Paper Reference(s)

## 6664/01

## Edexcel GCE

## Core Mathematics C2

Silver Level S3

## Time: 1 hour 30 minutes

## Materials required for examination papers <br> Mathematical Formulae (Green) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 11 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 63 | 56 | 48 | 41 | 34 |

1. 

$$
\mathrm{f}(x)=2 x^{3}-7 x^{2}-5 x+4
$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x-1)$.
(b) Use the factor theorem to show that $(x+1)$ is a factor of $\mathrm{f}(x)$.
(c) Factorise $\mathrm{f}(x)$ completely.
2. In the triangle $A B C, A B=11 \mathrm{~cm}, B C=7 \mathrm{~cm}$ and $C A=8 \mathrm{~cm}$.
(a) Find the size of angle $C$, giving your answer in radians to 3 significant figures.
(b) Find the area of triangle $A B C$, giving your answer in $\mathrm{cm}^{2}$ to 3 significant figures.

January 2011
3. (a) Find the first 4 terms, in ascending powers of $x$, of the binomial expansion of $(1+a x)^{10}$, where $a$ is a non-zero constant. Give each term in its simplest form.

Given that, in this expansion, the coefficient of $x^{3}$ is double the coefficient of $x^{2}$,
(b) find the value of $a$.
4.


Figure 1
Figure 1 shows the triangle $A B C$, with $A B=6 \mathrm{~cm}, B C=4 \mathrm{~cm}$ and $C A=5 \mathrm{~cm}$.
(a) Show that $\cos A=\frac{3}{4}$.
(b) Hence, or otherwise, find the exact value of $\sin A$.
5. The first three terms of a geometric series are $4 p,(3 p+15)$ and $(5 p+20)$ respectively, where $p$ is a positive constant.
(a) Show that $11 p^{2}-10 p-225=0$.
(b) Hence show that $p=5$.
(c) Find the common ratio of this series.
(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.
6.


Figure 2
Figure 2 shows the sector $O A B$ of a circle with centre $O$, radius 9 cm and angle 0.7 radians.
(a) Find the length of the arc $A B$.
(b) Find the area of the sector $O A B$.

The line $A C$ shown in Figure 2 is perpendicular to $O A$, and $O B C$ is a straight line.
(c) Find the length of $A C$, giving your answer to 2 decimal places.

The region $H$ is bounded by the arc $A B$ and the lines $A C$ and $C B$.
(d) Find the area of $H$, giving your answer to 2 decimal places.
7.


## Figure 3

The triangle $X Y Z$ in Figure 3 has $X Y=6 \mathrm{~cm}, Y Z=9 \mathrm{~cm}, Z X=4 \mathrm{~cm}$ and angle $Z X Y=\alpha$. The point $W$ lies on the line $X Y$.

The circular arc $Z W$, in Figure 3 is a major arc of the circle with centre $X$ and radius 4 cm .
(a) Show that, to 3 significant figures, $\alpha=2.22$ radians.
(b) Find the area, in $\mathrm{cm}^{2}$, of the major sector $X Z W X$.
(3)

The region enclosed by the major arc $Z W$ of the circle and the lines $W Y$ and $Y Z$ is shown shaded in Figure 3.

Calculate
(c) the area of this shaded region,
(d) the perimeter $Z W Y Z$ of this shaded region.
8. A trading company made a profit of $£ 50000$ in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio $r, r>1$.

The model therefore predicts that in 2007 (Year 2) a profit of $£ 50000 r$ will be made.
(a) Write down an expression for the predicted profit in Year $n$.

The model predicts that in Year $n$, the profit made will exceed $£ 200000$.
(b) Show that $n>\frac{\log 4}{\log r}+1$.
(3)

Using the model with $r=1.09$,
(c) find the year in which the profit made will first exceed $£ 200000$,
(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest $£ 10000$.

May 2007
9. The curve with equation

$$
y=x^{2}-32 \sqrt{ } x+20, \quad x>0
$$

has a stationary point $P$.
Use calculus
(a) to find the coordinates of $P$,
(b) to determine the nature of the stationary point $P$.

## END

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1. (a) | $\mathrm{f}(\mathrm{x})=2 \mathrm{x}^{3}-7 x^{2}-5 x+4$ |  |
|  | Remainder $=\mathrm{f}(1)=2-7-5+4=-6$ | M1 |
|  | $=-6$ | A1 |
|  |  | (2) |
| (b) | $f(-1)=2(-1)^{3}-7(-1)^{2}-5(-1)+4$ | M1 |
|  | and so ( $x+1$ ) is a factor. | A1 |
|  |  | (2) |
| (c) | $\mathrm{f}(\mathrm{x})=\{(x+1)\}\left(2 x^{2}-9 x+4\right)$ | M1 A1 |
|  | $=(x+1)(2 x-1)(x-4)$ | dM1 A1 |
|  |  | (4) <br> [8] |
| 2. (a) | $11^{2}=8^{2}+7^{2}-(2 \times 8 \times 7 \cos C)$ | M1 |
|  | $\begin{equation*} \cos C=\frac{8^{2}+7^{2}-11^{2}}{2 \times 8 \times 7} \tag{oe} \end{equation*}$ | A1 |
|  | $\{\hat{C}=1.64228 . ..\} \Rightarrow \hat{C}=$ awrt 1.64 | A1 cso |
|  |  | (3) |
| (b) | Use of Area $\triangle A B C=\frac{1}{2} a b \sin ($ their $C)$ | M1 |
|  | $=\frac{1}{2}(7 \times 8) \sin C$ | A1 ft |
|  | $\{=27.92848 \ldots$ or $27.93297 \ldots\}=$ awrt 27.9 | A1 cso <br> (3) |
|  |  | [6] |
| 3. (a) | $(1+a x)^{10}=1+10 a x \ldots \ldots$ | B1 |
|  | $+\frac{10 \times 9}{2}(a x)^{2}+\frac{10 \times 9 \times 8}{6}(a x)^{3}$ | M1 |
|  | $+45(a x)^{2},+120(a x)^{3}$ or $+45 a^{2} x^{2},+120 a^{3} x^{3}$ | A1A1 |
|  |  | (4) |
| (b) | $120 a^{3}=2 \times 45 a^{2} \quad a=\frac{3}{4} \text { or equiv. }\left(\text { e.g. } \frac{90}{120}, 0.75\right)$ | M1A1 |
|  |  | (2) [6] |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. (a) | $4^{2}=5^{2}+6^{2}-(2 \times 5 \times 6 \cos \theta)$ | M1 |
|  | $\cos \theta=\frac{5^{2}+6^{2}-4^{2}}{2 \times 5 \times 6}$ | A1 |
|  | $\left(=\frac{45}{60}\right)=\frac{3}{4} \quad(*)$ | A1cso |
|  |  | (3) |
|  | $\sin ^{2} A+\left(\frac{3}{4}\right)^{2}=1$ <br> (o.e. Pythag. method) | M1 |
|  | $\left(\sin ^{2} A=\frac{7}{16}\right) \quad \sin A=\frac{1}{4} \sqrt{ } 7$ <br> (o.e.) | A1 |
|  |  | (2) [5] |
| 5. (a) | $a=4 p, a r=(3 p+15)$ and $a r^{2}=5 p+20$ | B1 |
|  | $($ So $r=) \quad \frac{5 p+20}{3 p+15}=\frac{3 p+15}{4 p}$ or $4 p(5 p+20)=(3 p+15)^{2}$ oe | M1 |
|  | See $(3 p+15)^{2}=9 p^{2}+90 p+225$ | M1 |
|  | $20 p^{2}+80 p=9 p^{2}+90 p+225 \rightarrow 11 p^{2}-10 p-225=0 \quad *$ | A1* |
| (b) |  | (4) |
|  | $(p-5)(11 p+45)$ so $p=$ | M1 |
|  | $p=5$ only ( after rejecting -45/11) | A1 |
| (c) |  | (2) |
|  | $\frac{3 \times 5+15}{4 \times 5} \text { or } \frac{5 \times 5+20}{3 \times 5+15}$ | M1 |
|  | $r=\frac{3}{2}$ | A1 |
| (d) | $\begin{aligned} & S_{10}=\frac{20\left(1-\left(" \frac{3}{2} "\right)^{10}\right)}{\left(1-" \frac{3}{2} "\right)} \\ & (=2266.601568 \ldots)=2267 \end{aligned}$ | M1A1ft |
|  |  |  |
|  |  | A1 |
|  |  | $\begin{array}{r} (3) \\ {[11]} \end{array}$ |



| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| 8. (a) | $50000 r^{n-1} \quad \text { (o.e.) }$ | B1 <br> (1) |
| (b) | $50000 r^{n-1}>200000$ | M1 |
|  | $r^{n-1}>4 \quad \Rightarrow \quad(n-1) \log r>\log 4$ | M1 |
|  | $\begin{equation*} n>\frac{\log 4}{\log r}+1 \tag{*} \end{equation*}$ | A1cso |
| (c) | $r=1.09: n>\frac{\log 4}{\log 1.09}+1 \text { or } n-1>\frac{\log 4}{\log 1.09} \quad(n>17.086 \ldots)$ | (3) |
| (d) | Year 18 or 2023 | A1 <br> (2) |
|  | $S_{n}=\frac{a\left(1-r^{n}\right)}{1-r}=\frac{50000\left(1-1.09^{10}\right)}{1-1.09}$ | M1 A1 |
|  | £760 000 | A1 |
|  |  | $\begin{aligned} & \text { (3) } \\ & \text { [9] } \end{aligned}$ |
| 9. (a) | $\left\{\frac{\mathrm{d} y}{\mathrm{~d} x}=\right\} 2 x-16 x^{-\frac{1}{2}}$ | M1 A1 |
|  | $2 x-16 x^{-\frac{1}{2}}=0 \Rightarrow x^{\frac{3}{2}}=, x^{-\frac{3}{2}}=\text {, or } 2 x-=16 x^{-\frac{1}{2}}$ <br> then squared then obtain $x^{3}=$ | M1 |
|  | $\left(x^{\frac{3}{2}}=8 \Rightarrow\right) x=4$ | A1 |
|  | $x=4, y=4^{2}-32 \sqrt{4}+20=-28$ | M1 A1 <br> (6) |
| (b) | $\left\{\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=\right\} 2+8 x^{-\frac{3}{2}}$ | M1 A1 |
|  | $\left(\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}>0 \Rightarrow\right) y$ is a minimum | A1 |
|  |  | (3) <br> [9] |

## Examiner reports

## Question 1

Most candidates attempted this question and many achieved full marks. In part (a), a significant number used long division in order to find the remainder, many successfully but others making sign errors. Those that used the remainder theorem and found $f(1)$ almost always gained full marks.
In part (b), a significant number of candidates gained only one mark as they were able to show that $\mathrm{f}(-1)=0$ successfully but then did not make any comment to the effect that $(x+1)$ was then a factor. Others clearly did not know what was meant by the factor theorem and used long division for which they did not gain any marks.
Part (c) was completed successfully by many candidates. The majority found the quadratic factor by long division rather than inspection of coefficients. Some of those candidates who used a method of long division on occasion arrived at the incorrect quadratic factor because of sign errors. Nearly all candidates who arrived at the correct quadratic factor were then able to factorise it correctly. A number of candidates did not obtain the final mark as they did not write all 3 factors together on one line at the end of their solution.

## Question 2

This question was well answered with a considerable number of candidates gaining full marks. It was rare to see a solution assuming that the triangle was right-angled, although there were a few candidates who did not proceed beyond using right-angled trigonometric ratios.
In part (a), the majority of candidates were able to correctly state or apply the correct cosine rule formula. In rearranging to make $\cos C$ the subject a significant minority of candidates incorrectly deduced that $\cos C=\frac{1}{14}$. A negative sign leading to an obtuse angle appeared to upset these candidates. The more usual error, however, was to use the formula to calculate one of the other two angles. This was often in spite of a diagram with correctly assigned values being drawn by candidates, thus indicating a lack of understanding of how the labelling of edges and angles on a diagram relates to the application of the cosine rule formula. Although the question clearly stated that the answer should be given in radians, it was not unusual to see an otherwise completely correct solution losing just one mark due to candidates giving the answer to part (a) in degrees. It was also fairly common to see evidence of candidates preferring to have their calculator mode in degrees, by evaluating their answer in degrees and then converting their answer to radians.
Part (b) was a good source of marks, with most candidates showing competence in using $\frac{1}{2} a b \sin C$ correctly. Of those candidates who "really" found angle $A$ or $B$ in part (a), most assumed it was angle $C$ and applied $\frac{1}{2}(7)(8) \sin ($ their $C$ ), thus gaining 2 out of the possible 3 marks available. A few candidates correctly found the height of the triangle and applied $\frac{1}{2}$ (base)(height) to give the correct answer.

## Question 3

In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Coefficients were generally found using ${ }^{n} C_{r}$, but Pascal's triangle was also frequently seen. The most common mistake was to omit the powers of $a$, either completely or perhaps in just the simplified version of the answer.
Part (b) was often completed successfully, but a significant number of candidates included powers of $x$ in their 'coefficients', resulting in some very confused algebra and indicating misunderstanding of the difference between 'coefficients' and 'terms'. Sometimes the wrong coefficient was doubled and sometimes the coefficients were equated with no doubling. Some candidates, having lost marks in part (a) due to the omission of powers of $a$, recovered in part (b) and achieved the correct answer.

## Question 4

The typical response to this question scored full marks in part (a) and no marks in part (b). In part (a) the cosine rule was well known and most candidates managed to manipulate convincingly to achieve the correct (given) value of $\cos A$. A few experienced difficulty in making $\cos A$ the subject of their equation, and $61-60 \cos A$ occasionally became $1 \cos A$, but otherwise mistakes were uncommon. In part (b), however, the majority of candidates ignored the requirement for an exact value of $\sin A$. The most popular approach was to simply use a calculator to find $A$ and $\sin A(\approx 0.66)$. A significant number of candidates, having used the cosine rule in part (a), thought that they ought to use the sine rule in part (b) and invariably made no effective progress. Others seemed to assume that the triangle was right-angled. It was pleasing to see good candidates producing correct, concise solutions via $\sin ^{2} A+\cos ^{2} A=1$ or equivalent methods. The identity $\sin A+\cos A=1$ made the occasional appearance.

## Question 5

Part (a) caused the greatest variety of responses. The most common correct approach was to write the terms as ratios of each other (as in the second line of the mark scheme).This mostly led to the correct answer, with any marks lost being due to slips rather than to errors in the method. Another approach was a multi layered substitution, by squaring the middle term and dividing by the first term and then putting that equal to the third term leading to
$4 p\left(\frac{3 p+15}{4 p}\right)^{2}=5 p+20$ which then required more careful algebraic work. The geometric mean method was rarely seen.

Some students also chose to take out the 3 as a common factor on the middle term and the 5 as a common factor of the 3rd term and then manipulated as above. A sizeable minority attempted it incorrectly and then concluded with the final statement and 'hence proved', perhaps hoping that their errors would not be noticed.
In part (b) most were able to solve the quadratic by factorisation and quite a lot by formula, but many lost the second mark for not rejecting the second solution clearly. This was a printed answer.
A small number did it by verification and gained one of the two marks, as they had not shown that 5 was the only value which $p$ could take.
There were no difficulties finding the common ratio in part (c) and it was rare to see the value given as $\frac{2}{3}$ instead of $\frac{3}{2}$ (usually a common error)

The formula for the sum of a geometric series was well applied in part (d) and usually gave the correct answer. A few used $n=9$ or $n=20$ or put $a=5$ leading to errors and some did not give their answer to the nearest integer.

## Question 6

In parts (a) and (b) of this question, most candidates were able to quote and accurately use the formulae for length of an arc and area of a sector. Wrong formulae including $\pi$ were occasionally seen and it was sometimes felt necessary to convert 0.7 radians into degrees.
Despite the right-angled triangle, a very popular method in part (c) was to find the angle at $C$ and use the sine rule. For the angle at $C$, many candidates used 0.87 radians (or a similarly rounded version in degrees) rather than a more accurate value. This premature approximation resulted in an answer for $A C$ that was not correct to 2 decimal places, so the accuracy mark was lost.
In part (d), although a few candidates thought the region $H$ was a segment, most were able to make a fair attempt to find the required area. There was again an unwillingness to use the fact that triangle $O A C$ was right-angled, so that $\frac{1}{2} a b \sin C$ appeared frequently. Unnecessary calculations (such as the length of $O C$ ) were common and again premature approximation often led to the loss of the accuracy mark.

## Question 7

In part (a) the majority of candidates could establish the printed angle by using the cosine rule. Some candidates chose to verify that the angle was 2.22 radians by again using the cosine rule to show that $Z Y$ was 9 cm . A small number of candidates worked in degrees and converted to radians at the end.
Part (b) involved finding the area of the major sector $X Z W X$ but many candidates found the area of the minor sector. As an alternative correct method some candidates found the area of the minor sector and subtracted this from the area of the circle. Some candidates found the area of triangle $Z X Y$ and a minority of candidates made some attempt at the area of a segment.

In part (c), candidates recognised they needed to find the area of triangle $Z X Y$ and add the area from part (b). It was clear here that those with an incorrect part (b) did not understand the expression 'major sector' as they were able to score all the marks in part (c).
Part (d) was met with more success although a common error was to add 11 to the minor arc length. Some candidates misinterpreted the perimeter and as a final step, added an attempt at the length $Z W$.

## Question 8

Responses to this question were very mixed, with many candidates scoring marks in only one or two parts and with much misunderstanding of logarithms.
In part (a), most managed to write down $50000 r^{n-1}$ as the predicted profit in Year $n$, although $50000 r^{n}$ was a popular alternative. In part (b), showing the result $n>\frac{\log 4}{\log r}+1$ proved
difficult for the average candidate. Sometimes this was simply not attempted, sometimes candidates tried to 'work backwards' and sometimes there were mistakes in logarithmic theory such as $50000 r^{n-1}>200000 \Rightarrow(n-1) \log 50000 r>\log 200000$.

Disappointingly, many candidates failed to use the given result from part (b) in their solutions to part (c). Some worked through the method of part (b) again (perhaps successfully) but others used the sum formula for the geometric series, scoring no marks. Even those who correctly achieved $n>17.08 \ldots$ tended to give the answer as 'Year 17' or '2022' instead of 'Year 18' or 2023.
After frequent failure in parts (b) and (c), many candidates recovered to score two or three marks in part (d), where they had to use the sum formula for the geometric series. Occasionally here the wrong value of $n$ was used, but more often a mark was lost through failure to round the final answer to the nearest $£ 10000$.

## Question 9

This fairly standard turning point question saw a large number of excellent solutions, and was more accessible to weaker candidates than in some previous years, although the fractional powers caused difficulties for a significant number of candidates.
Although in most cases a correct first derivative was found in part (a), many candidates struggled to find a solution to $2 x-\frac{16}{\sqrt{x}}=0$. Some candidates spotted that $x=4$ is a solution, whilst some of those who saw how to solve the equation and achieved the stage $x^{\frac{3}{2}}=8$ still had issues, with many reaching the result of $16 \sqrt{ }$ 2, clearly having evaluated $8^{\frac{3}{2}}$. Candidates who correctly squared their equation to give $4 x^{2}=\frac{256}{x}$, as opposed to the occasionally seen $4 x^{2}+\frac{256}{x}=0$, were often more successful in finding $x=4$. Providing the $x$-coordinate found was positive, there was a method mark available for finding $y$, but often this was not attempted or, less frequently, lost because $x$ was substituted in the expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$. Other poor attempts saw the use of a second derivative equated to zero which led to a forfeit of the final method mark for finding a $y$ value using an $x$ value resulting from this incorrect process.
In part (b) many candidates were able to correctly differentiate their first derivative, with very few using the alternative gradient method. However, there were some common sign slips with the second term. Incorrect statements were seen such as ' $x>0$ so minimum' or use of $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=0$ leading to an alternative value of $x$ which was then used to determine the nature of the turning point. Others listed all possible outcomes for the second derivative (>0 so minimum, $<0$ so maximum, etc) but failed to identify whether the point $P$ was in fact a maximum or minimum.

## Statistics for C2 Practice Paper Silver Level S3

Mean score for students achieving grade:

| Qu | Max <br> score | Modal <br> score | Mean <br> \% | ALL | A* $^{\text {* }}$ | A | B | C | D | E | $\mathbf{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{8}$ |  | 80 | 6.40 | 7.77 | 7.56 | 7.26 | 6.88 | 6.33 | 5.52 | 3.34 |
| $\mathbf{2}$ | 6 |  | 77 | 4.64 | 5.81 | 5.65 | 5.09 | 4.53 | 3.72 | 2.86 | 1.98 |
| $\mathbf{3}$ | 6 |  | 69 | 4.11 |  | 5.72 | 5.06 | 4.28 | 3.41 | 2.54 | 1.19 |
| $\mathbf{4}$ | 5 |  | 61 | 3.07 |  | 4.10 | 3.45 | 3.14 | 2.77 | 2.39 | 1.43 |
| $\mathbf{5}$ | 11 |  | 86 | 9.44 | 10.89 | 10.64 | 9.91 | 8.71 | 8.08 | 7.40 | 3.39 |
| $\mathbf{6}$ | 9 |  | 66 | 5.95 | 8.64 | 8.13 | 7.18 | 6.33 | 5.23 | 3.99 | 1.89 |
| $\mathbf{7}$ | 12 |  | 60 | 7.20 | 11.04 | 10.22 | 8.15 | 6.20 | 4.69 | 3.25 | 1.77 |
| $\mathbf{8}$ | 9 |  | 58 | 5.18 |  | 7.75 | 6.29 | 5.13 | 3.98 | 2.97 | 1.51 |
| $\mathbf{9}$ | 9 |  | 60 | 5.38 | 8.72 | 8.18 | 7.05 | 5.75 | 4.38 | 3.12 | 1.30 |
|  | $\mathbf{7 5}$ |  | $\mathbf{6 8}$ | $\mathbf{5 1 . 3 7}$ |  | $\mathbf{6 7 . 9 5}$ | $\mathbf{5 9 . 4 4}$ | $\mathbf{5 0 . 9 5}$ | $\mathbf{4 2 . 5 9}$ | $\mathbf{3 4 . 0 4}$ | $\mathbf{1 7 . 8 0}$ |

