Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 1

Question:

As part of a statistics project, Gill collected data relating to the length of time, to the nearest minute, spent by shoppers in a supermarket and the amount of money they spent. Her data for a random sample of 10 shoppers are summarised in the table below, where t represents time and £m the amount spent over £20.

t (minutes)	£m
15	−3
23	17
5	−19
16	4
30	12
6	−9
32	27
23	6
35	20
27	6

a Write down the actual amount spent by the shopper who was in the supermarket for 15 minutes.

b Calculate S_{tt}, S_{mm} and S_{tm}.

(You may use $\Sigma t^2 = 5478$, $\Sigma m^2 = 2101$, and $\Sigma tm = 2485$)

c Calculate the value of the product moment correlation coefficient between t and m.

d Write down the value of the product moment correlation coefficient between t and the actual amount spent. Give a reason to justify your value.

On another day Gill collected similar data. For these data the product moment correlation coefficient was 0.178

e Give an interpretation to both of these coefficients.

f Suggest a practical reason why these two values are so different.

Solution:

a $20 - 3 =$ **£ 17**

b $\sum t = 212$ and $\sum m = 61$

$$S_{tm} = 2485 - \frac{61 \times 212}{10} = \underline{\mathbf{1191.8}}$$

$$S_{tt} = 5478 - \frac{212^2}{10} = \underline{\mathbf{983.6}}$$

$$S_{mm} = 2101 - \frac{61^2}{10} = \underline{\mathbf{1728.9}}$$

c

$$r = \frac{1191.8}{\sqrt{983.6 \times 1728.9}}$$
$$\underline{= \mathbf{0.914}}$$

d <u>**0.914**</u>

e.g. linear transformation, coding does not affect coefficient

e

0.914 suggests that the longer spent shopping the more money spent. (Idea more time, more spent)	Interpretation must be done in the context of the question
0.178 suggests that different amounts spent for same time.	

f e.g. might spend short time buying 1 expensive item <u>OR</u> might spend a long time checking for bargains, talking, buying lots of cheap items.

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 2

Question:

The random variable X has probability function

$P(X = x) = \frac{(2x-1)}{36}$ $x = 1, 2, 3, 4, 5, 6.$

a Construct a table giving the probability distribution of X.

Find

b $P(2 < X \le 5)$,

c the exact value of $E(X)$.

d Show that $Var(X) = 1.97$ to three significant figures.

e Find $Var(2 - 3X)$.

Solution:

a

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
	0.0278, 0.0833, 0.139, 0.194, 0.25, 0.306					

b $P(3) + P(4) + P(5) = \underline{\mathbf{\frac{21}{36}} \text{ or } \mathbf{\frac{7}{12}} \text{ or } \mathbf{0.58\dot{3}}}$

c $E(X) = \frac{1}{36}[1 + 2\times3 + 3\times5 + 4\times7 + 5\times9 + 6\times11], = \underline{\mathbf{\frac{161}{36}} \text{ or } \mathbf{4.47\dot{2}} \text{ or } \mathbf{4\frac{17}{36}}}$

d $E(X^2) = \frac{1}{36}[1 + 2^2\times3 + 3^2\times5 + 4^2\times7 + 5^2\times9 + 6^2\times11],$ Using $\Sigma x^2 p$

$= \underline{\mathbf{\frac{791}{36}} \text{ or } \mathbf{21.97\dot{2}} \text{ or } \mathbf{21\frac{35}{36}} \text{ or awrt } \mathbf{21.97}}$

You must show all the steps when you are asked to show that $Var(X)$ = 1.97

$Var(X) = \frac{791}{36} - \left(\frac{161}{36}\right)^2 = \underline{\mathbf{1.9714}\ldots}$

e $Var(2 - 3X) = 9 \times 1.97$ or $(-3)^2 \times 1.97 = \underline{\mathbf{17.73}}$ Using $Var(aX + b) = a^2 Var(X)$

more accurate: <u>17.74</u>

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 3

Question:

The measure of intelligence, IQ, of a group of students is assumed to be Normally distributed with mean 100 and standard deviation 15.

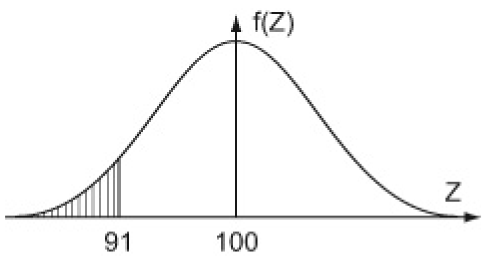
a Find the probability that a student selected at random has an IQ less than 91.

The probability that a randomly selected student as an IQ of at least 100 + *k* is 0.2090.

b Find, to the nearest integer, the value of *k*.

Solution:

a



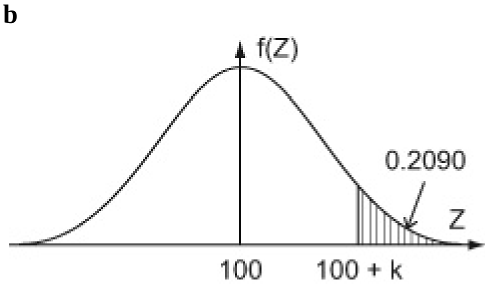
Drawing a diagram will help you to work out the correct area

$$\begin{aligned}P(X < 91) &= P\left(Z < \frac{91-100}{15}\right)\\ &= P(Z < -0.6)\\ &= 1 - 0.7257\\ &= \underline{\mathbf{0.2743}}\end{aligned}$$

Using $z = \frac{x-\mu}{\sigma}$. As 91 is to the left of 100 your z value should be negative.

The tables give P(Z < 0.6) = P(Z > − 0.6) so you want 1 – this probability.

b



As 0.2090 is not in the table of percentage points you must work out the largest area

$1 - 0.2090 = 0.7910$

$P(X > 100 + k) = 0.2090$ <u>or</u> $P(X < 100 + k) = 0.791$

$$\frac{100+k-100}{15} = 0.81$$

$\underline{\boldsymbol{k} \mathbf{= 12}}$

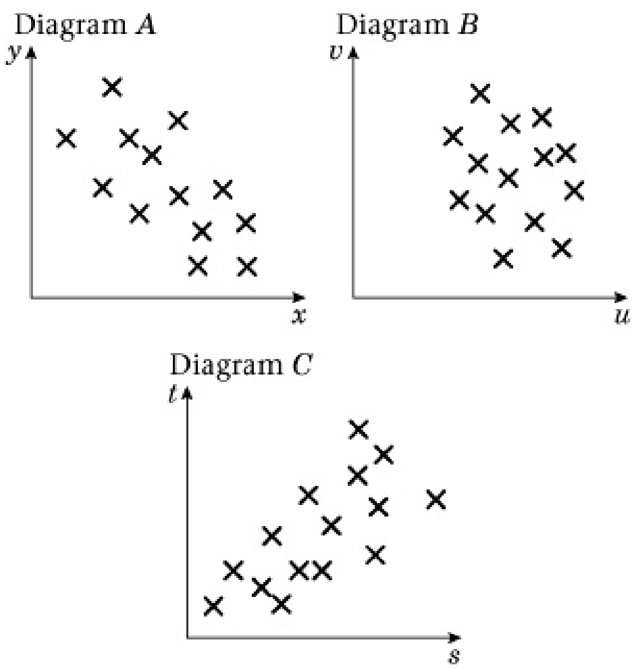
Use the first table or calculator to find the z value. It is positive as 100 + k is to the right of 100

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 4

Question:

The scatter diagrams below were drawn by a student.



The student calculated the value of the product moment correlation coefficient for each of the sets of data.

The values were

0.68 −0.79 0.08

Write down, with a reason, which value corresponds to which scatter diagram.

Solution:

Diagram A : y & x : $\mathbf{r} = -\mathbf{0.79}$; as x increases, y decreases or most points lie in the 2nd and 4th quadrant

You must identify clearly which diagram each value goes with. And use the letters labelled on the axes.

Diagram B: v & u: $\mathbf{r = 0.08}$; no real pattern. Several values of v for one value of u or points lie in all four quadrants, randomly selected

Diagram C : t and s: $\mathbf{r = 0.68}$; As s increases, t increases or most points lie in the 1st and 3rd quadrants.

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 5

Question:

A long distance lorry driver recorded the distance travelled, m miles, and the amount of fuel used, f litres, each day. Summarised below are data from the driver's records for a random sample of eight days.

The data are coded such that $x = m - 250$ and $y = f - 100$.

$\Sigma x = 130$	$\Sigma y = 48$
$\Sigma xy = 8880$	$S_{xx} = 20\,487.5$

a Find the equation of the regression line of y on x in the form $y = a + bx$.

b Hence find the equation of the regression line of f on m.

c Predict the amount of fuel used on a journey of 235 miles.

Solution:

a $S_{xx} = 20487.5$

$$S_{xy} = 8880 - \frac{130 \times 48}{8} = 8100$$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{8100}{20487.5} = 0.395$$

$$a = \frac{48}{8} - (0.395363\ldots)\frac{130}{8} = -0.425$$

$\underline{\boldsymbol{y = -0.425 + 0.395x}}$

b $f - 100 = -0.4246 \ldots.. + 0.395\ldots(m - 250)$ Just substitute in for x and y.

$\underline{\boldsymbol{f = 0.735 + 0.395\,m}}$ You must use the accurate values for a and b otherwise you get an incorrect answer of 0.825 instead of 0.735

c $m = 235 \Rightarrow \underline{\boldsymbol{f = 93.6}}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 6

Question:

The random variable X has probability function

$$P(X = x) = \begin{cases} kx & x = 1, 2, 3, \\ k(x+1) & x = 4, 5 \end{cases}$$

where k is a constant.

a Find the value of k.

b Find the exact value of E(X).

c Show that, to three significant figures, Var(X) = 1.47.

d Find, to one decimal place, Var (4 − 3X).

Solution:

a

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$5k$	$6k$

Draw a probability distribution table. Substitute 1, 2 and 3 into kx, and then 4 and 5 into $k(x+1)$ to work out the probabilities.

$$k + 2k + 3k + 5k + 6k = 1$$
$$17k = 1$$
$$k = \underline{\mathbf{\frac{1}{17}}}$$

The sum of the probabilities = 1

b $$E(X) = 1 \times \frac{1}{17} + 2 \times \frac{2}{17} + 3 \times \frac{3}{17} + 4 \times \frac{5}{17} + 5 \times \frac{6}{17}$$
$$= \frac{64}{17}$$
$$= \underline{\mathbf{3\frac{13}{17}}}$$

Question requires an exact answer therefore it is best to work in fractions

c $$E(X^2) = 1 \times \frac{1}{17} + 4 \times \frac{2}{17} + 9 \times \frac{3}{17} + 16 \times \frac{5}{17} + 25 \times \frac{6}{17}$$
$$= \mathbf{\frac{266}{17}}$$

Using $\Sigma x^2 p$

You must show all the steps when you are asked to show that Var(X) = 1.47.

$$\text{Var}(X) = \frac{266}{17} - \left(3\frac{13}{17}\right)^2$$
$$= 1.474 = \underline{\mathbf{1.47}}$$

d $$\text{Var}(4 - 3X) = 9\text{Var}(X)$$

Using Var (aX + b) = a^2Var(X)

$= 9 \times 1.47$

$=$ **<u>13.3</u>**

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 7

Question:

A scientist found that the time taken, M minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes.

Find

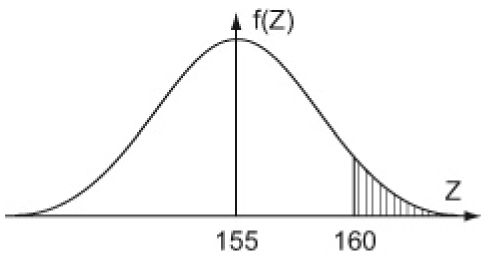
a $P(M > 160)$.

b $P(150 \leq M \leq 157)$.

c the value of m, to one decimal place, such that $P(M \leq m) = 0.30$.

Solution:

a $M \sim N(155, 3.5^2)$



Drawing a diagram will help you to work out the correct area

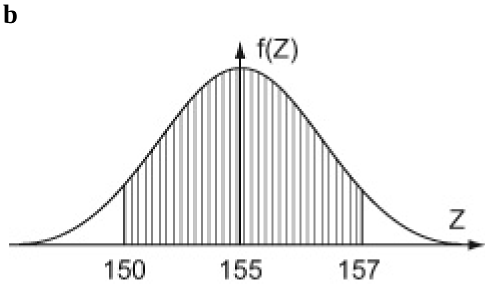
$$P(M > 160) = P\left(z > \frac{160-155}{3.5}\right)$$

Using $z = \frac{x-\mu}{\sigma}$. As 160 is to the right of 155 your z value should be positive

$$= P(z > 1.43)$$
$$= 1 - 0.9236$$
$$= \underline{\mathbf{0.0764}} \qquad \underline{\mathbf{(0.0766 \text{ if calc used})}}$$

The tables give $P(Z < 1.43)$ so you want 1 − this probability.

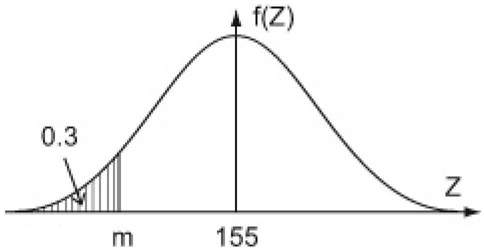
b



$$P(150 < M < 157) = P\left(\frac{150-155}{3.5} < z < \frac{157-155}{3.5}\right)$$
$$= P(-1.43 \leq z \leq 0.57)$$
$$= P(z \leq 0.57) - P(z \leq -1.43)$$
$$= 0.7157 - (1 - 0.9236)$$
$$= \underline{\mathbf{0.6393}} \qquad \underline{\mathbf{(0.6396 \text{ if calc used})}}$$

The tables give $P(Z > -1.43)$ so you want 1 − this probability.

c



$P(M \le m) = 0.3 \Rightarrow \frac{m-155}{3.5} = -0.5244$

<u>***m* = 153.2**</u>

Use the table of percentage points or calculator to find z. You must use at least the 4 decimal places given in the table. It is a negative value since *m* is to the left of 155

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 8

Question:

The random variable X has probability distribution

x	1	2	3	4	5
$\mathbf{P}(X = x)$	0.1	p	0.20	q	0.30

a Given that E(X) = 3.5, write down two equations involving p and q.

Find

b the value of p and the value of q,

c Var (X),

d Var (3 − 2X).

Solution:

a $0.1 + p + 0.2 + q + 0.3 = 1$ $\sum p = 1$

$\mathbf{p + q = 0.4 \quad (1)}$

$1 \times 0.1 + 2 \times p + 3 \times 0.2 + 4 \times q + 5 \times 0.3 = 3.5$ $\mathrm{E}(X) = \sum x\mathrm{P}(X = x) = 3.5$

$\mathbf{2p + 4q = 1.3 \quad (2)}$

b Solving simultaneously

Multiplying **(1)** by **(2)**

$2p + 2q = 0.8$
$2q = 0.5$
$\underline{\mathbf{q = 0.25}}$

subst in to **(1)** $p + 0.25 = 0.4$

$\underline{\mathbf{p = 0.15, q = 0.25}}$

c $\mathrm{E}(X^2) = 1^2 \times 0.10 + 2^2 \times 0.15 + 3^2 \times 0.2 + 4^2 \times 0.25 + 5^2 \times 0.30 = \underline{\mathbf{14}}$ Using $\Sigma x^2 p$

$\mathrm{Var}(X) = 14 - 3.5^2 = \underline{\mathbf{1.75}}$

d $\mathrm{Var}(3 - 2X) = 4\mathrm{Var}(X) = 4 \times 1.75 = \underline{\mathbf{7.00}}$ Using $\mathrm{Var}(aX + b) = a^2\ \mathrm{Var}(X)$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 9

Question:

A manufacturer stores drums of chemicals. During storage, evaporation takes place. A random sample of 10 drums was taken and the time in storage, x weeks, and the evaporation loss, y ml, are shown in the table below.

x	3	5	6	8	10	12	13	15	16	18
y	36	50	53	61	69	79	82	90	88	96

a On graph paper, draw a scatter diagram to represent these data.

b Give a reason to support fitting a regression model of the form $y = a + bx$ to these data.

c Find, to two decimal places, the value of a and the value of b.

(You may use $\Sigma x^2 = 1352$, $\Sigma y^2 = 53112$ and $\Sigma xy = 8354$.)

d Give an interpretation of the value of b.

e Using your model, predict the amount of evaporation that would take place after

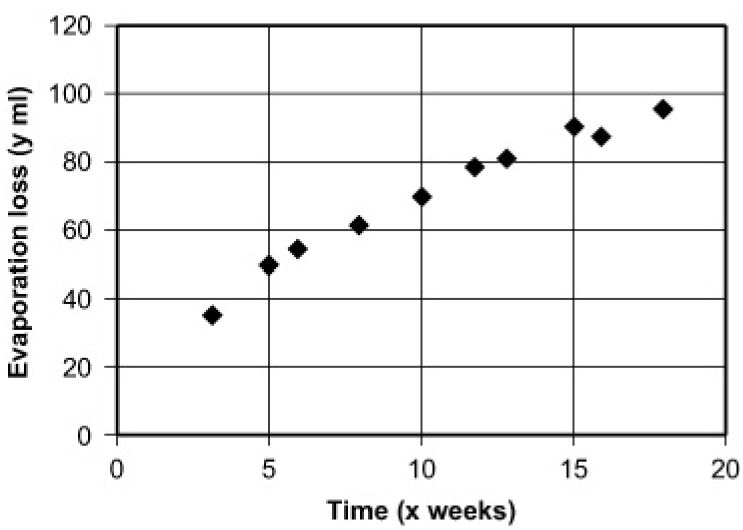
i 19 weeks,

ii 35 weeks.

f Comment, with a reason, on the reliability of each of your predictions.

Solution:

a



b Points lie close to a straight line

c $\sum x = 106, \sum y = 704, \sum xy = 8354$

$$S_{xy} = 8354 - \frac{106 \times 704}{10} = 891.6$$

$$S_{xx} = 1352 - \frac{106^2}{10} = 228.4$$

$$b = \frac{891.6}{228.4} = \underline{\mathbf{3.90}}$$

$$a = \frac{704}{10} - b\frac{106}{10} = \underline{\mathbf{29.02}} \text{ (2 dp required)}$$

d For every extra week in storage, another 3.90 ml of chemical evaporates

Interpretation must be done in the context of the question

e (i) $y = 29.0 + 3.90 \times 19 =$ **<u>103.1</u>** ml (ii) $y = 29.0 + 3.90 \times 35 =$ **<u>165.5</u>** ml

f (i) Close to range of x**, so reasonably reliable.**

(ii) **Well outside range of** x**, could be unreliable since no evidence that model will continue to hold.**

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 10

Question:

a Write down two reasons for using statistical models.

b Give an example of a random variable that could be modelled by

i a normal distribution,

ii a discrete uniform distribution.

Solution:

a To simplify a real world problem

To improve understanding / describe / analyse a real world problem

Quicker and cheaper than using real thing

To predict possible future outcomes

Refine model / change parameters possible

b (i) e.g. height, weight (ii) score on a face after tossing a fair die

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 11

Question:

The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete,

a is taller than 188 cm,

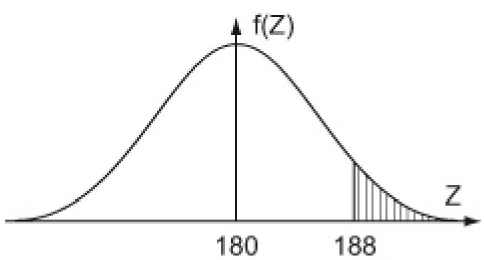
b weighs less than 97 kg.

c Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

d Comment on the assumption that height and weight are independent.

Solution:

a Let H be the random variable ~ height of athletes, so $H \sim N(180, 5.2^2)$



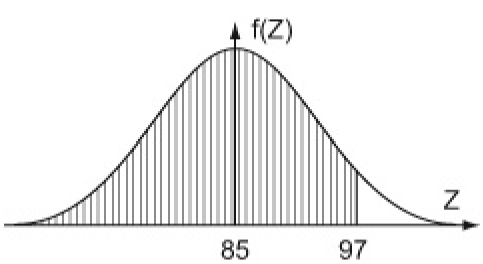
Drawing a diagram will help you to work out the correct area

$$\begin{aligned} P(H > 188) &= P\left(Z > \frac{188-180}{5.2}\right) \\ &= P(Z > 1.54) \\ &= 1 - 0.9382 \\ &= \underline{\mathbf{0.0618}} \end{aligned}$$

Using $z = \frac{x-\mu}{\sigma}$. As 188 is to the right of 180 your z value should be positive

The tables give $P(Z < 1.54)$ so you want 1 − this probability.

b Let W be the random variable ~ weight of athletes, so $W \sim N(85, 7.1^2)$



$$\begin{aligned} P(W < 97) &= P\left(Z < \frac{97-85}{7.1}\right) \\ &= P(Z < 1.69) \\ &= \underline{\mathbf{0.9545}} \end{aligned}$$

Using $z = \frac{x-\mu}{\sigma}$. As 97 is to the right of 85 your z value should be positive

c

$\text{P}(H > 188 \ \& \ W > 97) = 0.0618(1 - 0.9545)$ $= 0.00281$	$\text{P}(\text{W} > 97) = 1 - \text{P}(\text{W} < 97)$

d

Evidence suggests height and weight are positively correlated / linked	Use the context of the question when you are commenting

Assumption of independence is not sensible

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 12

Question:

A metallurgist measured the length, l mm, of a copper rod at various temperatures, t°C, and recorded the following results.

t	l
20.4	2461.12
27.3	2461.41
32.1	2461.73
39.0	2461.88
42.9	2462.03
49.7	2462.37
58.3	2462.69
67.4	2463.05

The results were then coded such that $x = t$ and $y = l - 2460.00$.

a Calculate S_{xy} and S_{xx}.

(You may use $\Sigma x^2 = 15\ 965.01$ and $\Sigma xy = 757.467$)

b Find the equation of the regression line of y on x in the form $y = a + bx$.

c Estimate the length of the rod at 40°C.

d Find the equation of the regression line of l on t.

e Estimate the length of the rod at 90°C.

f Comment on the reliability of your estimate in **e**.

Solution:

a

$\Sigma x = \Sigma t = 337.1$, $\Sigma y = 16.28$

Σy is found by subtracting 2460 form all the l values to get y.

$$S_{xy} = 757.467 - \frac{337.1 \times 16.28}{8} = \underline{\mathbf{71.4685}}$$

$$S_{xx} = 15965.01 - \frac{337.1^2}{8} = \underline{\mathbf{1760.45875}}$$

b $b = \frac{71.4685}{1760.45875} = 0.04059652$

$a = \frac{16.28}{8} - 0.04059652 \times \frac{337.1}{8} = 0.324364$

$y = 0.324 + 0.0406x$

Remember to write the equation of the line at the end.

c $t = 40$ therefore $x = 40$,

$y = 0.324 + 0.0406 \times 40 = 1.948$,

$l = 2460 + 1.948 = \underline{\mathbf{2461.948}}$ mm

Calculate the value of y and then decode.

d

$l - 2460 = 0.324 + 0.0406t$

Just substitute in the coding for for x and y.

$l = 2460.324 + 0.0406t$

e At $t = 90$, $l = 2463.978$ mm

f 90°C outside range of data therefore unlikely to be reliable

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 13

Question:

The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \quad x = 1, 2, 3, 4, 5.$$

a Write down the value of E(X) and show that Var(X) = 2.

Find

b E($3X - 2$),

c Var($4 - 3X$).

Solution:

a E(X) = 3;

$$\text{Var}(X) = \frac{25-1}{12} = \mathbf{2}$$

This method is using the formulae for the uniform distribution

Or $\text{Var}(X) = 1 \times \frac{1}{5} + 2^2 \times \frac{1}{5} + 3^2 \times \frac{1}{5} + 4^2 \times \frac{1}{5} + 5^2 \times \frac{1}{5} - \text{E}(X)^2$

$= 2$

b

$\text{E}(3X - 2) = 3\text{E}(X) - 2 = \mathbf{7}$

Using $\text{E}(aX + b) = a\text{E}(X) + b$

c

$\text{Var}(4 - 3X) = 9\text{Var}(X) = 18$

Using $\text{Var}(aX + b) = a^2\text{Var}(X)$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 14

Question:

From experience a high jumper knows that he can clear a height of at least 1.78 m once in five attempts. He also knows that he can clear a height of at least 1.65 m on seven out of 10 attempts.

Assuming that the heights the high jumper can reach follow a Normal distribution,

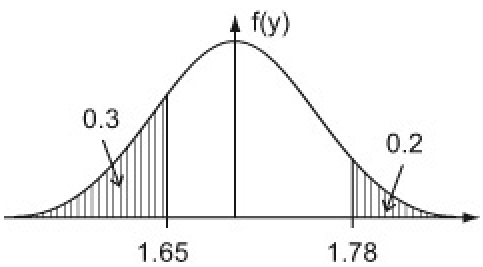
a draw a sketch to illustrate the above information,

b find, to three decimal places, the mean and the standard deviation of the heights the high jumper can reach,

c calculate the probability that he can jump at least 1.74 m.

Solution:

a



b

$$\begin{aligned} P(Z > a) &= 0.2 \\ a &= 0.8416 \\ P(Z < b) &= 0.3 \\ b &= -0.5244 \end{aligned}$$

Use the table of percentage points or calculator to find z. You must use at least the 4 decimal places given in the table. 0.5244 is negative since 1.65 is to the left of the centre. 0.8416 is positive as 1.78 is to the right of the centre.

Using $z = \frac{x-\mu}{\sigma}$.

$$\frac{1.78-\mu}{\sigma} = 0.8416 \Rightarrow 1.78-\mu = 0.8416\sigma \quad \textbf{(1)}$$

$$\frac{1.65-\mu}{\sigma} = -0.5244 \Rightarrow 1.65-\mu = -0.5244\sigma \quad \textbf{(2)}$$

Solving simultaneously **(1)–(2)**

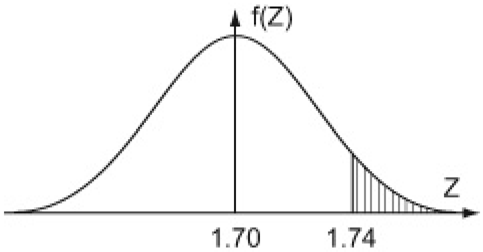
$0.13 = 1.366\sigma$

$\underline{\boldsymbol{\sigma = 0.095,}}$ metres

subst in $1.78 - \mu = 0.8416 \times 0.095$

$\underline{\boldsymbol{\mu = 1.70}}$ metres

c



$$\text{P(height} \geq 1.74) = \text{P}\left(z > \frac{1.74 - 1.70}{0.095}\right)$$
$$= \text{P}(z > 0.42)$$
$$= 1 - 0.6628$$
$$\underline{= \mathbf{0.3372}} \quad \textbf{(calc gives 0.3369)}$$

Using $z = \frac{x - \mu}{\sigma}$.

The tables give $P(Z < 0.42)$ so you want 1 − this probability.

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 15

Question:

A young family were looking for a new three bedroom semi-detached house.

A local survey recorded the price x, in £1000s, and the distance y, in miles, from the station, of such houses. The following summary statistics were provided

$S_{xx} = 113573$, $S_{yy} = 8.657$,

$S_{xy} = -808.917$

a Use these values to calculate the product moment correlation coefficient.

b Give an interpretation of your answer to **a**.

Another family asked for the distances to be measured in km rather than miles.

c State the value of the product moment correlation coefficient in this case.

Solution:

a $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{-808.917}{\sqrt{113573 \times 8.657}}$

$= -0.816 \ldots$

b

Houses are cheaper further away from the town centre or equivalent statement	Use the context of the question when you are asked to interpret

c

-0.816	To change miles to km you multiply by $\frac{8}{5}$. Coding makes no difference to the product moment correlation

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 16

Question:

A student is investigating the relationship between the price (y pence) of 100 g of chocolate and the percentage (x%) of cocoa solids in the chocolate.

The following data are obtained

Chocolate brand	x (% cocoa)	y (pence)
A	10	35
B	20	55
C	30	40
D	35	100
E	40	60
F	50	90
G	60	110
H	70	130

(You may use: $\Sigma x = 315$, $\Sigma x^2 = 15\ 225$, $\Sigma y = 620$, $\Sigma y^2 = 56550$, $\Sigma xy = 28\ 750$)

a Draw a scatter diagram to represent these data.

b Show that $S_{xy} = 4337.5$ and find S_{xx}.

The student believes that a linear relationship of the form $y = a + bx$ could be used to describe these data.

c Use linear regression to find the value of a and the value of b, giving your answers to one decimal place.

d Draw the regression line on your diagram.

The student believes that one brand of chocolate is overpriced.

e Use the scatter diagram to

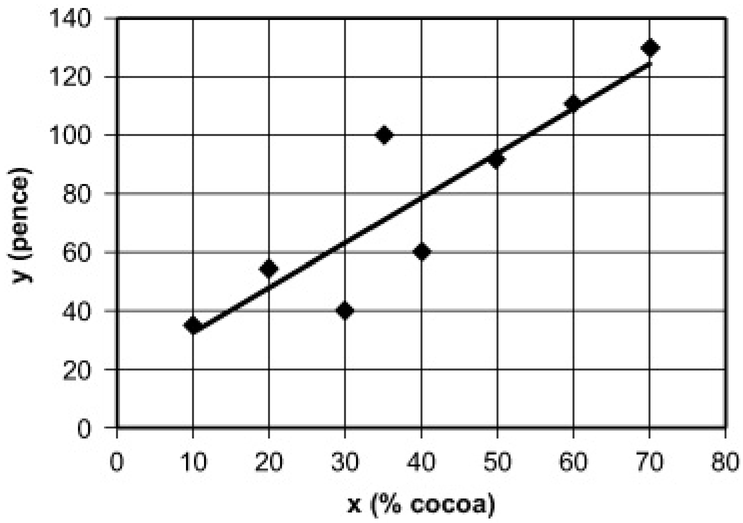
i state which brand is overpriced,

ii suggest a fair price for this brand.

Give reasons for both your answers.

Solution:

a, d



b $S_{xy} = 28750 - \dfrac{315 \times 620}{8} = 4337.5$

$S_{xx} = 15225 - \dfrac{315^2}{8} = 2821.875$

c $b = \dfrac{S_{xy}}{S_{xx}} = 1.537\ldots. = 1.5$

$a = \overline{y} - b\overline{x} = \dfrac{620}{8} - b\dfrac{315}{8} = 16.97\ldots = 17.0$

d on graph draw the line y = 17.0 + 1.5*x*

e i Brand D, since a long way above the line

ii Using the line: $y = 17 + 35 \times 1.5 = 69.5$ pence

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 17

Question:

The random variable X has a normal distribution with mean 20 and standard deviation 4.

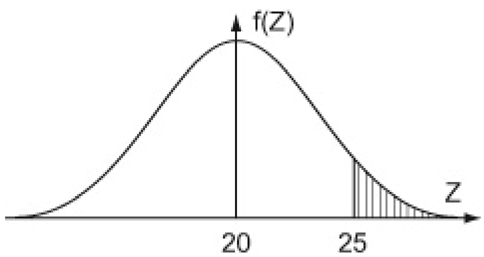
a Find $P(X > 25)$.

b Find the value of d such that

$P(20 < X < d) = 0.4641$.

Solution:

a



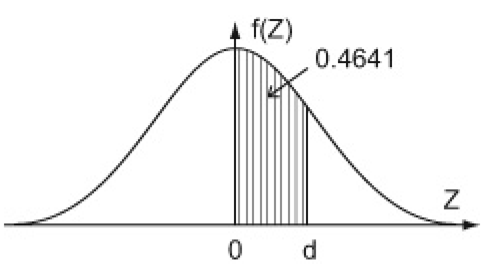
Drawing a diagram will help you to work out the correct area

$$P(X > 25) = P\left(z > \frac{25-20}{4}\right) = P(Z > 1.25) = 1 - 0.8944 = \underline{\mathbf{0.1056}}$$

Using $z = \frac{x-\mu}{\sigma}$. As 25 is to the right of 20 your z value should be positive.

The tables give $P(Z < 1.25)$ so you want 1 – this probability.

b



$P(X < 20) = 0.5$ so $P(X < d) = 0.5 + 0.4641 = 0.9641$

The area to the right of d is 0.0359. This is not in the table so you need to work out the area to the left of d.

$P(Z < z) = 0.9641,\ z = 1.80$

$$\frac{d-20}{4} = 1.80$$

$\underline{\mathbf{d = 27.2}}$

Use the first table or calculator to find the z value. It is positive as d is to the right of 0

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 18

Question:

The random variable X has probability distribution

x	1	3	5	7	9
$\mathrm{P}(X = x)$	0.2	p	0.2	q	0.15

a Given that E(X) = 4.5, write down two equations involving p and q.

Find

b the value of p and the value of q,

c $\mathrm{P}(4 < X \le 7)$.

Given that $\mathrm{E}(X^2) = 27.4$, find

d Var(X),

e E(19 − 4X),

f Var(19 − 4X).

Solution:

a $0.2 + p + 0.2 + q + 0.15 = 1$ Sum of the probabilities = 1

$\underline{p + q = 0.45}$ (1)

$1 \times 0.2 + 3 \times p + 5 \times 0.2 + 7 \times q + 9 \times 0.15 = 4.5$ $\mathrm{E}(X) = \Sigma x\mathrm{P}(X = x) = 4.5$

$3p + 7q = 1.95$ (2)

b Solving the two equations simultaneously

$$\begin{aligned} 3p + 7q &= 1.95 \\ 3p + 3q &= 1.35 \quad \textbf{(1)} \times 3 \\ 4q &= 0.6 \end{aligned}$$

$q = \underline{0.15}$

subst $3p + 7 \times 0.15 = 1.95$

$p = \underline{0.3}$

c $\mathrm{P}(4 < X \le 7) = \mathrm{P}(5) + \mathrm{P}(7)$

$$\begin{aligned} &= 0.2 + q \\ &= \underline{\mathbf{0.35}} \end{aligned}$$

d $\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2 = 27.4 - 4.5^2$

$= \underline{\mathbf{7.15}}$

e $\text{E}(19 - 4X) = 19 - 4 \times 4.5$ Using $\text{E}(aX + b) = a\text{E}(X) + b$

$= \underline{\mathbf{1}}$

f $\text{Var}(19 - 4X) = 16\text{Var}(X)$ Using $\text{Var}(aX + b) = a^2\ \text{Var}(X)$

$= 16 \times 7.15$

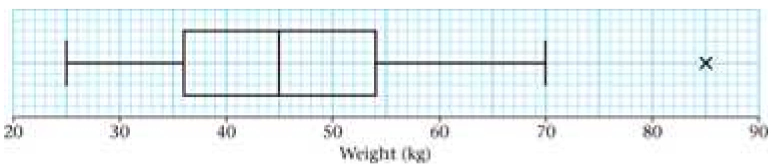
$= \underline{\mathbf{114.4}}$

Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 19

Question:

The box plot in Figure 1 shows a summary of the weights of the luggage, in kg, for each musician in an orchestra on an overseas tour.



The airline's recommended weight limit for each musicians' luggage was 45 kg.

Given that none of the musician's luggage weighed exactly 45 kg,

a state the proportion of the musicians whose luggage was below the recommended weight limit.

A quarter of the musicians had to pay a charge for taking heavy luggage.

b State the smallest weight for which the charge was made.

c Explain what you understand by the × on the box plot in Figure 1, and suggest an instrument that the owner of this luggage might play.

d Describe the skewness of this distribution. Give a reason for your answer.

One musician in the orchestra suggests that the weights of the luggage, in kg, can be modelled by a normal distribution with quartiles as given in Figure 1.

e Find the standard deviation of this normal distribution. ***E***

Solution:

a $\frac{1}{2}$

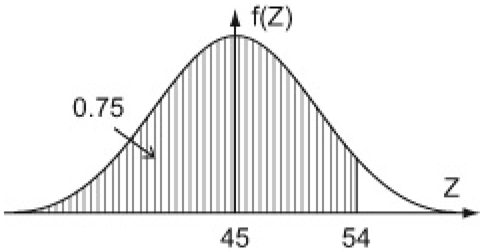
b <u>54</u> kg

c + is an 'outlier' or 'extreme' value

Any heavy musical instrument such as double-bass, drums.

d $Q_3 - Q_2 (9) = Q_2 - Q_1 (9)$ so symmetrical or no skew

e $P(W < 54) = 0.75$ or $P(W > 54) = 0.25$



$$\frac{54-45}{\sigma} = 0.67$$

$$\sigma = \underline{\mathbf{13.4\ \ (calc\ gives\ \ 13.3)}}$$

Using the normal distribution table

$\Phi(0.67) = 0.7486$

$\Phi(0.68) = 0.7517$

0.7486 is closer to 0.75 therefore use $z = 0.67$.

or use the calculator to get 13.343…