mark scheme

Practice Paper A : Further Pure 1

Question Number	General Scheme		Marks
1 (a)	$(3r-1)^2=9r^2-6r+1$	**B1:** correct expansion	**B1**
	$\sum_{r=1}^{n}(3r-1)^2 = 9\sum_{r=1}^{n}r^2-6\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$	**M1:** correct partitioning	**M1**
	$=\frac{9n}{6}(n+1)(2n+1)-\frac{6n}{2}(n+1)+n$ $=\frac{3n}{2}(n+1)(2n+1)-3n(n+1)+n$	**M1:** correct use of standard formula, including the $(+)\ n$	**M1**
	$=\frac{n}{2}\left[3(n+1)(2n+1)-6(n+1)+2\right]$ $=\frac{n}{2}\left[3\left(2n^2+3n+1\right)-6n-6+2\right]$ $=\frac{n}{2}\left(6n^2+3n-1\right)$	**M1:** factorising of $\frac{n}{2}$ at any stage **A1:** cao	**M1** **A1** **(6)**
(b)	$2^2+5^2+8^2+11^2+...+149^2=\sum_{r=1}^{50}(3r-1)^2$	**B1:** correct limits seen or implied through a correct substitution	**B1**
	$\frac{50}{2}\left[6(50)^2+3(50)-1\right]=378725$	**A1:** cao	**A1** **(2)**
	Total		**8**

2 (a)	$f(-2)=(-2)^3-4(-2)+2=2$ $f(-2.5)=(-2.5)^3-4(-2.5)+2=-3.6(25)$ $f(-2.25)=(-2.25)^3-4(2.25)+2=-0.39(0625)$	**M1:** an attempt to evaluate $f(-2)$, $f(-2.5)$ and $f(-2.25)$. At least **one** must be correct for this mark.	**M1**
	$f(-2.125)=(-2.125)^3-4(-2.125)+2$ $=0.90(429...)$	**dM1:** an attempt to evaluate the midpoint of **their** interval from the first iteration	**M1**
	Since there is a <u>change of sign</u> across the interval $\left[-2.25,-2.125\right]$, α must lie within this interval.	**A1:** cao, including a justification that mentions the idea of a change of sign. **Use of a table with correct working scores 3/3**	**A1** **(3)**
(b)	$f(0)=2$, $f(1)=-1$	**B1:** $f(0)$ and $f(1)$ correctly evaluated	**B1**
	$\dfrac{1-\beta}{1}=\dfrac{\beta-0}{2}$ $2-2\beta=\beta \rightarrow 3\beta=2$	**M1:** a correct method using linear interpolation to find β.	**M1**
	$\beta=\dfrac{2}{3}=0.667$	**A1:** correct value of β **to three significant figures**	**A1** **(3)**
		Total	**6**

3 (a)	Let $f(x)=2x^4-14x^3+51x^2-98x+85$ If $2-\mathrm{i}$ is a root of $f(x)$, then $2+\mathrm{i}$ is also a root	**B1:** identifies $2+\mathrm{i}$ is also a root at any stage	**B1**
	$\therefore (x-2)^2=-1$ $\therefore x^2-4x+5$ is a factor of $f(x)$ $\begin{array}{r} 2x^2-6x+17 \\ x^2-4x+5\overline{)2x^4-14x^3+51x^2-98x+85} \\ \underline{2x^4-8x^3+10x^2} \\ -6x^3+41x^2-98x \\ \underline{-6x^3+24x^2-30x} \\ 17x^2-68x+85 \\ \underline{17x^2-68x+85} \\ 0 \end{array}$	**M1:** correct attempt to work out a factor of $f(x)$ **dM1:** use of algebraic division (or any alternate method, i.e. inspection) to find the other factor	**M1** **M1**
	$\therefore f(x)=\left(x^2-4x+5\right)\left(2x^2-6x+17\right)$ Other roots given when, $2x^2-6x+17=0$ $x^2-3x+\frac{17}{2}=0 \rightarrow \left(x-\frac{3}{2}\right)^2=\frac{9}{4}-\frac{17}{2}$ $\left(\therefore x=\frac{3}{2}\pm\frac{5}{2}\mathrm{i}\right)$	**ddM1:** correct method to solve 3TQ	**M1**
	$\therefore\ x=\frac{3}{2}\pm\frac{5}{2}\mathrm{i},\ x=2\pm\mathrm{i}$	**A1:** correct roots worked out and stated **at any stage of the working**	**A1** **(5)**
Note	**Do not penalise candidates who do not define $f(x)$ or use a less substantiated method – provided enough is present for the award of the method marks, full marks should be awarded. Answer only scores 0/0.**		
(b)	**B3:** all roots correctly plotted (deduct one mark for every incorrect plot)		**B3**

4 (a)	$\det\mathbf{O} = ad - bc$ $\therefore \det\mathbf{O} = 3 - 2 = 1$	**M1:** correct substitution of elements of **O** into the formula **A1:** cao	**M1** **A1** **(2)**
(b)	$\mathbf{N} = \mathbf{M}^{-1} \times \det\mathbf{M} \times \det\mathbf{O} \times \mathbf{O}$ $\mathbf{N} = \mathbf{M}^{-1} \times \det\mathbf{M} \times \det\mathbf{O} \times \mathbf{O}$	**M1:** correct application of inverse matrices to find **N**	**M1**
	$\mathbf{M}^{-1} = \frac{1}{15-0}\begin{pmatrix} 3 & 2 \\ 0 & 5 \end{pmatrix}$ $\therefore \mathbf{N} = \frac{\cancel{1}}{\cancel{15}} \times \begin{pmatrix} 3 & 2 \\ 0 & 5 \end{pmatrix} \times \cancel{\det\mathbf{M}} \times \det\mathbf{O} \times \mathbf{O}$ $\therefore \mathbf{N} = \begin{pmatrix} 3 & 2 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ $\therefore \mathbf{N} = \begin{pmatrix} 11 & 6 \\ 5 & 0 \end{pmatrix}$	**M1:** an attempt to find $\mathbf{M}^{-1}$. **See the note about the award of this mark** **A1:** correct matrix for **N**	**M1** **A1**
	$\therefore \begin{pmatrix} a+6 & b-a \\ a & b-2a-1 \end{pmatrix} = \begin{pmatrix} 11 & 6 \\ 5 & 0 \end{pmatrix}$ $a = 5$, $b = 11$	**M1:** correct comparison of elements **and** suitable verifications using substitutions **A1:** cao	**M1** **A1** **(5)**
Note	**Some candidates may realise from an early stage that the $\det\mathbf{M}$ from the inverse matrix cancels directly with the $\det\mathbf{M}$ from the question. Hence do not penalise if $\mathbf{M}^{-1}$ is not explicitly stated or is stated without the $\frac{1}{\det\mathbf{M}}$.**		
	Total		7
ALT	An alternative method for (b) would be as follows: $\begin{pmatrix} 5 & -2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} a+6 & b-a \\ a & b-2a-1 \end{pmatrix} = (15)(1)\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ **M1**		

	$\begin{pmatrix} 3a+30 & 3b-a+2 \\ 3a & 3b-6a-3 \end{pmatrix} = 15\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ **M1 A1** $\therefore 3a = 15 \rightarrow a = 5,\ 3b-5+2 = 30 \rightarrow b = 11$ **M1 A1**	

5 (a)	$x = \dfrac{-11 \pm \sqrt{11^2 - 4(5)(-17)}}{2(5)}$ $\alpha = 1.047091,\ \beta = -3.247091$	**M1:** correct method to solve 3TQ **A1:** α and β correctly evaluated **to six decimal places**	**M1** **A1** **(2)**
(b)	$f'(x) = 10x + 11$ $f(1.25) = 5(1.25)^2 + 11(1.25) - 17 = 4.5625$ $f'(1.25) = 23.5$	**B1:** correct differential **M1:** attempts to evaluate $f(1.25)$ and $f'(1.25)$ **at any stage** **A1ft**: $f(1.25)$ and $f'(1.25)$ correct, ft *their* differential	**B1** **M1** **A1**
	$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ $\therefore\ x_2 = 1.25 - \dfrac{f(1.25)}{f'(1.25)}$ $\therefore\ x_2 = 1.25 - \dfrac{4.5625}{23.5} = 1.055851$	**M1:** correct use of formula **A1:** cao **to six decimal places**. No ft	**M1** **A1** **(5)**
(c)	$\alpha = 1.047091$ Our estimate states that $\alpha = 1.055851$ The decimal places agree up to 2 decimal places. Hence our approximation is correct to 2 decimal places	**B1:** cao **Answer only (without justification) is enough for B1**	**B1** **(1)**

(d)	$\frac{1.055851-1.047091}{1.04791}(\times 100)$ $=0.84\%$	**M1:** correct method based on their values. Condone the numerator in the form $\alpha - x_2$ if *their* $x_2 < \alpha$ **A1:** 0.84%	**M1** **A1** **(2)**
		Total	**10**

6 (a)	$z_1=\sqrt{2}+\mathrm{i}\sqrt{2}$ $z_3=2\sqrt{2}-\mathrm{i}\left(2\sqrt{2}\right)$	**B1:** z_1 correct **B1:** z_3 correct	**B1** **B1** **(2)**
(b)	$\frac{\sqrt{2}+\mathrm{i}\sqrt{2}}{2\sqrt{2}-\mathrm{i}\left(2\sqrt{2}\right)}\times\frac{2\sqrt{2}+\mathrm{i}\left(2\sqrt{2}\right)}{2\sqrt{2}+\mathrm{i}\left(2\sqrt{2}\right)}$ $=\frac{8\mathrm{i}}{16}=\frac{1}{2}\mathrm{i}$	**M1:** correct realisation of the denominator **A1:** correct manipulation of complex terms arriving at the correct answer	**M1** **A1** **(2)**
(c)	$\arg\left(\frac{1}{2}\mathrm{i}-\frac{\lambda\sqrt{3}}{2}-\frac{\lambda}{2}\mathrm{i}\right)=\pi$ If $\arg\left(\frac{z_1}{z_3}-z_2\right)=\pi$, then $\operatorname{Im}\left(\frac{z_1}{z_3}-z_2\right)=0$ $\therefore \frac{1}{2}-\frac{\lambda}{2}=0 \rightarrow \lambda=1$	**M1:** attempts to evaluate and use z_2 **A1:** correct z_2 **M1:** sets imaginary components equal to 0 **A1:** cao	**M1** **A1** **M1** **A1** **(3)**
		Total	**8**

7	$\frac{dy}{dx}=\frac{2a}{y}$		**M1:** attempts to find $\frac{dy}{dx}$. This can be done by numerous methods (it is done here implicitly)	**M1**
	$\therefore\ m_{T_m}=\frac{2a}{2am}=\frac{1}{m}$	$\therefore\ m_{T_n}=\frac{2a}{2an^2}=\frac{1}{n^2}$	**A1:** correct gradient of tangent at M or N	**A1**
	$y-2am=\frac{1}{m}\left(x-am^2\right)$ $my-x=am^2$	$y-2an^2=\frac{1}{n^2}\left(x-an^4\right)$ $n^2y-x=an^4$	**M1:** correct method to find equation of tangent at M or N **A1: both** equations for tangents correct	**M1** **A1**
	To find intersections: $my-n^2y=am^2-an^4$ $y\left(m-n^2\right)=a\left(m^2-n^4\right)$ $y=\frac{a\cancel{\left(m-n^2\right)}\left(m+n^2\right)}{\cancel{\left(m-n^2\right)}}=a\left(m+n^2\right)$ $\therefore am\left(m+n^2\right)-x=am^2$ $\therefore x=amn^2$		**M1:** correct attempt at simultaneous equations **A1:** correct x	**M1** **A1**
	$\therefore 3a=amn^2\ \rightarrow\ 3=m^2n$ $\therefore m=\sqrt{\frac{3}{n}}$		**M1:** sets x coordinate = $3a$ **A1:** cao oe	**M1** **A1**
			Total	**8**

8	Let $f(n)=4^{n+1}+5^{2n-1}$ When $n=1$: $f(1)=4^2+5=21$ $\therefore$ The result is true for $n=1$	**B1:** candidate shows that the result is true for $n=1$	**B1**
	Assume that when $n=k$, $f(k)$ divides 21 / $f(k)\vert\ 21$ When $n=k+1$: $f(k+1)=4^{k+2}+5^{2k+1}$ $f(k+1)=4\left(4^{k+1}\right)+25\left(5^{2k-1}\right)$ $f(k+1)=4\left(4^{k+1}\right)+\left(21+4\right)\left(5^{2k-1}\right)$ $f(k+1)=4\left(4^{k+1}\right)+4\left(5^{2k-1}\right)+21\left(5^{2k-1}\right)$ $f(k+1)=4\left[4^{k+1}+5^{2k-1}\right]+21\left(5^{2k-1}\right)$ $f(k+1)=4f(k)+21\left(5^{2k-1}\right)$	**M1:** assumption made **M1:** correct method to use the assumption in the inductive stage **A1:** reduces $f(k+1)$ to a multiple of 21	**M1** **M1** **A1**
	If the result is true for $n=k$, then it has been shown to be true for $n=k+1$. As it was shown to be true for $n=1$, it is thus true for all $n\left(\in\mathbb{Z}^+\right)$. **A1:** a conclusive statement that conveys the consensus of all the underlined elements		**A1**
		Total	**5**
ALT	There are many alternatives methods to prove this result. Many candidates will evaluate $f\left(k+1\right)-f\left(k\right)$ and try to show $f\left(k+1\right)$ is a multiple of 21 from that. This method is shown below, but in whatever method is used, the same mark scheme from above should be employed. $f\left(k+1\right)-f\left(k\right)=4^{k+2}+5^{2k+1}-4^{k+1}-5^{2k-1}$ $f\left(k+1\right)-f\left(k\right)=\left(4^{k+2}-4^{k+1}\right)+\left(5^{2k+1}-5^{2k-1}\right)$ $f\left(k+1\right)-f\left(k\right)=\left(4^{k+2}-4^{k+1}\right)+\left(5^{2k+1}-5^{2k-1}\right)$		

	$f\left(k+1\right)-f\left(k\right)=4^{k+1}\left(4-1\right)+5^{2k-1}\left(5^2-1\right)$ $f\left(k+1\right)-f\left(k\right)=3\left(4^{k+1}\right)+5^{2k-1}\left(21+3\right)$ $f\left(k+1\right)-f\left(k\right)=3\left(4^{k+1}\right)+21\left(5^{2k-1}\right)+3\left(5^{2k-1}\right)$ $f\left(k+1\right)-f\left(k\right)=3\left(4^{k+1}+5^{2k-1}\right)+21\left(5^{2k-1}\right)$ $f\left(k+1\right)-f\left(k\right)=3f\left(k\right)+21\left(5^{2k-1}\right)$ $f\left(k+1\right)=4f\left(k\right)+21\left(5^{2k-1}\right)$ This should then be followed by the required conclusion.	

9 (a)	$\frac{dy}{dx}=-\frac{y}{x}$ or $-\frac{c^2}{x^2}$ $\left.\frac{dy}{dx}\right\|_{x=ct,y=\frac{c}{t}}=-\frac{1}{t^2}$	**B1:** correct $\frac{dy}{dx}$ **B1ft**: $\frac{dy}{dx}$ correctly evaluated at P	**B1** **B1**
	$y-\frac{c}{t}=-\frac{1}{t^2}(x-ct)$ $t^2y-ct=-x+ct$ $t^2y+x=2ct$	**M1:** correct method to find equation of tangent **A1:** cso	**M1** **A1** **(4)**
(b)	$y-\frac{c}{t}=t^2(x-ct)$	**B1ft:** correct gradient of normal **M1:** use of $y-y_1=m(x-x_1)$ or $y=mx+c$ **A1:** cao oe	**B1** **M1** **A1** **(3)**
(c)	$A(2ct,0)$ $B\left(0,\ \frac{c-ct^4}{t}\right)$	**B1:** correct coordinates of A **B1:** correct coordinates of B	**B1** **B1**
	Area of triangle $=\frac{2ct\times\left(\frac{c-ct^4}{t}\right)}{2}$ $=c^2(1-t^4)$	**M1:** correct method to work out the area of the triangle **A1:** correct expression oe	**M1** **A1** **(4)**
		Total	**11**

9	When $i=1$, LHS: $\sum_{n=1}^{1} 1\cdot 1!=1$ RHS: $2!-1=1$ $\therefore$ The statement is true for $i=1$.	**B1:** candidate shows that the result is true for $i=1$	**B1**
	Assume that when $i=k$, $\sum_{n=1}^{k} n\cdot n!=(k+1)!-1$ When $i=k+1$: $\sum_{n=1}^{k+1} n\cdot n!=\sum_{n=1}^{k} n\cdot n!+(k+1)\cdot(k+1)!$ $=(k+1)!\ -1+(k+1)\cdot(k+1)!$ $=(k+1)![1+k+1]-1$ $=(k+1)![k+2]-1$ $=(k+2)!\ -1$	**M1:** assumption made **M1:** correct method to use the assumption in the inductive stage **A1:** convincingly shows that $\sum_{n=1}^{k+1} n\cdot n!=(k+2)!-1$ cso	**M1** **M1** **A1**
	If the result is true for $i=k$, <u>then it has been shown to be true</u> for $i=k+1$. <u>As it was shown to be true </u>for $i=1$, <u>it is thus true for all</u> $i\left(\in \mathbb{Z}^{+}\right)$. **A1:** a conclusive statement that conveys the consensus of all the underlined elements		**A1**
		Total	**6**