Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2
Gold Level G3

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
60	53	45	37	29	21

1. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\, dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

May 2007

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2. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(*b*) find the value of k.

(2)

June 2009

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3. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(4)

May 2013 (R)

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4. (*a*) Find, to 3 significant figures, the value of x for which $5^x = 7$.

(2)

(*b*) Solve the equation $5^{2x} - 12(5^x) + 35 = 0$.

(4)

June 2008

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5. (*a*) Given that $5 \sin \theta = 2 \cos \theta$, find the value of $\tan \theta$.

(1)

(*b*) Solve, for $0 \leq x < 360°$,

$$5 \sin 2x = 2 \cos 2x \text{ ,}$$

giving your answers to 1 decimal place.

(5)

June 2010

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6. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(*a*) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(*b*) Show that PQ is a diameter of C.

(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(*c*) Find the coordinates of R.

(4)

June 2009

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7. (*a*) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin (x + 45°) = 2.$$

(4)

(*b*) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7\cos x,$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

May 2011

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8. (*a*) Find the value of y such that

$$\log_2 y = -3.$$

(2)

(*b*) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x.$$

(5)

June 2009

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9. (*a*) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(*b*) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(*c*) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places.

(5)

May 2007

10.

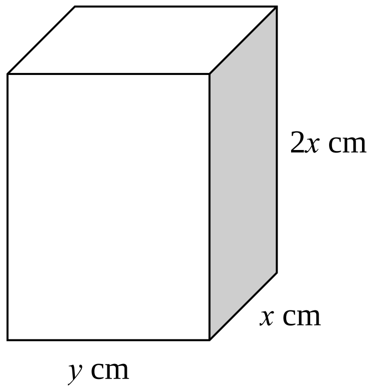


Figure 1

Figure 1 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(*a*) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(*b*) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(*c*) Justify that the value of V you have found is a maximum.

(2)

May 2007

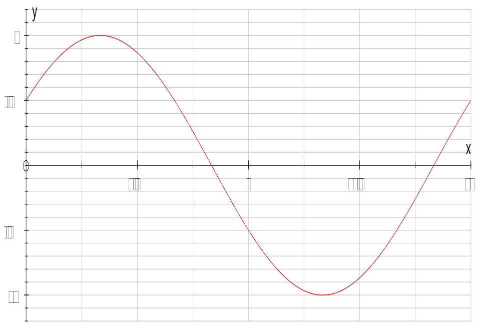
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\int x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (o.e.)	M1 A1
	$\left[\frac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8} - 2 = -2 + 4\sqrt{2}$	M1 A1
	[or $4\sqrt{2}-2$, or $2(2\sqrt{2}-1)$, or $2(-1+2\sqrt{2})$]	**[4]**
2. (a)	$(7 \times \ldots \times x)$ or $(21 \times \ldots \times x^2)$	M1
	$(2+kx)^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$]	B1; A1 A1 (4)
(b)	$6 \times 448k = 672k^2$	M1
	$k = 4$	A1 (2) **[6]**
3.	$\left(2-\frac{1}{2}x\right)^8 = 2^8 + \binom{8}{1}.2^7\left(-\frac{1}{2}x\right) + \binom{8}{2}2^6\left(-\frac{1}{2}x\right)^2 + \binom{8}{3}2^5\left(-\frac{1}{2}x\right)^3$	
	First term of 256	B1
	$\left({}^8C_1 \times \ldots \times x\right) + \left({}^8C_2 \times \ldots \times x^2\right) + \left({}^8C_3 \times \ldots \times x^3\right)$	M1
	$= (256) - 512x + 448x^2 - 224x^3$	A1 A1 **[4]**
4. (a)	$x = \frac{\log 7}{\log 5}$ or $x = \log_5 7$	M1
	1.21	A1 (2)
(b)	$(5^x - 7)(5^x - 5)$	M1 A1
	$(5^x = 7$ or $5^x = 5)$ $x = 1.2$ (awrt)	A1 ft
	$x = 1$	B1 (4) **[6]**

Question Number	Scheme	Marks
5. (a)	$\tan\theta = \frac{2}{5}$ (or 0.4)	B1
		(1)
(b)	awrt 21.8 (α)	B1
	$180+\alpha$ $(=201.8)$, or $90+(\alpha/2)$	M1
	$360+\alpha$ $(=381.8)$, or $180+(\alpha/2)$ **or** $540+\alpha$ $(=561.8)$, or $270+(\alpha/2)$	M1
	Dividing at least one of the angles by 2	M1
	$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt)	A1
		(5) **[6]**
6. (a)	$(x-3)^2-9+(y+2)^2-4=12$	M1 A1
	Centre is (3, −2)	A1
	$(x-3)^2+(y+2)^2=12+"9"+"4"$	M1
	$r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$ **)**	A1
		(5)
(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
	= 10 = $2\times$radius, $\therefore$diam.	A1
		(2)
(c)	*R* must lie on the circle (angle in a semicircle theorem)…	B1
	$x=0\Rightarrow\quad y^2+4y-12=0$	M1
	$(y-2)(y+6)=0\quad y=.....$	dM1
	$y=-6$ or 2	A1
		(4) **[11]**

Question Number	Scheme	Marks
7. (a)	$\sin(x+45^\circ)=\frac{2}{3}$, so $(x+45^\circ)=41.8103...$ $\quad(\alpha=41.8103...)$	M1
	So, $x+45^\circ=\{138.1897...,\ 401.8103...\}$	M1
	and $x=\{93.1897...,\ 356.8103...\}$	A1 A1
		(4)
(b)	$2(1-\cos^2 x)+2=7\cos x$	M1
	$2\cos^2 x+7\cos x-4=0$	A1 **oe**
	$(2\cos x-1)(\cos x+4)\ \{=0\}$, $\cos x=...$	M1
	$\cos x=\frac{1}{2}$, $\{\cos x=-4\}$	
	$\left(\beta=\frac{\pi}{3}\right)$	
	$x=\frac{\pi}{3}$ or $1.04719...^{c}$	B1
	$x=\frac{5\pi}{3}$ or $5.23598...^{c}$	B1 ft
		(6) **[10]**
8. (a)	$\log_2 y=-3 \Rightarrow y=2^{-3}$	M1
	$y=\frac{1}{8}$ or 0.125	A1
		(2)
(b)	$32=2^5$ or $16=2^4$ or $512=2^9$	M1
	[or $\log_2 32=5\log_2 2$ or $\log_2 16=4\log_2 2$ or $\log_2 512=9\log_2 2$]	
	[or $\log_2 32=\frac{\log_{10}32}{\log_{10}2}$ or $\log_2 16=\frac{\log_{10}16}{\log_{10}2}$ or $\log_2 512=\frac{\log_{10}512}{\log_{10}2}$]	
	$\log_2 32+\log_2 16=9$	A1
	$(\log x)^2=...$ or $(\log x)(\log x)=...$	M1
	$\log_2 x=3 \Rightarrow x=2^3=8$	A1
	$\log_2 x=-3 \Rightarrow x=2^{-3}=\frac{1}{8}$	A1ft
		(5) **[7]**

Question Number	Scheme	Marks
9. (a)	 Sine wave (anywhere) with at least 2 turning points.	M1
	Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis…	A1
		(2)
(b)	$\left(0, \frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, 0\right)$, $\left(\frac{11\pi}{6}, 0\right)$	B1 B1 B1
		(3)
(c)	awrt 0.71 radians (0.70758…), or awrt 40.5° (40.5416…) (α)	B1
	$(\pi - \alpha)$ (2.43…) or $(180 - \alpha)$	M1
	Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)… or subtract 30	M1
	0.18 (or 0.06π), 1.91 (or 0.61π)	A1 A1
		(5) **[10]**
10. (a)	$4x^2 + 6xy = 600$	M1 A1
	$V = 2x^2 y = 2x^2\left(\frac{600 - 4x^2}{6x}\right)$ $V = 200x - \frac{4x^3}{3}$ (*)	M1 A1cso
		(4)
(b)	$\frac{dV}{dx} = 200 - 4x^2$	B1
	Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$	M1 A1
	Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943 \text{ cm}^3$	M1 A1
		(5)
(c)	$\frac{d^2V}{dx^2} = -8x$ Negative $\therefore$ Maximum	M1 A1ft
		(2) **[11]**

Examiner reports

Question 1

Although many candidates scored full marks on this question, others had difficulty dealing with $\frac{1}{\sqrt{x}}$. This was sometimes misinterpreted as $x^{\frac{1}{2}}$ or x^{-1}, leading to incorrect integration but still allowing the possibility of scoring a method mark for use of limits. Sometimes there was no integration attempt at all and the limits were simply substituted into $\frac{1}{\sqrt{x}}$. The candidates who performed the integration correctly were usually able to deal with their surds and proceeded score full marks, although a few left their answer as $-2 + 2\sqrt{8}$. Occasionally the answer was given as a decimal.

Question 2

In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Those who used the $(a+b)^n$ formula were usually able to pick up accuracy marks but many of those who attempted to use the $(1+x)^n$ version made mistakes in simplifying terms, often taking out 2 as a factor rather than 2^7. Coefficients were generally found using ${}^nC_r$, but Pascal's triangle was also frequently seen. The simplified third term was often given as $672kx^2$ instead of $672k^2x^2$, but this mistake was much less common than in similar questions on previous C2 papers.

Part (b) was often completed successfully, but some candidates included powers of x in their 'coefficients'. Compared with recent papers, there seems to be some improvement in the understanding of the difference between 'coefficients' and 'terms'. Accuracy mistakes, including multiplying the wrong coefficient by 6, were common.

Question 3

This question was done well and the majority of candidates gained full marks. The method used was equally divided between candidates working with the expression given and those taking out a factor of 2 at the start. Although answers using the second approach were more likely to have errors, most candidates could work accurately with this method. The common error of applying the power of a bracket to the x but not to the $-\frac{1}{2}$ was only seen occasionally. The most common error seen was where candidates used $\frac{1}{2}x$ instead of $-\frac{1}{2}x$. Some candidates gave every term of the expansion, and not just the ones required by the question, which would cost them time in an examination.

Question 4

Most candidates completed part (a) successfully (sometimes by 'trial and error'), but sometimes a mark was lost through incorrectly rounding to 3 decimal places instead of 3 significant figures.

Responses to part (b) varied considerably. Many candidates failed to appreciate that $5^{(2x)}$ is equivalent to $(5^x)^2$ and either substituted the answer to part (a) into the given equation or took logs of each separate term, resulting in expressions such as $2x\log5 - x\log60 + \log 35 = 0$. The candidates who managed to form the correct quadratic in 5^x were usually able to proceed to a correct solution, but sometimes the final answers were left as 5 and 7. Notation was sometimes confusing, especially where the substitution $x = 5^x$ appeared.

Some candidates wasted a significant amount of time on part (b), producing a number of different wrong responses with a variety of logarithmic mistakes.

Question 5

While many struggled with this question, strong candidates often produced clear, concise, well-structured responses.

Finding the value of $\tan\theta$ in part (a) proved surprisingly difficult. The most common wrong answer was $\frac{5}{2}$ instead of $\frac{2}{5}$, but many candidates failed to obtain *any* explicit value of $\tan\theta$. Some, not recognising the link between the two parts of the question, failed in part (a) but went on to find a value of $\tan 2x$ in part (b) before solving the equation. Most candidates achieved an acute value for $2x$ and then used the correct method to find the second solution. At this stage some omitted to halve their angles and some did not continue to find the other two solutions in the given range. Alternative methods using double angle formulae were occasionally seen, but were rarely successful. Some candidates resorted to using interesting 'identities' such as $\cos 2x = 1 - \sin 2x$.

Question 6

Many candidates had difficulty with this question, with part (c) being particularly badly answered.

In part (a) the method of completing the square was the most popular approach, but poor algebra was often seen, leading to many incorrect answers. Although the correct centre coordinates $(3, -2)$ were often achieved (not always very convincingly), the radius caused rather more problems and answers such as $\sqrt{12}$ appeared frequently. Some candidates inappropriately used the information about the diameter in part (b) to find their answers for part (a), scoring no marks.

There were various possible methods for part (b), the most popular of which were either to show that the mid-point of *PQ* was the centre of the circle or to show that the length of *PQ* was twice the radius of the circle. Provided that either the centre or the radius was correct in part (a), candidates therefore had at least two possible routes to success in part (b), and many scored both marks here. Some, however, thought that it was sufficient to show that both *P* and *Q* were on the circle.

Part (c) could have been done by using the fact that the point *R* was on the circle (angle in a semicircle result), or by consideration of gradients, or by use of Pythagoras' Theorem. A common mistake in the 'gradient' method was to consider the gradient of *PQ*, which was not directly relevant to the required solution.

Many candidates were unable to make any progress in part (c), perhaps omitting it completely, and time was often wasted in pursuing completely wrong methods such as finding an equation of a line perpendicular to *PQ* (presumably thinking of questions involving the tangent to a circle).

Question 7

Candidates were generally more successful in answering part (b) than part (a), but it was felt that a significant number of candidates were unsure about solving trigonometric equations and would have benefited from a more methodical approach of either using a CAST diagram technique or solution curve technique. Although part (a) required answers in degrees and part (b) required answers in radians; this did not appear to be a problem for the majority of candidates.

In part (a), a significant number of candidates did not know how to deal with the 45 in $\sin(x + 45°)$ or with the order of operations to use. A fair number thought that $3\sin(x + 45°)$ simplified to $3\sin x + 3\sin 45°$. Of those who correctly found $\sin^{-1}\left(\frac{2}{3}\right)$, many were then unsure about how to proceed, with some candidates believing that 41.8° was one of the values of x, whilst others subtracted 45 from this answer to achieve –3.2° and at this point could not progress any further. Work to find solutions inside the required range was often muddled, although some candidates were able to find one of the two solutions required for x. Only a minority of candidates were able to find both solutions correctly, but a number of these candidates were penalised 1 mark by offering at least one extra solution in the required range.

In part (b), many candidates were able to correctly substitute $1 - \cos^2 x$ for $\sin^2 x$, and manipulate their resulting equation to find a correct quadratic equation in $\cos x$, with a few candidates either making sign or bracketing errors. It was disappointing, however, to see a fair number of candidates who thought that $\cos x$ could be replaced by $1 - \sin x$ in the initial equation and then went on to attempt to solve a quadratic equation in $\sin x$. Although the majority were able to factorise $2\cos^2 x + 7\cos x - 4$ to give $(2\cos x - 1)(\cos x + 4)$, a minority incorrectly factorised to give $(2\cos x + 1)(\cos x - 4)$. Many candidates went on to solve $\cos x = \frac{1}{2}$ to give $x = \frac{\pi}{3}$, but the second solution sometimes ignored or incorrectly found. A minority of candidates worked in degrees, but most gave their answers in radians in terms of π.

Question 8

In part (a), the majority of candidates showed an understanding of the definition of a logarithm, although the answers –8 and 8 appeared occasionally instead of $\frac{1}{8}$.

Part (b), however, was often badly done. Not realising that $\log_2 32$ and $\log_2 16$ could be immediately written as 5 and 4 respectively, most candidates launched unnecessarily into laws of logarithms, replacing the numerator by $\log_2 512$ (or sometimes by $\log_2 48$). Although it would have been quite possible to proceed from here to correct answers, confusion often followed at the next stage, with $(\log_2 x) \times (\log_2 x)$ becoming $\log_2 x^2$ and then, perhaps, $2\log_2 x$. Other unfortunate mistakes included writing $\frac{\log 512}{\log x}$ as either $\log_2(512 - x)$ or $\frac{512}{x}$. Those candidates who managed to reach $(\log_2 x)^2 = 9$ were usually able to find one correct answer from $\log_2 x = 3$, but the second answer (from $\log_2 x = -3$) appeared much less frequently.

Question 9

Sketches of the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$ in part (a) were generally disappointing. Although most candidates were awarded a generous method mark for the shape of their graph, many lost the accuracy mark, which required a good sketch for the full domain with features such as turning points, scale and intersections with the axes 'in the right place'. In part (b), the exact coordinates of the points of intersection with the axes were required. Many candidates were clearly uncomfortable working in radians and lost marks through giving their *x* values in degrees, and those who did use radians sometimes gave rounded decimals instead of exact values. The intersection point (0, 0.5) was often omitted.

Part (c) solutions varied considerably in standard from the fully correct to those that began with $\sin\left(x + \frac{\pi}{6}\right) = \sin x + \sin\frac{\pi}{6} = 0.65$. The most common mistakes were: failing to include the 'second solution', subtracting from π <u>after</u> subtracting $\frac{\pi}{6}$, leaving answers in degrees instead of radians, mixing degrees and radians, and approximating prematurely so that the final answers were insufficiently accurate.

Question 10

Responses to this question that were blank or lacking in substance suggested that some candidates were short of time at the end of the examination. Although many good solutions were seen, it was common for part (b) to be incomplete.

The algebra in part (a) was challenging for many candidates, some of whom had difficulty in writing down an expression for the total surface area of the brick and others who were unable to combine this appropriately with the volume formula. It was common to see several attempts at part (a) with much algebraic confusion.

Working with the given formula, most candidates were able to score the first three marks in part (b), but surprisingly many, having found $x \approx 7.1$, seemed to think that this represented the maximum value of V. Failing to substitute the value of x back into the volume formula lost them two marks.

Almost all candidates used the second derivative method, usually successfully, to justify the maximum value in part (c), but conclusions with a valid reason were sometimes lacking.

Statistics for C2 Practice Paper Gold Level G3

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		68	2.70		3.64	3.25	2.87	2.52	2.00	0.97
2	6		68	4.05		5.54	4.86	4.23	3.54	2.77	1.48
3	4		90	3.60	3.99	3.92	3.66	3.64	3.25	3.18	2.25
4	6		55	3.28		4.83	3.63	3.02	2.48	2.05	1.33
5	6		46	2.77	5.84	4.87	3.67	2.72	1.83	1.07	0.33
6	11		46	5.11		8.71	6.39	4.80	3.28	2.03	0.66
7	10		56	5.61	9.79	9.00	7.40	5.72	3.89	2.26	0.64
8	7		44	3.11		4.64	3.32	2.83	2.47	2.09	1.22
9	10		46	4.57		7.93	5.73	4.14	2.91	1.84	0.78
10	11		48	5.24		9.41	6.72	4.55	2.81	1.58	0.52
	75		**53**	**40.04**		**62.49**	**48.63**	**38.52**	**28.98**	**20.87**	**10.18**