(2)**
(b)	$$\begin{array}{r} 2x^2+5x-3 \\ x-1\overline{)2x^3+3x^2-8x+3} \\ \underline{2x^3-2x^2} \\ 5x^2-8x \\ \underline{5x^2-5x} \\ -3x+3 \\ \underline{-3x+3} \\ 0 \end{array}$$	**M2:** uses *their* factor from (a) to find other factors by long division or inspection **or** uses the factor theorem. **A1:** correct resultant quadratic from *their* division	**M2** **A1**
Note	**Award M1 for one error in working and M0 for more than one error.**		
	$2x^3+3x^2-8x+3=(x-1)(2x^2+5x-3)$ $=(x-1)(ax+b)(x+c)$ $=(x-1)(2x-1)(x+3)$	**M1:** factorises *their* quadratic to obtain two linear factors **A1:** correct factors	**M1** **A1** **(5)**
(c)	$4\sin^3 2\theta+6\sin^2 2\theta-16\sin 2\theta+6=0 \ (\div 2)$ $2\sin^3 2\theta+3\sin^2 2\theta-8\sin 2\theta+3=0$ $(\sin 2\theta-1)(2\sin 2\theta-1)(\sin 2\theta+3)=0$	**B1ft:** correct use of *their* (b) to factorise the trigonometric cubic	**B1**
	$\sin 2\theta=1$, $\sin 2\theta=\frac{1}{2}$, $\sin 2\theta\neq -3$		

	$2\theta = \sin^{-1}(1)$ $= \dfrac{\pi}{2}$	$2\theta = \sin^{-1}\left(\dfrac{1}{2}\right)$ $= \dfrac{\pi}{6}$, $\pi - \dfrac{\pi}{6}$	**M1:** attempts to find the principal values of **both** equations **M1:** $\pi -$ their $\sin^{-1}\left(\dfrac{1}{2}\right)$	**M1** **M1**
	$= 2\pi + \dfrac{\pi}{2}$	$= 2\pi + \dfrac{\pi}{6}$, $2\pi + \left(\text{their } \pi - \dfrac{\pi}{6}\right)$	**M1:** a clear method to find the additional values for $2\theta \geq 2\pi$	**M1**
	$2\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{5\pi}{2}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$ $\therefore \theta = \dfrac{\pi}{12}, \dfrac{\pi}{4}, \dfrac{5\pi}{12}, \dfrac{5\pi}{4}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}$		**M1:** divides *their* values of 2θ by 2. Condone arithmetic slips, provided the intention is clear **A1: all** values of θ given. Ignore additional values of θ outside of the given range, but **if they are incorrect**, award **A0**.	**M1** **A1** **(6)**
			Total	**13**

Question Number	General Scheme			Marks
8 (a)	Gradient of the normal at $P = \frac{4-7}{2-6} = \frac{3}{4}$		**M1:** correct method to work out the gradient of the normal at P **A1:** $\frac{3}{4}$	**M1** **A1**
	$\therefore$ Gradient of $T = -\frac{4}{3}$		**B1ft:** gradient of $T = -\frac{1}{m_{\text{their normal}}}$	**B1**
	$\therefore y-7 = -\frac{4}{3}(x-6)$		**M1:** correct method to work out the equation of T **A1:** equation of T oe	**M1** **A1** **(5)**
(b)	**B1:** circle with centre in the correct quadrant and radius 5 correctly drawn + tangent drawn at the point $(6,7)$ **B1:** correct illustration that the tangent does not intersect the circle again			**B1** **B1** **(2)**
(c)	$(x-2)^2 + \left(-\frac{4}{3}x+15-4\right)^2 = 5^2$ $(x-2)^2 + \left(-\frac{4}{3}x+11\right)^2 = 5^2$		**M1:** attempts to find coordinates of intersection	**M1**
	$x^2-4x+4-\frac{16}{9}x^2-\frac{88}{3}+121=25$ $9x^2-36x+36+16x^2-264x+1089=225$ $\therefore 25x^2-300x+900=0$ $\therefore x^2-12x+36=0$		**M1:** a good attempt to form 3TQ **A1:** correct 3TQ oe	**M1** **A1**
	$\therefore (x-6)^2 = 0$ $\therefore$ There is only one intersection between C and T	$b^2-4ac = 144-144$ $=0$ $\therefore$ There is only one intersection between	**M1:** a valid method to show that there is only one intersection **A1:** cso **including** a conclusive statement	**M1** **A1**

	when $x = 6$	C and T when $x = 6$		**(5)**
			Total	**12**
ALT	Some candidates may attempt part (a) using implicit differentiation. That method is shown here: $$2(x-2)+2(y-4)\frac{dy}{dx}=0 \textbf{ M1}$$ $$\frac{dy}{dx}=-\frac{x-2}{y-4} \textbf{ A1}$$ $$\left.\frac{dy}{dx}\right\|_{x=6,y=7}=-\frac{4}{3} \textbf{ A1}$$ Then the rest is as the given method. In this case, the final **B1** becomes **A1.** Ascribe the marks as shown.			

Question Number	General Scheme		Marks
9 (a)	$= \dfrac{3\cdot 2^x \cdot 12^{2x} - 5\cdot 4^x \cdot 6^{2x}}{2^x \cdot 6^{2x}}$	**M1:** one attempt to reduce a term into base 2	**M1**
	$= \dfrac{3\cdot 2^x \cdot (6\cdot 2)^{2x}}{2^x \cdot 6^{2x}} - \dfrac{5\cdot (2\cdot 2)^x \cdot 6^{2x}}{2^x \cdot 6^{2x}}$	**M1:** all terms reduced into base 2, where appropriate	**M1**
	$= \dfrac{3\cdot 2^x \cdot 6^{2x} \cdot 2^{2x}}{2^x \cdot 6^{2x}} - \dfrac{5\cdot 2^x \cdot 2^x}{2^x}$		
	$= 3\cdot 2^{2x} - 5\cdot 2^x$	**A1:** cao	**A1 (3)**
(b)	$3\cdot 2^{2x} - 5\cdot 2^x - 2 = 0$ $\left(3\cdot 2^x + 1\right)\left(2^x - 2\right) = 0$	**M1:** correct method to solve 3TQ (either directly or by substitution)	**M1**
	$3\cdot 2^x + 1 \neq 0$, since $\ln x \geq 0$	**B1:** clear or implied rejection of the $3\cdot 2^x + 1$ factor	**B1**
	$\therefore 2^x = 2$ $x = 1$	**A1:** $x = 1$	**A1** **(3)**
		Total	**6**