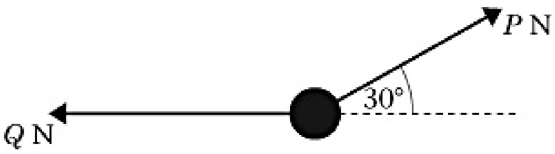
Solutionbank M1
Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 1

Question:



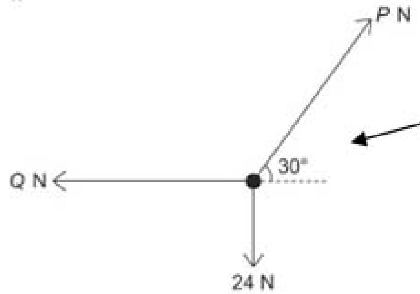
A particle of weight 24 N is held in equilibrium by two light inextensible strings. One string is horizontal. The other string is inclined at an angle of 30° to the horizontal, as shown. The tension in the horizontal string is Q newtons and the tension in the other string is P newtons. Find

a the value of P,

b the value of Q.

Solution:

a



Draw a diagram, showing the weight as well as the tensions in the strings.

R(↑)

$P\sin 30^\circ - 24 = 0$

Resolve vertically first, as there is only one unknown in the resulting equation.

$$\therefore P = \frac{24}{\sin 30^\circ} = 48$$

b R(→)

$$P\cos 30^\circ - Q = 0$$
$$\therefore 48\cos 30^\circ - Q = 0$$

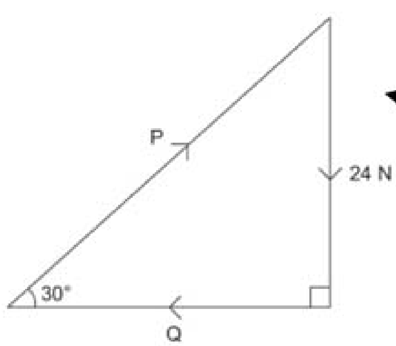
Then resolve horizontally and substitute the value of P, found in **a.**

$$\therefore Q = 48\cos 30^\circ = 41.6 \text{ (3s.f.)}$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces

This is another method of solving problems involving equilibrium under 3 forces.



You can draw a triangle with sides representing each of the three forces.

From the triangle

a

$$\sin 30^\circ = \frac{24}{P}$$
$$\therefore P = \frac{24}{\sin 30^\circ} = 48$$

b

$$\tan 30^\circ = \frac{24}{Q}$$
$$\therefore Q = \frac{24}{\tan 30^\circ} = 41.6 \text{ (3s.f.)}$$

Use trigonometry of the triangle to find P and to find Q.

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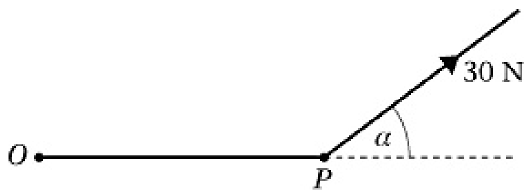
2 Review Exercise
Exercise A, Question 2

Question:



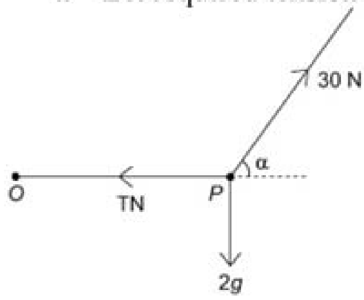
A particle P, of mass 2 kg, is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP horizontal, by a force of magnitude 30 N applied at an angle α to the horizontal, as shown.

a Find, to the nearest degree, the value of α.

b Find, in N to 3 significant figures, the magnitude of the tension in the string.

Solution:

a Let required tension be T N.



Draw a diagram, showing the weight $2g$ N as well as the force 30 N and the tension T N.

R(↑)

$30\sin\alpha - 2g = 0$

Resolve vertically first, as there is only one unknown in the resulting equation.

$\therefore \sin\alpha = \dfrac{2g}{30}$

$= 0.65\dot{3}$

$\therefore \alpha = 40.8^\circ = 41^\circ$

Find arcsin $0.65\dot{3}°$.

b R(→)

$30\cos\alpha - T = 0$

Resolve horizontally and substitute the value of α found in part **a**.

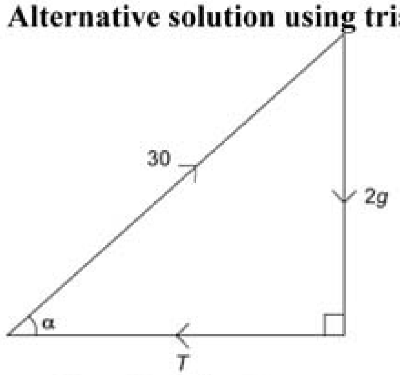
$\therefore T = 30\cos\alpha$

$= 22.7$

Tension is 22.7N (3 s.f.).

Give your answer to 3 s.f.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

a From the triangle

$$\sin\alpha = \frac{2g}{30}$$
$$= 0.65\dot{3}$$
$$\therefore \alpha = 40.8^\circ = 41^\circ$$

Use trigonometry of the triangle to find α.

b

$$30^2 = (2g)^2 + T^2$$
$$\therefore T^2 = 900 - 384.16$$
$$= 515.84$$
$$\therefore T = 22.7\,\text{N}$$

Use Pythagoras' Theorem to find T.

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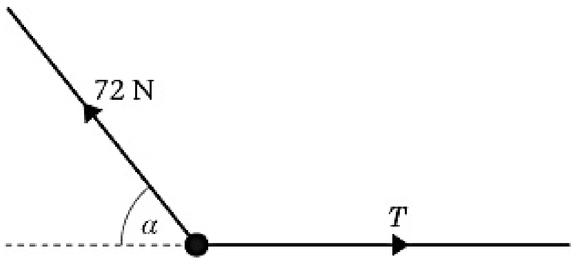
2 Review Exercise
Exercise A, Question 3

Question:



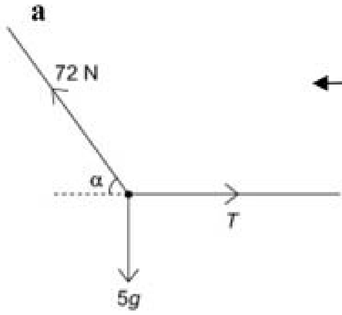
A body of mass 5 kg is held in equilibrium under gravity by two inextensible light ropes. One rope is horizontal, the other is at an angle α to the horizontal, as shown. The tension in the rope inclined at α to the horizontal is 72 N. Find

a the angle α, giving your answer to the nearest degree,

b the tension T in the horizontal rope, giving your answer to the nearest N.

Solution:

a



Draw a diagram, showing the weight 5th g N as well as the forces 72 N and T.

R(↑)

Resolve vertically first as there is only one unknown in the resulting equation.

$$
\begin{aligned}
72\sin\alpha - 5g &= 0 \\
\therefore \sin\alpha &= \frac{5g}{72} \\
&= 0.680\dot{5} \\
\therefore \alpha &= 42.9^\circ \\
&= 43^\circ
\end{aligned}
$$

Find $\arcsin 0.680\dot{5}$.

b R(→)

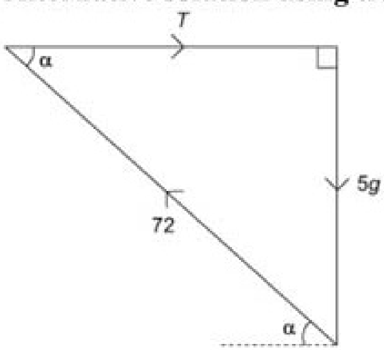
Resolve horizontally and substitute the value of α found in **a**.

$T - 72\cos\alpha = 0$

$$
\begin{aligned}
\therefore T &= 72\cos\alpha \\
&= 52.8 \\
\text{tension} &= 53\text{ N (2 s.f.)}
\end{aligned}
$$

Give your answer to the nearest Newton.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

From the triangle

a

$$\sin\alpha = \frac{5g}{72}$$
$$= 0.680\dot{5}$$
$$\therefore \alpha = 42.9^\circ \text{ (3 s.f.)}$$

Use trigonometry of the triangle to find α and to find T.

b

$$\cos\alpha = \frac{T}{72}$$
$$\therefore T = 72\cos\alpha$$
$$= 53\text{ N (2 s.f.)}$$

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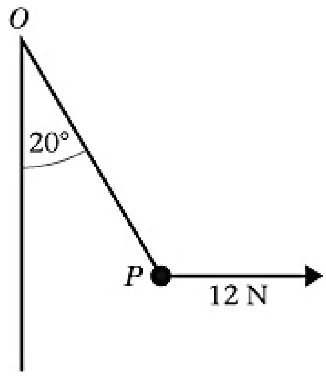
2 Review Exercise
Exercise A, Question 4

Question:



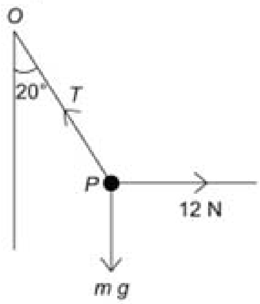
A particle P is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 12 N is applied to P. The particle P is in equilibrium with the string taut and OP making an angle of 20° with the downward vertical, as shown. Find

a the tension in the string,

b the weight of P.

Solution:

a



Draw a diagram, showing the tension T in the string and the weight mg of the particle.

Let T be the tension in the string, and let mg be the weight of P.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

$T\sin 20^\circ - 12 = 0$

$$\therefore T = \frac{12}{\sin 20} = 35.1$$

Tension in the string = 35.1 N (3 s.f.)

Give your answer to 3 s.f.

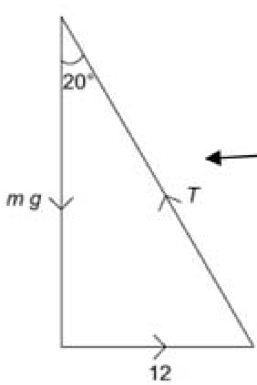
b R(↑)

Then resolve vertically and substitute the value of T found in **a.**

$$T\cos 20^\circ - mg = 0$$
$$\therefore 35.1\cos 20^\circ = mg$$
$$\therefore \text{weight} = 33.0\text{ N (3 s.f.)}$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces



You can draw a triangle with sides representing each of the three forces.

a From the triangle: $\sin 20^\circ = \frac{12}{T}$

$\therefore T = \frac{12}{\sin 20^\circ} = 35.1 \text{ (3 s.f.)}$

Use trigonometry of the triangle to find T and to find mg.

b

$$\cos 20^\circ = \frac{mg}{T}$$
$$\therefore mg = 35.1\cos 20^\circ = 33.0 \text{ (3 s.f.)}$$

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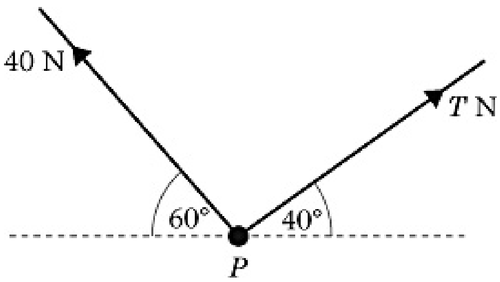
2 Review Exercise
Exercise A, Question 5

Question:



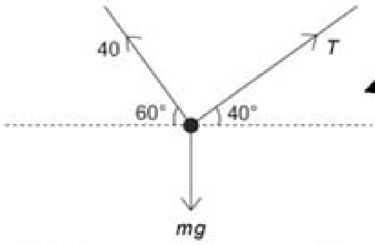
A particle P is held in equilibrium under gravity by two light, inextensible strings. One string is inclined at an angle of 60° to the horizontal and has a tension of 40 N. The other string is inclined at an angle of 40° to the horizontal and has a tension of T newtons, as shown. Find, to three significant figures,

a the value of T,

b the weight of P.

Solution:

a Let the weight of P be mg N.



Draw a diagram showing the tension T in the string and the weight mg of the particle.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

$$T\cos 40^\circ - 40\cos 60^\circ = 0$$

$$\therefore T = \frac{40\cos 60^\circ}{\cos 40^\circ}$$

$$= 26.1 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

b R(↑)

$$40\sin 60^\circ + T\sin 40^\circ - mg = 0$$

$$\therefore 40\sin 60^\circ + 26.1\sin 40^\circ - mg = 0$$

Then resolve vertically and substitute the value of T found in **a**.

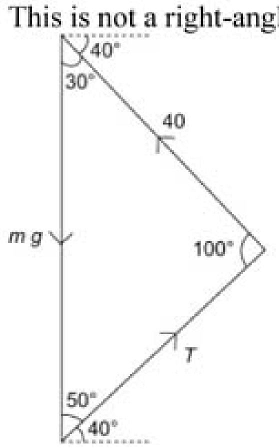
$$\therefore mg = 40\sin 60^\circ + 26.1\sin 40^\circ$$

$$= 51.4 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

Alternative solution using triangle of forces

This is not a right-angled triangle so use the sine rule.



You can draw a triangle with sides representing each of the three forces.

Calculate the angles in the triangle using $90^\circ - 60^\circ = 30; 90^\circ - 40^\circ = 50^\circ$

and angles of a triangle add up to 180° to give 100 as the third angle.

$$\frac{T}{\sin 30^\circ} = \frac{mg}{\sin 100^\circ} = \frac{40}{\sin 50^\circ}$$

$$\therefore T = \frac{40\sin 30^\circ}{\sin 50^\circ} = 26.1 \text{ (3 s.f.)}$$

$$mg = \frac{40\sin 100^\circ}{\sin 50^\circ} = 51.4 \text{ (3 s.f.)}$$

Use the sine rule to find T and mg.

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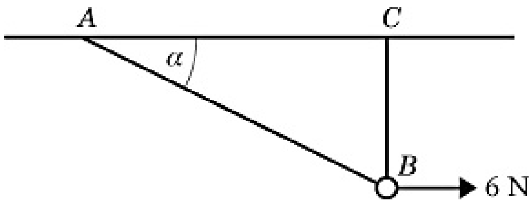
2 Review Exercise
Exercise A, Question 6

Question:



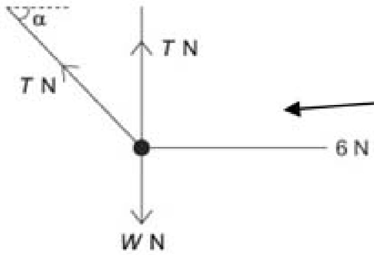
A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 6 N acting parallel to AC. The bead B is vertically below C and $\angle BAC = \alpha$, as shown in the diagram. Given that $\tan \alpha = \frac{3}{4}$, find

a the tension in the string,

b the weight of the bead.

Solution:

a



Draw a diagram showing the forces on the bead.
The tension in both parts of the string is the same.

Let the tension in the string be T N and let the weight of the bead be W N.

R(→)

Resolve horizontally first as there is only one unknown in the resulting equation.

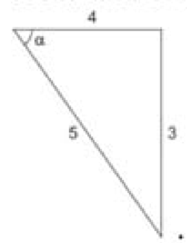
$6 - T\cos\alpha = 0$

$\therefore T = \dfrac{6}{\cos\alpha}$

As $\tan\alpha = \dfrac{3}{4}, \cos\alpha = \dfrac{4}{5}$

$\therefore T = 6 \div \dfrac{4}{5} = 7.5$

Use this triangle to find $\cos\alpha$ and $\sin\alpha$



b R(↑)

$T + T\sin\alpha - W = 0$

As $\tan\alpha = \dfrac{3}{4}, \sin\alpha = \dfrac{3}{5}$

$\therefore 7.5 + 7.5 \times \dfrac{3}{5} - W = 0$

$\therefore W = 12$

Resolve vertically and substitute the value of T found in part **a**.

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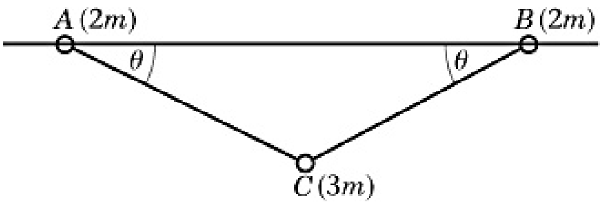
2 Review Exercise
Exercise A, Question 7

Question:



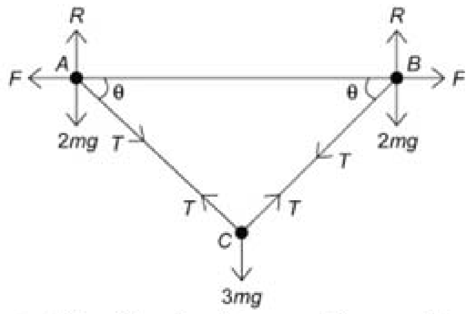
Two small rings, *A* and *B*, each of mass 2*m*, are threaded on a rough horizontal pole. The coefficient of friction between each ring and the pole is μ. The rings are attached to the ends of a light inextensible string. A smooth ring *C*, of mass 3*m*, is threaded on the string and hangs in equilibrium below the pole. The rings *A* and *B* are in limiting equilibrium on the pole, with

$\angle BAC = \angle ABC = \theta$, where $\tan\theta = \frac{3}{4}$, as shown in the diagram.

a Show that the tension in the string is $\frac{5}{2}mg$.

b Find the value of μ.

Solution:



Show all the forces acting on A, B and C.
The normal reactions R and the friction forces F will be the same at A as at B by symmetry.

a Consider the forces acting on the ring C.

Consider ring C first as there is only one unknown force at C.

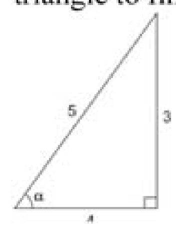
R(↑)

$2T\sin\theta - 3\,mg = 0$

$\therefore T = \dfrac{3\,mg}{2\sin\theta}$

As $\tan\theta = \dfrac{3}{4}, \sin\theta = \dfrac{3}{5}$

As $\tan\theta = \dfrac{3}{4}$, use 3, 4, 5 triangle to find $\sin\theta$ and $\cos\theta$.



$$\therefore T = \frac{3\,mg}{2\times\frac{3}{5}} = \frac{5\,mg}{2}$$

b Consider the forces acting on the ring A.

You could consider ring A or ring B.

R(→)

$T\cos\theta - F = 0$

$\therefore F = \dfrac{5\,mg}{2}\times\dfrac{4}{5} = 2\,mg$

Use the value of T given from part **a** to find F.

R(↑)

$$R - 2\,mg - T\sin\theta = 0$$

$$\therefore R = 2\,mg + \frac{5\,mg}{2}\times\frac{3}{5} = \frac{7\,mg}{2}$$

Use the value of T to find R.

The ring A is in limiting equilibrium

You were told that friction was limiting at A and B.

$$\therefore F = \mu R$$

$$\therefore 2\,mg = \mu\times\frac{7\,mg}{2}$$

$$\therefore \mu = \frac{4}{7}$$

Give μ as a fraction as this is an exact answer.

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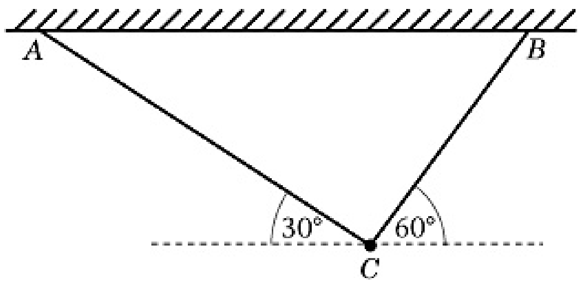
2 Review Exercise
Exercise A, Question 8

Question:

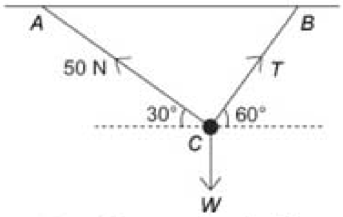


A particle of weight W newtons is attached at C to the ends of two light inextensible strings AC and BC. The other ends of the strings are attached to two fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 60° respectively, as shown. Given the tension in AC is 50 N, calculate

a the tension in BC, to three significant figures,

b the value of W.

Solution:



Draw a diagram showing the three forces acting on the particle.

a Let the tension in the string BC be T N.

R($\rightarrow$)

$T\cos 60^\circ - 50\cos 30^\circ = 0$

Resolve horizontally first, as there is only one unknown in the resulting equation.

$$\therefore T = \frac{50\cos 30^\circ}{\cos 60^\circ} = 86.6 \text{ (3 s.f.)}$$

Give your answer to 3 s.f.

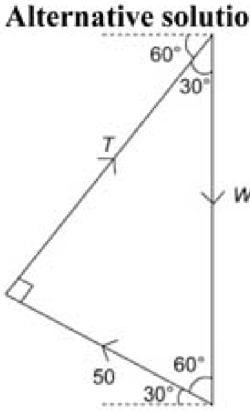
b R($\uparrow$)

$50\sin 30^\circ + T\sin 60^\circ - W = 0$

$$\therefore W = 50\sin 30^\circ + T\sin 60^\circ = 50\sin 30^\circ + 86.6\sin 60^\circ = 100$$

Resolve vertically and substitute the value of T found in part **a**.

Alternative solution using triangle of forces



Calculate the angles of the triangle using $90^\circ - 60^\circ = 30^\circ, 90^\circ - 30^\circ = 60^\circ$ and angles of a triangle add up to 180° to give 90° as the third angle.

From the triangle

$$\tan 60^\circ = \frac{T}{50}$$

$$\therefore T = 50\tan 60^\circ = 86.6 \text{ (3 s.f.)}$$

$$\cos 60^\circ = \frac{50}{W}$$

$$\therefore W = \frac{50}{\cos 60^\circ} = 100$$

Use trigonometry of the right angled triangle to find T and W.

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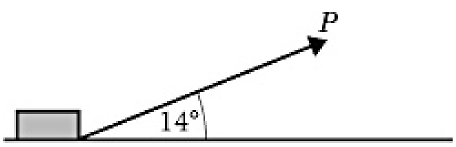
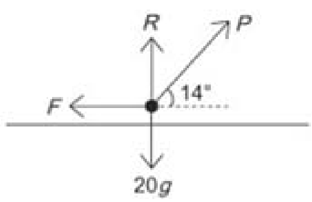
2 Review Exercise
Exercise A, Question 9

Question:



A small box of mass 20 kg rests on a rough horizontal floor. The coefficient of friction between the box and the floor is 0.25. A light inextensible rope is tied to the box and pulled with a force of magnitude P newtons at 14° to the horizontal as shown in the diagram. Given that the box is on the point of sliding, find the value of P, giving your answer to 1 decimal place.

Solution:



Draw a diagram showing the normal reaction R, the friction force F and the weight $20g$.

R(↑)

$R + P\sin 14^\circ - 20g = 0$ (1)

R(→)

$P\cos 14^\circ - F = 0$ (2)

As friction is limiting

$F = 0.25R$ (3)

The box is on the point of sliding and so the friction is limiting.

Substituting $F = 0.25R$ into (2)

$0.25R = P\cos 14^\circ$

Eliminate F and then eliminate R from the simultaneous equations to find P.

From equation (1)

$R = 20g - P\sin 14^\circ$

$\therefore 0.25(20g - P\sin 14^\circ) = P\cos 14^\circ$

Collect the terms in P together and factorise.

$$\therefore 5g = P(\cos 14^\circ + 0.25\sin 14^\circ)$$

$$\therefore P = \frac{5g}{\cos 14^\circ + 0.25\sin 14^\circ} = 47.5 \text{ (1 d.p.)}$$

Make P the subject of the formula.

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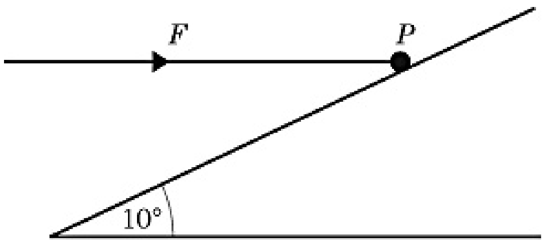
2 Review Exercise
Exercise A, Question 10

Question:



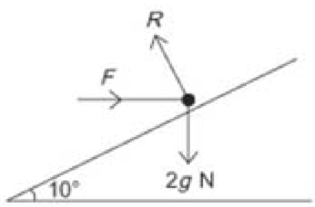
A smooth plane is inclined at an angle 10° to the horizontal. A particle *P* of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude *F* newtons, as shown.

Find, to three significant figures,

a the normal reaction exerted by the plane on *P*.

b the value of *F*.

Solution:



Show the normal reaction and the weight $2g$. There is no friction as the plane is smooth.

a R(↑)

$R\cos 10^\circ - 2g = 0$

Resolving vertically eliminates force F and the equation has one unknown R.

$$\therefore R = \frac{2g}{\cos 10^\circ}$$
$$= 19.9 \text{ N (3 s.f.)}$$

b R(↑)

$$F - R\sin 10^\circ = 0$$
$$\therefore F = R\sin 10^\circ$$
$$= 3.46 \text{ (3 s.f.)}$$

Substitute the value of R found in **a**.

Alternative method

b Resolve along the plane

$F\cos 10^\circ - 2g\sin 10^\circ = 0$

Resolving along the plane gives an equation in one unknown F.

$$\therefore F = \frac{2g\sin 10^\circ}{\cos 10^\circ}$$
$$= 3.46 \text{ (3 s.f.)}$$

This is answer to **b**.

a Resolve perpendicular to the plane

$$R - F\sin 10^\circ - 2g\cos 10^\circ = 0$$
$$\therefore R = F\sin 10^\circ + 2g\cos 10^\circ$$
$$= 19.9 \text{ (3 s.f.)}$$

Substitute the value of F to find R.

This is answer to **a**.

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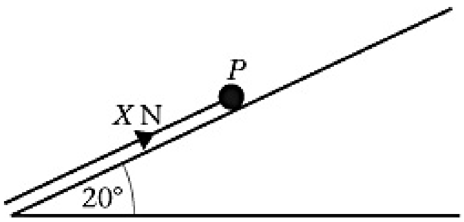
2 Review Exercise
Exercise A, Question 11

Question:



A particle *P* of mass 2.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude *X* newtons acting up a line of greatest slope of the plane, as shown in the diagram. The plane is inclined at 20° to the horizontal. The coefficient of friction between *P* and the plane is 0.4. The particle is in limiting equilibrium and is on the point of moving up the plane. Calculate

a the normal reaction of the plane on *P*,

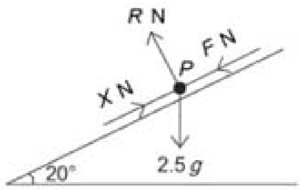
b the value of *X*.

The force of magnitude *X* newtons is now removed.

c Show that *P* remains in equilibrium on the plane.

Solution:

a



Draw a diagram showing the normal reaction R N, the friction force down the plane F N and the weight 2 *mg*.

R(↖)

$$R - 2.5g\cos 20^\circ = 0$$
$$\therefore R = 2.5g\cos 20^\circ$$
$$= 23\,(2\,\text{s.f.})$$

Resolve perpendicular to the plane first, as R is the only unknown in the resulting equation.

The normal reaction is 23 N.

Give your answer to 2 s.f. as you used $g = 9.8$ in your calculation.

b R(↗)

$$X - F - 2.5g\sin 20^\circ = 0$$
$$\therefore X = F + 2.5g\sin 20^\circ \quad *$$

Resolve parallel to the plane to find X in terms of F.

As friction is limiting,

$$F = \mu R$$
$$\text{i.e. } F = 0.4R$$

and as $R = 23.0$, $F = 9.21$

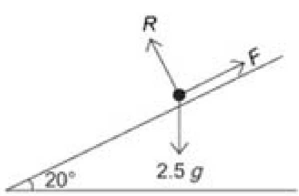
Use limiting friction and the value of R from part **a** to find the force F N.

substitute into equation *

$$\therefore X = 17.6$$
$$= 18\,(2\,\text{s.f.})$$

Then X may be calculated. Again give X to 2 s.f.

c The force X is removed and the friction force will now act up the plane.



Draw a new diagram with F acting up the plane.

R(↗)

For equilibrium

$$F - 2.5g\sin 20^\circ = 0$$
$$\therefore F = 8.38\ (3\,\text{s.f.})$$

Find the value of F which would maintain equilibrium.

R(↖)

$R - 2.5g\cos 20^\circ = 0$, as before

Check to see whether $F \le \mu R$. Use 3 s.f. in your calculation.

$$\therefore R = 23.0$$
$$\mu R = 0.4 \times 23.0 = 9.21$$

This shows that $F < \mu R$, so the friction is not limiting, and equilibrium is maintained.

It is not possible for $F > \mu R$. If $F \le \mu R$ equilibrium is maintained.

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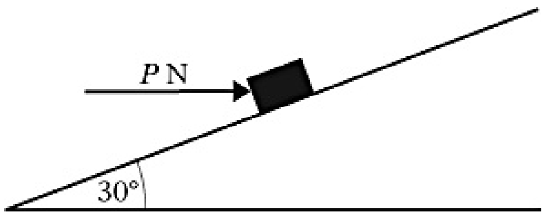
2 Review Exercise
Exercise A, Question 12

Question:



A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

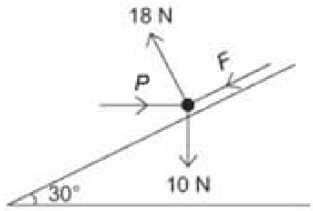
a the value of P,

b the value of μ.

The horizontal force is removed.

c Determine whether or not the parcel moves.

Solution:



Draw a diagram showing the weight, the normal reaction and the friction force F.

F acts down the plane as the parcel is on the point of slipping up the plane.

a R(↖)

$18 - 10\cos 30^\circ - P\sin 30^\circ = 0$
$\therefore P\sin 30^\circ = 18 - 10\cos 30^\circ$

Resolve perpendicular to the plane and make P the subject of the equation.

$$\therefore P = \frac{18 - 10\cos 30^\circ}{\sin 30^\circ}$$
$$= 18.68 \text{ (4 s.f.)}$$
$$= 18.7 \text{ (3 s.f.)}$$

Answer is given to 3 s.f. as g is not used in the calculation.

b R(↗)

$P\cos 30^\circ - F - 10\sin 30^\circ = 0$

Resolve along the plane to find F.

$\therefore F = P\cos 30^\circ - 10\sin 30^\circ$
$= 11.2$

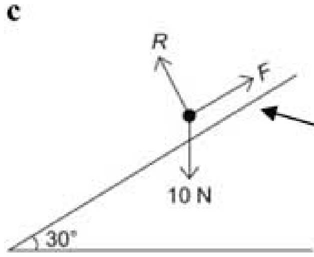
As the friction is limiting,

$$F = \mu R$$
$$\therefore \mu = \frac{11.2}{18}$$
$$= 0.62 \text{ (2 s.f.)}$$

Use $\mu = \dfrac{F}{R}$ and give your answer to 2 s.f.

P is now removed.

c



Draw a new diagram.

F now acts up the plane as any motion would be down the plane.

The friction now acts up the plane and the normal reaction will be different. Let the normal reaction be R.

R(↗)

If equilibrium is maintained

$$F - 10\sin 30^\circ = 0$$
$$\therefore F = 10\sin 30^\circ$$
$$= 5$$

Find the force F which would maintain equilibrium.

R(↖)

$$R - 10\cos 30^\circ = 0$$
$$\therefore R = 10\cos 30^\circ$$
$$= 8.66$$

Find the normal reaction force R and use the value of μ found in part **b** to calculate μR.

The value of μ is unchanged

$$\therefore \mu R = 0.599 \times 8.66$$
$$= 5.19 \text{ (3 s.f.)}$$

as $F < \mu R$, equilibrium is maintained, and the parcel does not move

Check that $F \le \mu R$. As $F < \mu R$ the equilibrium is not limiting.

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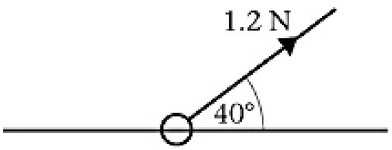
2 Review Exercise
Exercise A, Question 13

Question:

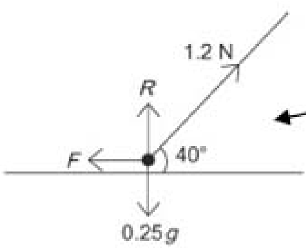


A small ring of mass 0.25 kg is threaded on a fixed rough horizontal rod. The ring is pulled upwards by a light string which makes an angle 40° with the horizontal, as shown. The string and the rod are in the same vertical plane. The tension in the string is 1.2 N and the coefficient of friction between the ring and the rod is μ. Given that the ring is in limiting equilibrium, find

a the normal reaction between the ring and the rod,

b the value of μ.

Solution:



Draw a diagram showing the forces acting on the ring. Include the weight 0.25*g*, the normal reaction *R* and the friction force *F*.

a R(↑)

$R + 1.2\sin 40^\circ - 0.25g = 0$

$\therefore R = 0.25g - 1.2\sin 40^\circ$

$= 1.68$

∴ Normal reaction is 1.7 N (2 s.f.)

Resolve vertically first as *R* is the only unknown force in the resulting equation.

Give answer to 2 s.f. as $g = 9.8$ has been used in the calculation.

b R(→)

$1.2\cos 40^\circ - F = 0$

$\therefore F = 0.919$

Then resolve horizontally to find *F*.

as the ring is on limiting equilibrium $F = \mu R$

$\therefore \mu = 0.548$

$= 0.55$ (2 s.f.)

Use $F = \mu R$ to find $\mu = \frac{F}{R}$. Give final answer to 2 s.f. but use values of *F* and *K* to 3 s.f. in calculation of μ.

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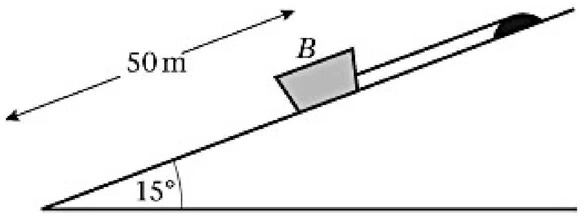
2 Review Exercise
Exercise A, Question 14

Question:



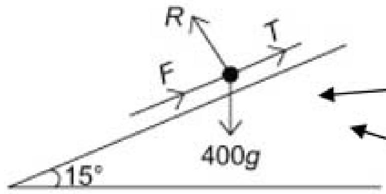
The diagram shows a boat *B* of mass 400 kg held at rest on a slipway by a rope. The boat is modelled as a particle and the slipway as a rough plane inclined at 15° to the horizontal. The coefficient of friction between *B* and the slipway is 0.2. The rope is modelled as a light, inextensible string, parallel to a line of greatest slope of the plane. The boat is in equilibrium and on the point of sliding down the slipway.

a Calculate the tension in the rope.

The boat is 50 m from the bottom of the slipway. The rope is detached from the boat and the boat slides down the slipway.

b Calculate the time taken for the boat to slide to the bottom of the slipway.

Solution:



Draw a diagram showing the forces acting.

The friction acts up the plane, as the motion would be down the plane.

Let T be the tension, F the friction and R the normal reaction.

a R(↖)

$$R - 400\,g\cos 15^\circ = 0$$
$$\therefore R = 400\,g\cos 15^\circ$$
$$= 3786$$

First find force R.

As friction is limiting, $F = \mu R$

$$\therefore \text{F} = 0.2 \times 3786$$
$$= 757$$

Use $F = \mu R$ to find F.

R(↗)

$$T + F - 400\,g\sin 15^\circ = 0$$

Then resolve along the plane to find T.

$$\therefore T = 400\,g\sin 15^\circ - F$$
$$= 257.3 = 257 \text{ (3 s.f.) or } 260 \text{ (2 s.f.)}$$

b When the rope is detached, the resultant force is 257 N acting down the plane.

The forces were in equilibrium, when T is removed the resultant force is equal and opposite to T.

Using Newton's Second Law $F = ma$,

$$257 = 400\,a$$
$$\therefore a = \frac{257}{400} = 0.643$$

Find the constant acceleration a, then use one of the constant acceleration formulae to find t.

Consider the motion down the plane.
$u = 0, a = 0.643, s = 50, t = ?$
Use

Choose this equation as it relates u, a, s and t.

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 50 = 0 + \frac{1}{2} \times 0.643t^2$$

$$\therefore t^2 = \frac{2 \times 50}{0.643}$$

$$= 155.5\ldots$$

$$\therefore t = 12.5 \text{ (3 s.f.) or } t = 12 \text{ (1 s.f.)}$$

Make t^2 the subject of the equation, then square root to obtain t.

Time to slide down is 12.5 s.

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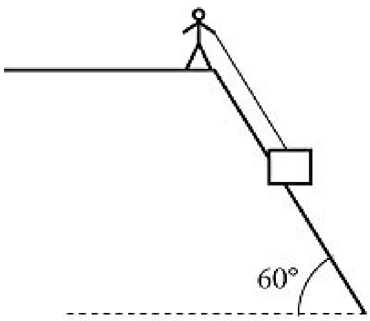
2 Review Exercise
Exercise A, Question 15

Question:



A heavy package is held in equilibrium on a slope by a rope. The package is attached to one end of the rope, the other end being held by a man standing at the top of the slope. The package is modelled as a particle of mass 20 kg. The slope is modelled as a rough plane inclined at 60° to the horizontal and the rope as a light inextensible string. The string is assumed to be parallel to a line of greatest slope of the plane, as shown in the diagram. At the contact between the package and the slope, the coefficient of friction is 0.4.

a Find the minimum tension in the rope for the package to stay in equilibrium on the slope.

The man now pulls the package up the slope. Given that the package moves at constant speed,

b find the tension in the rope.

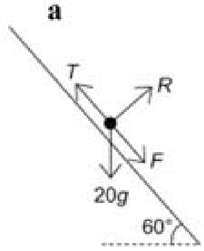
c State how you have used, in your answer to part **b**, the fact that the package moves

i up the slope,

ii at constant speed.

Solution:

a



Draw a diagram, showing the forces acting.

The friction is directed up the plane to prevent slipping down the plane.

Let T be the minimum tension, F the force of friction and R the normal reaction.

R(↗)

$$R - 20g\cos 60^\circ = 0$$
$$\therefore R = 20g\cos 60^\circ$$
$$= 98$$

Resolve perpendicular to the plane first as the resulting equation has only one unknown.

As the friction is limiting, $F = \mu R$.

$$\therefore F = 0.4 \times 98$$
$$= 39.2$$

When the tension is a minimum the friction is limiting.

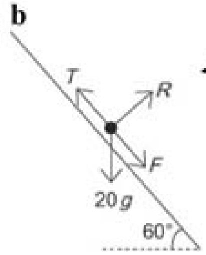
R(↖)

$$T + F - 20g\sin 60^\circ = 0$$
$$\therefore T = 20g\sin 60^\circ - F$$
$$= 130.5$$

Resolve along the plane and substitute the value for F, to give T.

Tension = 131 N (3 s.f.) or 130 N (2 s.f.)

b



Draw a new diagram with F directed down the plane.

R(↗)
$R = 98$ as before

As the 'particle' is moving, friction is limiting, so $F = 39.2$ as before.

Use $F = \mu R$ with μ and R the same as in part **a**.

R(↖)
$T - F - 20g\sin 60^\circ = 0$
$\therefore T = 39.2 + 20g\sin 60^\circ$

Tension = 209 N (3 s.f.) or 210 N (2 s.f.)

N.B. for $131 < T < 209$ the package is in equilibrium.

c
i friction acts down the slope and has magnitude 0.4 R

Friction opposes motion.

ii there is no acceleration so the net force on the package is zero

This is a result of Newton's First and Second Laws.

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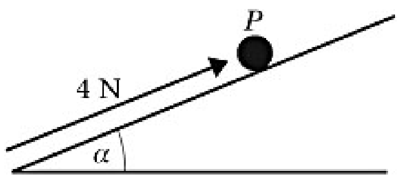
2 Review Exercise
Exercise A, Question 16

Question:



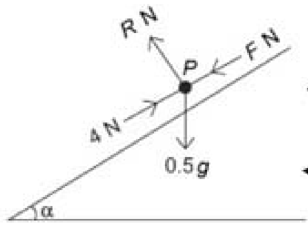
A particle *P* of mass 0.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is held at rest on the plane by the action of a force of magnitude 4 N acting up the plane in a direction parallel to a line of greatest slope of the plane, as shown. The particle is on the point of slipping up the plane.

a Find the coefficient of friction between *P* and the plane.

The force of magnitude 4 N is removed.

b Find the acceleration of *P* down the plane.

Solution:



Draw a diagram showing the forces acting on P.

Friction acts down the plane to oppose slipping up the plane.

a Let the normal reaction be R N, and the friction be F N.

R(↖)

$$R - 0.5g\cos\alpha = 0$$
$$\therefore R = 0.5g \times \frac{4}{5}$$
$$= 3.92\ (3\text{s.f.})$$

Find R by resolving perpendicular to the plane.

R(↗)

$$4 - F - 0.5g\sin\alpha = 0$$
$$\therefore F = 4 - 0.5g\sin\alpha$$
$$= 1.06\ (3\text{s.f.})$$

Find F by resolving parallel to the plane.

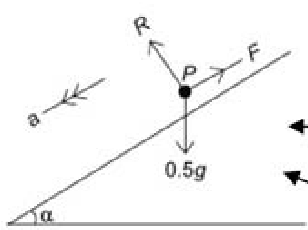
Use $F = \mu R$, as the friction is limiting.

$$\therefore \mu = \frac{F}{R} = \frac{1.06}{3.92}$$
$$\therefore \mu = 0.270\ (3\text{s.f.})$$

As the particle is on the point of slipping, $F = \mu R$.

b The force of 4 N is removed.

The friction will now act up the plane



Friction acts up the plane, in the opposite direction to the motion.

The forces perpendicular to the plane are unaltered and so R is unchanged.

As before: $R = 3.92$ and

$$F = \mu R$$
$$= 1.06$$

R(↗)

$0.5\,g\sin\alpha - F = 0.5\,a$

∴

$$\therefore a = \left(0.5\,g \times \frac{3}{5} - 1.06\right) \div 0.5$$
$$= 3.76 \text{ (3s.f.)}$$

Also μ and therefore F are unchanged.

use Newton's Second Law along the plane $F = ma$.

Rearrange the equation to find a.

∴ Acceleration is 3.76 m s^{-2} (3 s.f.) down the plane or 3.8 m s^{-2} (2 s.f.).

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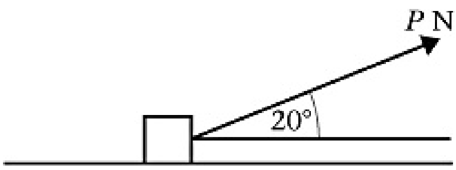
2 Review Exercise
Exercise A, Question 17

Question:



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

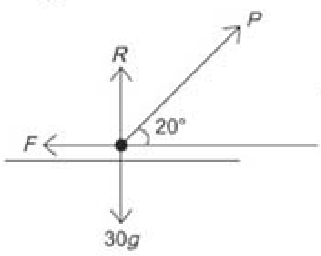
a Find the value of P.

The tension in the rope is now increased to 150 N.

b Find the acceleration of the box.

Solution:

a



Draw a diagram showing the forces acting on the box.

As the box is moving $F = \mu R$
i.e. $F = 0.4\,R$ (1)

The friction is limiting.

R(↑)

Resolve vertically and horizontally to give two equations.

$R + P\sin 20^\circ - 30\,g = 0$

$\therefore R = 30\,g - P\sin 20^\circ$ (2)

R(→)

$P\cos 20^\circ - F = 0$ (3)

$\therefore F = P\cos 20^\circ$ (3)

As the speed is constant, the resultant force is zero

Substitute R from equation (2) and F from equation (3) into equation (1).
Then $P\cos 20^\circ = 0.4(30\,g - P\sin 20^\circ)$

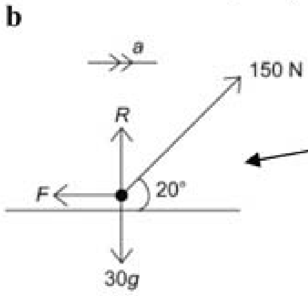
$\therefore P\cos 20^\circ + 0.4\,P\sin 20^\circ = 0.4 \times 30\,g$

$\therefore P(\cos 20^\circ + 0.4\sin 20^\circ) = 0.4 \times 30\,g$

$$\therefore P = \frac{12\,g}{(\cos 20^\circ + 0.4\sin 20^\circ)}$$
$$= 125.3$$
$$= 125\,(3\text{s.f.}) \text{ or } 130\,(2\text{s.f.})$$

Solve simultaneous equations to find the value of P.

b



Draw a new diagram.

Again $F = 0.4\,R$

R(↑)

$R + 150\sin 20^\circ - 30\,g = 0$

$\therefore R = 30\,g - 150\sin 20^\circ$

$= 242.7$

As $F = 0.4\,R$

$F = 97.1$

Resolve vertically to find R.

Use limiting equilibrium to find F.

R(→)

$150\cos 20^\circ - 97.1 = 30\,a$

$\therefore 30\,a = 43.88$

i.e. $a = 1.46$

$\therefore$ acceleration of the box is 1.46 m s^{-2}

Use Newton's Second Law $F = ma$ along the plane.

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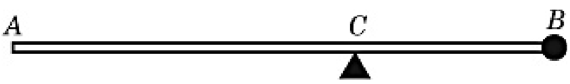
2 Review Exercise
Exercise A, Question 18

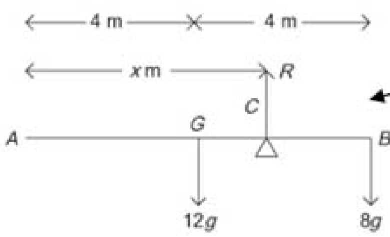
Question:



A uniform rod *AB* has length 8 m and mass 12 kg. A particle of mass 8 kg is attached to the rod at *B*. The rod is supported at a point *C* and is in equilibrium in a horizontal position, as shown.

Find the length of *AC*.

Solution:



Draw a diagram showing weights and distances.

Let the length $AC = x$ m.
Let G be the mid-point of AB.

The distance $GC = (x-4)$ m
The distance $BC = (8-x)$ m.

You will need these distances in the moments equation.

As the beam is uniform the weight $12g$ acts through G.

Take moments about C.

Anticlockwise moment $= 12g(x-4)$

Use moment = force × distance.

Clockwise moment $= 8g(8-x)$

$$\therefore 12g(x-4) = 8g(8-x)$$
$$\therefore 12gx - 48g = 64g - 8gx$$

Use anticlockwise moment = clockwise moment.

$$\therefore 20gx = 112g$$
$$\therefore x = \frac{112}{20} = 5.6$$

Make x the subject of the formula.

$\therefore$ The length AC is 5.6 m.

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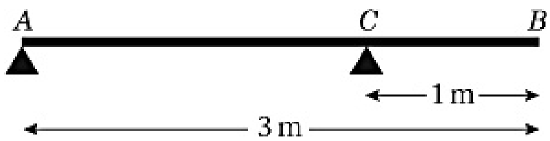
2 Review Exercise

Exercise A, Question 19

Question:



A uniform beam *AB* has mass 12 kg and length 3 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end *A*, the other at a point *C* on the beam, where *BC* = 1 m, as shown in the diagram. The beam is modelled as a uniform rod.

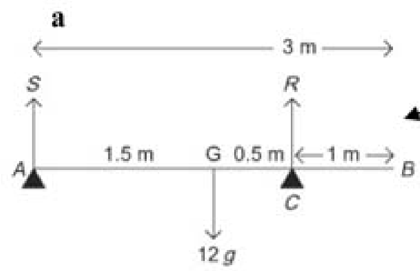
a Find the reaction on the beam at *C*.

A woman of mass 48 kg stands on the beam at the point *D*. The beam remains in equilibrium. The reactions on the beam at *A* and *C* are now equal.

b Find the distance *AD*.

Solution:

a



Draw a diagram showing the reactions at the supports and the weight of the beam.

As the beam is uniform the weight acts through G, the mid-point of AB.
Let the reaction at A be S N and the reaction at C be R N.

Take moments about A.

Distance $AC = 3\text{ m} - 1\text{ m} = 2\text{ m}$

$$R \times 2 = 12g \times 1.5$$

$$R = \frac{12g \times 1.5}{2}$$

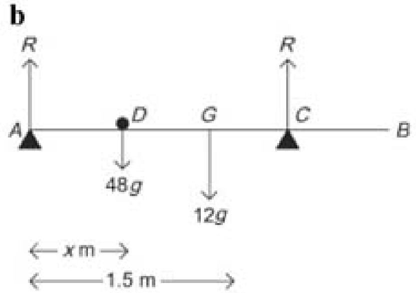
$$\therefore R = 88.2\text{ N (3 s.f.)}$$

The weight of the beam acts at its mid-point G.
$AG = \frac{1}{2} \times 3\text{ m} = 1\frac{1}{2}\text{ m}$

Use anticlockwise moment = clockwise moment.

Make R the subject of the equation.

b



Draw a new diagram showing the forces acting and the distances required.

Let the distance AD be x m and let the reactions at A and C be R N.

R(↑)

$$R + R - 12g - 48g = 0$$

$$\therefore R = 30g$$

Resolve to find the new value of R.

Take moments about A.
$12g \times 1.5 + 48g \times x - 30g \times 2 = 0$

Use clockwise moment = anticlockwise moment.

$$\therefore 48gx = 60g - 18g$$

$$= 42g$$

$$\therefore x = \frac{42}{48} = 0.875\text{ m}$$

Make x the subject of the formula.

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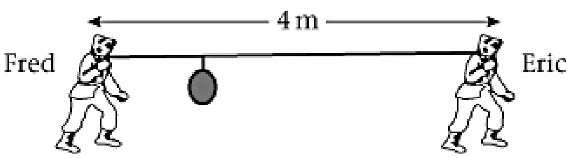
2 Review Exercise
Exercise A, Question 20

Question:

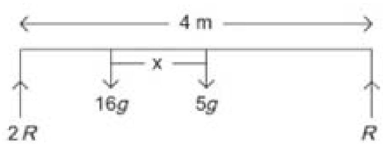


Two men, Eric and Fred, set out to carry a water container across a desert, using a long uniform pole. The length of the pole is 4 m and its mass is 5 kg. The ends of the pole rest in equilibrium on the shoulders of the two men, with the pole horizontal. The water container has mass 16 kg and is suspended from the pole by means of a light rope, which is short enough to prevent the container reaching the ground, as shown. Eric has a sprained ankle, so Fred fixes the rope in such a way that the vertical force on his shoulder is twice as great as the vertical force on Eric's shoulder.

a Find the vertical force on Eric's shoulder.

b Find the distance from the centre of the pole to the point at which the rope is fixed.

Solution:



Draw a diagram showing the forces acting.

Let the force on Eric's shoulder be R and the force on Fred's shoulder be $2R$.

R(↑)

$3R - 16g - 5g = 0$

$\therefore 3R = 21g$

$R = 7g$

Resolve vertically to find R.

Take moments about Fred's shoulder.
Let distance from centre of pole to point at which rope is fixed = x m.
Then

$5g \times 2 + 16g \times (2 - x) = R \times 4$

$\therefore 10g + 32g - 16gx = 28g$

$\therefore 16gx = 14g$

$\therefore x = \frac{7}{8}$ m

As the pole is uniform the weight acts at the middle i.e. 2 m from left hand end (Fred's shoulder).

Use clockwise moment = anticlockwise moment

$(2 - x)$ is distance from Fred's shoulder to weight $16g$.

Make x the subject of the formula.

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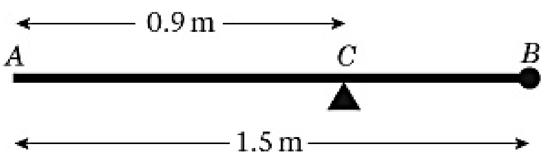
2 Review Exercise
Exercise A, Question 21

Question:



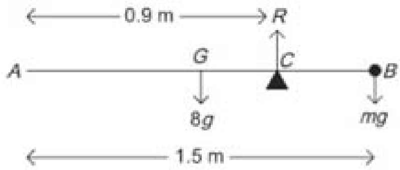
A uniform rod *AB* has length 1.5 m and mass 8 kg. A particle of mass *m* kg is attached to the rod at *B*. The rod is supported at the point *C*, where *AC* = 0.9 m, and the system is in equilibrium with *AB* horizontal, as shown.

a Show that $m = 2$.

A particle of mass 5 kg is now attached to the rod at *A* and the support is moved from *C* to a point *D* of the rod. The system, including both particles, is again in equilibrium with *AB* horizontal.

b Find the distance *AD*.

Solution:



Draw a diagram, showing the forces and distances given.

a The weight $8g$ N acts at the mid-point of AB.

$$\text{So } AG = 0.75 \text{ m}$$
$$\therefore GC = 0.9\text{ m} - 0.75\text{ m} = 0.15\text{ m}$$
$$\text{Also } CB = 0.75\text{ m} - 0.15\text{ m} = 0.6\text{ m}$$
$$\text{or } CB = 1.5\text{ m} - 0.9\text{ m} = 0.6\text{ m}$$

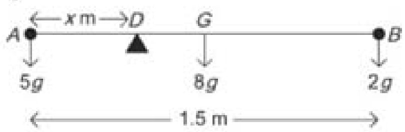
Calculate the other lengths needed in the moments equation.

Take moments about C.

$$mg \times 0.6 = 8\,g \times 0.15$$
$$\therefore m = \frac{8 \times 0.15}{0.6}$$
$$\therefore m = 2$$

Use clockwise moment = anticlockwise moment.

b



Draw a new diagram to illustrate the new situation.

Let the distance $AD = x$ m.

$$\text{Then } DG = (0.75 - x)\text{ m}$$
$$\text{and } BD = (1.5 - x)\text{ m}$$

Express distances from D in terms of x.

Take moments about D.

$$2\,g \times (1.5 - x) + 8\,g \times (0.75 - x) = 5\,gx$$
$$\therefore 3\,g - 2\,gx + 6\,g - 8\,gx - 5\,gx = 0$$
$$\therefore 9\,g = 15\,gx$$
$$\therefore x = \frac{9}{15} = 0.6\text{ m}$$

Use clockwise moment = anticlockwise moment.

Make x the subject of the formula.

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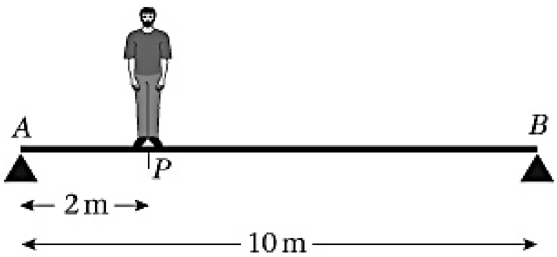
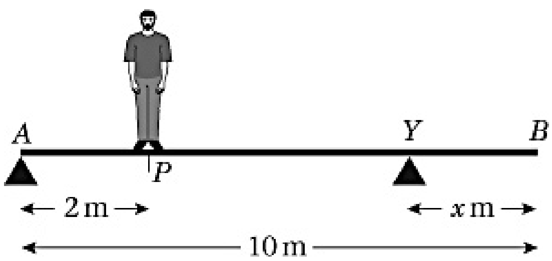
2 Review Exercise
Exercise A, Question 22

Question:



A uniform steel girder *AB*, of mass 150 kg and length 10 m, rests horizontally on two supports at *A* and *B*. A man of mass 90 kg stands on the girder at the point *P*, where *AP* = 2 m, as shown. By modelling the girder as a uniform rod and the man as a particle.

a find the magnitude of the reaction at *B*.



The support *B* is moved to a point *Y* on the girder, where *BY* = *x* metres, as shown. The man remains on the girder at *P*. The magnitudes of the reactions at the two supports are now equal.

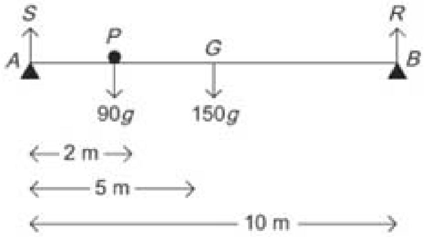
Find

b the magnitude of the reaction at each support,

c the value of *x*.

Solution:

a



Draw a diagram showing the weights acting, the reaction at R and distances.

Let G be the mid-point of AB. As the girder is uniform the weight acts through G.
Let the reaction at B be R N.

The reaction at A is not needed in this part of the question.

Take moments about A.

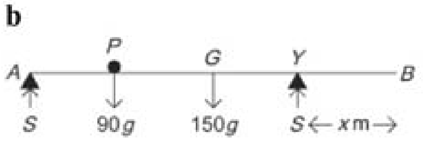
$R \times 10 - 150\,g \times 5 - 90\,g \times 2 = 0$

$$\begin{aligned} \therefore 10\,R &= 750\,g + 180\,g \\ \therefore R &= 930\,g \div 10 \\ &= 93\,g \\ &= 911\text{ N} (3\text{ s.f.}) \end{aligned}$$

or 910 N (2 s.f.)

Make R the subject of the equation.

b



Draw a new diagram showing the weights acting and the two equal reactions.

Let the reactions at A and Y be S N.

$R(\uparrow)$

$$\begin{aligned} 2\,S - 90\,g - 150\,g &= 0 \\ \therefore 2\,S = 240\,g \Rightarrow S &= 120\,g = 1176\text{ N} \\ \therefore\ S &= 1200\text{ N } (2\text{ s.f.}) \end{aligned}$$

Resolve to find S.

c Take moments about A.

Take moments about either point A or point Y.

$$\begin{aligned} S \times (10 - x) &= 150\,g \times 5 - 90\,g \times 2 \\ \therefore 10 - x &= \frac{750\,g + 180\,g}{S} = \frac{930\,g}{120\,g} \\ \therefore x &= 10 - \frac{93}{12} \\ &= 2.25\text{ m} \end{aligned}$$

Make x the subject of the formula.

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2 Review Exercise
Exercise A, Question 23

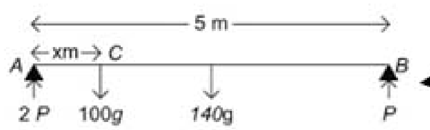
Question:

A footbridge across a stream consists of a uniform horizontal plank *AB* of length 5 m and mass 140 kg, supported at the ends *A* and *B*.
A man of mass 100 kg is standing at a point *C* on the footbridge. Given that the magnitude of the force exerted by the support at *A* is twice the magnitude of the force exerted by the support at *B*, calculate

a the magnitude, in N, of the force exerted by the support at *B*,

b the distance *AC*.

Solution:



Draw a diagram showing forces and distances.

a R(↑)
$2P + P - 140g - 100g = 0$

Let the reaction at *B* be *P* and at *A* be 2*P*.
Let the distance $AC = x$ m.

$\therefore 3P = 240g$
i.e. $P = 80g$
$= 784$ N
$= 780$ N (2 s.f.)

Resolve vertically to find *P*.

b Let distance *AC* be *x* m.

Take moments about *A* (or about *B*) to find the distance *x*.

Take moments about *A*.
$P \times 5 = 140g \times 2.5 + 100gx$

Substituting $P = 784$
$784 \times 5 - 140 \times 9.8 \times 2.5 = 100 \times 9.8x$
$\therefore x = 0.5$
The distance *AC* is 0.5 m.

Use total anticlockwise moment = clockwise moment.

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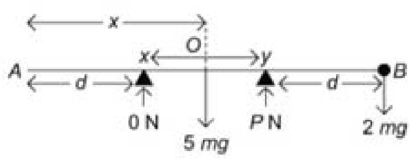
2 Review Exercise
Exercise A, Question 24

Question:

A non-uniform thin straight rod *AB* has length 3*d* and mass 5*m*. It is in equilibrium resting horizontally on supports at the points *X* and *Y*, where *AX* = *XY* = *YB* = *d*. A particle of mass 2*m* is attached to the rod at *B*. Given that the rod is on the point of tilting about *Y*, find the distance of the centre of mass of the rod from *A*.

Solution:



Let the required distance be x m.

When the rod begins to tilt about Y the force exerted by the support at $X = 0$ N.

This is an important fact which you should know.

Take moments about Y.

$$5\,mg(2d - x) = 2\,mg\,d$$
$$\therefore 10\,mgd - 5\,mg\,x = 2\,mg\,d$$
$$\therefore 10\,mgd - 2\,mg\,d = 5\,mg\,x$$

Total of the moments anti-clockwise = total of the moments clockwise.

$$\text{i.e. } 8\,mgd = 5\,mgx$$
$$\therefore x = \frac{8}{5}d$$
$$= 1.6\,d$$

Make x the subject of the formula.

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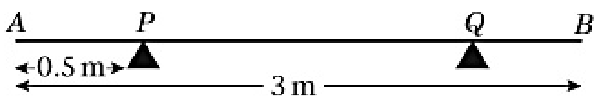
2 Review Exercise
Exercise A, Question 25

Question:

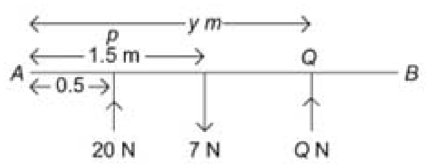


A uniform rod *AB* has weight 70 N and length 3 m. It rests in a horizontal position on two smooth supports placed at *P* and *Q*, where *AP* = 0.5 m as shown in the diagram. The reaction on the rod at *P* has magnitude 20 N. Find

a the magnitude of the reaction on the rod at *Q*,

b the distance *AQ*.

Solution:



Let the reaction at Q be Q N and let the distance $AQ = y$ m.

a $R(\uparrow)$

$20 + Q - 70 = 0$

$\therefore Q = 70 - 20$

$= 50$

Resolve to find Q.

$\therefore$ The reaction at Q is 50 N.

b Take moments about P.

$70 \times 1 = Q(y - 0.5)$

Total clockwise moment = total anticlockwise moment.

But $Q = 50$

$\therefore 50(y - 0.5) = 70$

i.e. $50y = 95$

$\therefore y = 1.9$

Substitute Q and make y the subject of the formula.

The distance AQ is 1.9 m.

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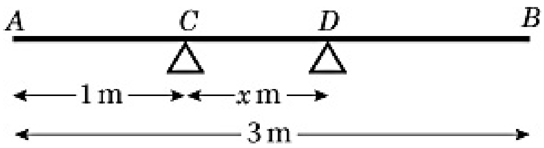
2 Review Exercise
Exercise A, Question 26

Question:



A uniform plank *AB* has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports *C* and *D*, where $AC = 1$ m and $CD = x$ m, as shown. The reaction of the support on the plank at *D* has magnitude 80 N. Modelling the plank as a rod.

a show that $x = 0.75$.

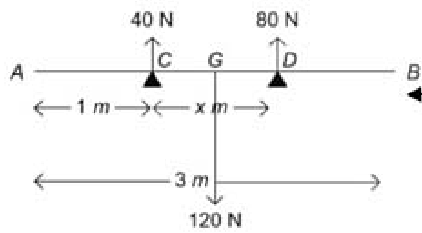
A rock is now placed at *B* and the plank is on the point of tilting about *D*. Modelling the rock as a particle, find

b the weight of the rock,

c the magnitude of the reaction of the support on the plank at *D*.

d State how you have used the model of the rock as a particle.

Solution:



Draw a diagram showing weights, distances and reactions relevant to the problem.

The reaction at C is not relevant to the problem because we are taking moments about C.

a Let G be the mid-point of AB.
The weight of the plank acts through G as the plank is modelled as a uniform rod.

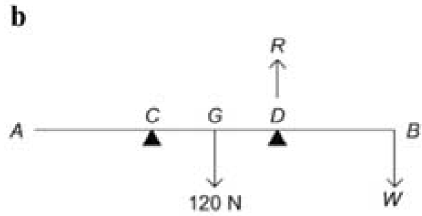
Take moments about C.

The weight of the plane acts at the mid-point.

$$80x = 120 \times 0.5$$
$$\therefore x = \frac{120 \times 0.5}{80}$$
$$\therefore x = 0.75$$

Make x the subject of the equation.

b



Draw a new diagram, showing the rock of weight W at point B.

When the plank is on the point of tilting about D, the reaction at C is zero.

Take moments about D.

Take moments about D as the resulting equation has one unknown W.

$$120 \times 0.25 = W \times 1.25$$
$$\therefore W = \frac{120 \times 0.25}{1.25} = 24 \text{ N}$$

Distance $DB = CB - CB = 2\text{m} - 0.75\text{ m}$

c $R(\uparrow)$

Resolve vertically or take moments about point C to find R.

$$R - 120 - W = 0$$
$$\therefore R = 120 + 24 = 144 \text{ N}$$

d The weight of the rock acts precisely at B.

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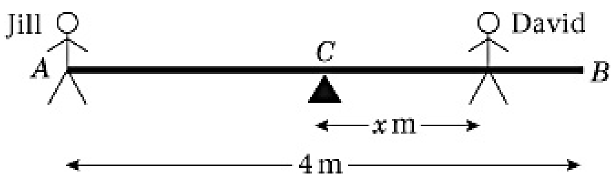
2 Review Exercise
Exercise A, Question 27

Question:



A seesaw in a playground consists of a beam *AB* of length 4 m which is supported by a smooth pivot at its centre *C*. Jill has mass 25 kg and sits on the end *A*. David has mass 40 kg and sits at a distance *x* metres from *C*, as shown. The beam is initially modelled as a uniform rod. Using this model,

a find the value of *x* for which the seesaw can rest in equilibrium in a horizontal position.

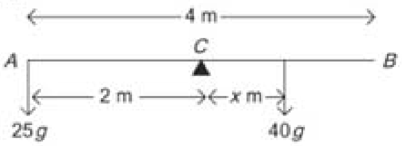
b State what is implied by the modelling assumptions that the beam is uniform.

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from *C* for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

c find the distance of the centre of mass of the beam from *C*.

Solution:

a



Draw a diagram, showing weights and distances.

Take moments about C.

As C is the mid-point the distance $AC = 2$ m.

$$25\,g \times 2 = 40\,g \times x$$

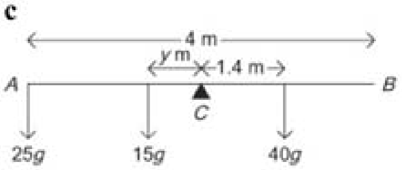
$$\therefore x = \frac{50\,g}{40\,g}$$

$$x = 1.25$$

b The weight of the beam acts through its mid-point at C.

The weight has zero moment about C.

c



Draw a new diagram showing weights and distances.

Let the distance of the centre of mass from C be y m.
Take moments about C.
$25\,g \times 2 + 15\,g \times y = 40\,g \times 1.4$

Use total anticlockwise moment = total clockwise moment.

$$\therefore 15\,gy = 56\,g - 50\,g$$

$$\therefore y = \frac{6\,g}{15\,g}$$

$$= 0.4$$

Make y the subject of the formula.

$\therefore$ Distance of centre of mass from C is 0.4 m.

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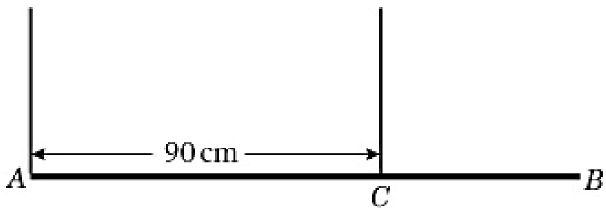
2 Review Exercise
Exercise A, Question 28

Question:



A steel girder *AB* has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end *A*. The other cable is attached to the point *C* on the girder, where *AC* = 90 cm, as shown. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at *C* is twice the tension in the cable at *A*, find

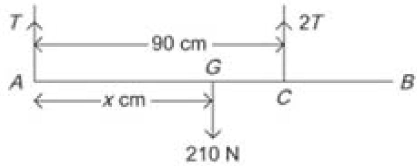
a the tension in the cable at *A*,

b show that *AB* = 120 cm.

A small load of weight *W* newtons is attached to the girder at *B*. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at *C* is now three times the tension in the cable at *A*.

c Find the value of *W*.

Solution:



Draw a diagram showing forces and distances.

Let the weight act through G at x cm from A.

a Let the tension in the cable at A be T N and at C be $2T$ N.

R(↑)

$$2T + T - 210 = 0 \Rightarrow 3T - 210 = 0$$
$$\therefore T = 70 \text{ N}$$

Resolve vertically as T is the only unknown in this equation.

b Take moments about A.

$$2T \times 90 = 210 \times x$$
$$\therefore x = \frac{180T}{210}$$

Substitute $T = 70$

$$\therefore x = \frac{180 \times 70}{210} = 60 \text{ cm}$$

It is also valid to take moments about C.

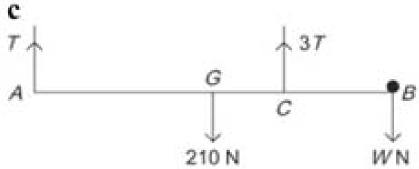
Express x in terms of T and use the value of T found in part **a** to give x.

As G is the mid-point of AB

$\therefore AB = 120$ cm

You were told that the girder is modelled as a uniform rod, so $AB = 2x$ cm.

c



Draw a new diagram, showing the load at B.

R(↑)

$$T + 3T - 210 - W = 0$$
$$\therefore 4T - W = 210 \qquad (1)$$

Resolve and take moments to give two equations in T and W.

Take moments about A.

$$3T \times 90 = W \times 120 - 210 \times 60$$

You could take moments about C to give a second equation.

Divide equation by 30.

$$\therefore 9T - 4W = 420 \qquad (2)$$

Solve the simultaneous equations.

$9 \times (1) - 4 \times (2)$ gives

$$-9W + 16W = 1890 - 1680$$
$$\text{i.e. } 7W = 210$$
$$\therefore W = 30$$

Solve the equation to find W.

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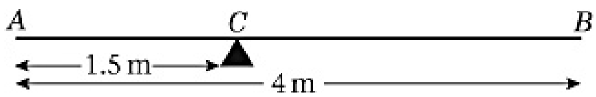
2 Review Exercise
Exercise A, Question 29

Question:

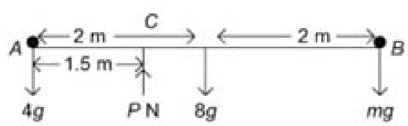


A uniform rod *AB* has mass 8 kg and length 4 m. A particle of mass 4 kg is attached to the rod at *A* and a particle of mass *M* kg is attached to the rod at *B*. The rod is supported at the point *C*, where *AC* = 1.5 *m*, and rests in equilibrium in a horizontal position, as shown in the diagram.

a Find the value of *M*.

b State how you used the information that the rod is uniform.

Solution:



a Take moments about *C*.

Using total of anti-clockwise moments = total of clockwise moments.

$$4g \times 1.5 = 8g \times 0.5 + Mg \times 2.5$$
$$\therefore 6g - 4g = 2.5\,Mg$$
$$\therefore \frac{2g}{2.5g} = M$$
$$\therefore M = 0.8$$

Make *M* the subject of the formula.

b The weight acts through the mid-point of the rod.

For a uniform rod its weight acts through the mid-point.

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2 Review Exercise
Exercise A, Question 30

Question:

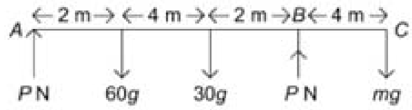
A uniform plank *ABC*, of length 12 m and mass 30 kg, is supported in a horizontal position at the points *A* and *B*, where *AB* = 8 m and *BC* = 4 m, as shown in the diagram. A woman of mass 60 kg stands on the plank at a distance of 2 m from *A*, and a rock of mass *M* kg is placed on the plank at the end *C*. The plank remains in equilibrium. The plank is modelled as a uniform rod, and the woman and the rock as particles.

Given that the forces exerted by the supports on the plank at *A* and *B* are equal in magnitude,

a find **i** the value of *M*, **ii** the magnitude of the force exerted by the support at *A* on the plank.

b State how you used the modelling assumption that the rock is a particle.

Solution:



Let the reactions at A and B be P N.

a i R(↑)

$2P - 60g - 30g - Mg = 0$

$\therefore 2P - Mg = 90g$ (1)

Take moments about A.

$P \times 8 = 60g \times 2 + 30g \times 6 + Mg \times 12$

$\therefore 8P - 12Mg = 300g$

÷ through by 4

Then $2P - 3Mg = 75g$ (2)

Solve by subtracting equation (2) from equation (1).

Then $2Mg = 15g$

$\therefore M = \dfrac{15g}{2g}$

$= 7.5$ kg

As there are two unknowns, P and M, you need two equations.

You can resolve and you can take moments about A or B.

Using anticlockwise moment = clockwise moment.

Then solve simultaneous equations to find M.

ii Substitute into equation (1)

$\therefore 2P = 90g + 7.5g$

$\therefore P = \dfrac{97.5g}{2}$

$= 48.75g$

$\therefore$ Force has magnitude 477.75 N = 480 N (2 s.f.).

Substitute into one of the equations to find P.

b Assumed that the centre of mass acts at the point C.

A particle is a point mass and its weight acts at C.

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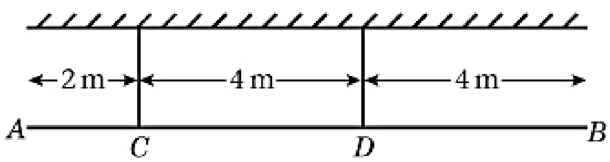
2 Review Exercise
Exercise A, Question 31

Question:



A light rod AB has length 10 m. It is suspended by two light vertical cables attached to the rod at the points C and D, where $AC = 2$ m, $CD = 4$ m, and $DB = 4$ m, as shown in the diagram. A load of weight 60 N is attached to the rod at A and a load of weight XN is attached to the rod at B. The rod is hanging in equilibrium in a horizontal position. Find, in terms of X,

a the tension in the cable at C,

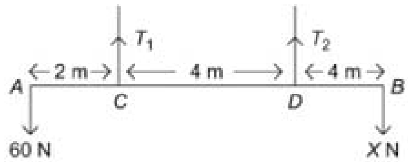
b the tension in the cable at D,

c Hence show that $15 \leq X \leq 90$.

If the tension in either cable exceeds 120 N that cable breaks.

d Find the maximum possible value of X.

Solution:



Let the tensions in the tables be T_1 and T_2 respectively.

a Take moments about *D*.

$60 \times 6 = T_1 \times 4 + X \times 4$

$\therefore T_1 \times 4 = 360 - 4X$

$\therefore T_1 = 90 - X$

Using anticlockwise moment = clockwise moment.

b Take moments about *C*.

$T_2 \times 4 + 60 \times 2 = X \times 8$

$\therefore T_2 \times 4 = 8X - 120$

$\therefore T_2 = 2X - 30$

Again using anticlockwise moment = clockwise moment.

c As T_1, T_2 are both positive

As the cables are under tension, T_1 and T_2 are both positive.

$90 - X \geq 0 \Rightarrow X \leq 90$
and $2X - 30 \geq 0 \Rightarrow X \geq 15$
so $15 \leq X \leq 90$

d T_1 cannot reach 120.

$T_1 = 90 - X$ which cannot exceed 90 (from part **a**).

If $T_2 > 120$, then

$2X - 30 > 120$

$\therefore 2X > 150$

i.e. $X > 75$

$T_2 = 2x - 30$, was found in part **b**.

The maximum possible value of X is 75.

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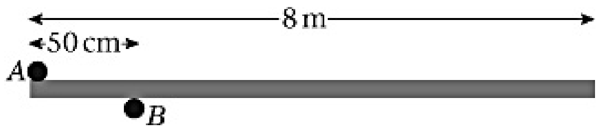
Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 32

Question:



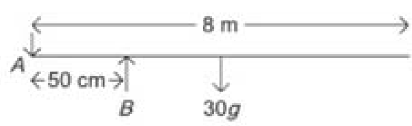
A large uniform plank of wood of length 8 m and mass 30 kg is held in equilibrium by two small steel rollers *A* and *B*, ready to be pushed into a saw-mill. The centres of the rollers are 50 cm apart. One end of the plank presses against roller *A* from underneath, and the plank rests on top of roller *B*, as shown in the diagram. The rollers are adjusted so that the plank remains horizontal and the force exerted on the plank by each roller is vertical.

a Suggest a suitable model for the plank to determine the forces exerted by the rollers.

b Find the magnitude of the force exerted on the plank by the roller at *B*.

c Find the magnitude of the force exerted on the plank by the roller at *A*.

Solution:



Let the forces at *A* and *B* be *A* N and *B* N.

a Model the plank as a uniform rod.

This implies the plank has negligible thickness.

b Take moments about *A*.

$$B \times 0.5 = 30g \times 4$$
$$\therefore B = \frac{120g}{0.5}$$
$$= 240g = 2400 \text{ (2 s.f.)}$$

Using total moment anticlockwise = total moment clockwise.

c $R(\uparrow)$

$$B - A - 30g = 0$$
$$\therefore A = B - 30g$$
$$= 240g - 30g$$
$$= 210g = 2100 \text{ (2 s.f.)}$$

Resolve or take moments about *B*, to find the force exerted at *A*.

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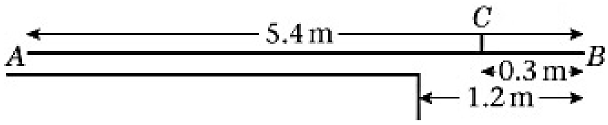
2 Review Exercise
Exercise A, Question 33

Question:



A plank of wood *AB* has length 5.4 m. It lies on a horizontal platform, with 1.2 m projecting over the edge, as shown in the diagram. When a girl of mass 50 kg stands at the point *C* on the plank, where *BC* = 0.3 m, the plank is on the point of tilting. By modelling the plank as a uniform rod and the girl as a particle,

a find the mass of the plank.

The girl places a rock on the end of the plank at *A*. By modelling the rock also as a particle,

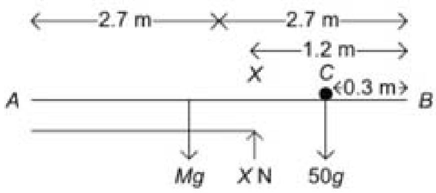
b find, to two significant figures, the smallest mass of the rock which will enable the girl to stand on the plank at *B* without tilting.

c State briefly how you have used the modelling assumption that

i the plank is uniform,

ii the rock is a particle.

Solution:



Let the edge of the platform be point X. Let the mass of the plank be M kg.

a When it is on the point of tilting the force of the platform on the rod acts at X (the edge of the platform).

The weight of the plank acts at its mid-point.

Take moments about X.
$Mg \times (2.7 - 1.2) = 50\,g(1.2 - 0.3)$

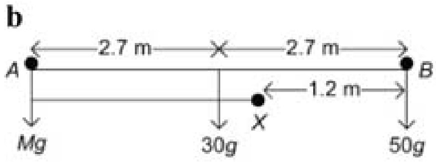
Using anti-clockwise moment = clockwise moment.

$$\therefore Mg \times 1.5 = 50\,g \times 0.9$$
$$\therefore 1.5\,Mg = 45\,g$$
$$\therefore Mg = \frac{45\,g}{1.5}$$
$$\therefore M = 30$$

Make M the subject of the formula.

$\therefore$ The mass of the plank is 30 kg.

b



This is a new situation so draw a new diagram. Let the mass of the rock be m kg.

Take moments about X again.

Using anticlockwise moment = clockwise moment.

$$30\,g \times (2.7 - 1.2) + mg \times (5.4 - 1.2) = 50\,g \times 1.2$$
$$\therefore mg \times 4.2 = 50\,g \times 1.2 - 30\,g \times 1.5$$
$$\text{i.e. } 4.2\,mg = 60\,g - 45\,g$$
$$= 15\,g$$

Make m the subject and evaluate to 2 s.f.

$$\therefore m = \frac{15\,g}{4.2\,g}$$
$$= 3.6 \text{ (2 s.f.)}$$
$$\therefore \text{mass} = 3.6 \text{ kg (2 s.f.)}$$

c i Plank is uniform $\rightarrow$ weight acts through the mid-point.
ii Rock is a particle $\rightarrow$ mass of rock acts through the end-point A.

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2 Review Exercise
Exercise A, Question 34

Question:

Three forces F_1, F_2 and F_3 act on a particle.

$F_1 = (-3i + 7j)$ N , $F_2 = (i - j)$ N , $F_3 = (pi + qj)$ N

a Given that the particle is in equilibrium, determine the value of p and the value of q.

The resultant of the forces F_1 and F_2 is **R.**

b Calculate, in N, the magnitude of **R.**

c Calculate to the nearest degree, the angle between the line of action of **R** and the vector **j**.

Solution:

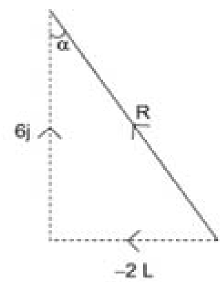
a If the particle is in equilibrium then $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

Use the condition for equilibrium.

$$\therefore -3\mathbf{i} + 7\mathbf{j} + \mathbf{i} - \mathbf{j} + p\mathbf{i} + q\mathbf{j} = 0$$
$$\therefore (p-2)\mathbf{i} + (q+6)\mathbf{j} = 0$$
$\therefore p = 2$ and $q = -6$

Collect **i** terms and **j** terms.

b The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is



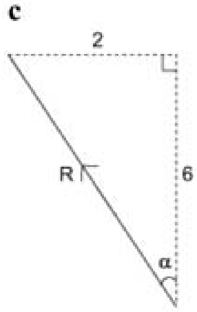
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$
$$\therefore \mathbf{R} = -3\mathbf{i} + 7\mathbf{j} + \mathbf{i} - \mathbf{j}$$
$$= -2\mathbf{i} + 6\mathbf{j}$$

$\therefore$ The magnitude of **R** is $\sqrt{(-2)^2 + 6^2}$

$$= \sqrt{4+36}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10} \text{ or } 6.32 \text{ (3 s.f.)}$$

Use Pythagoras' Theorem to find the magnitude of **R**.

c



Let the angle between **R** and **j** be α.
Then

$$\tan\alpha = \frac{2}{6}$$
$$\therefore \alpha = \arctan\left(\frac{1}{3}\right)$$
$$= 18^\circ \text{ (nearest degree)}$$

Use trigometry to find the required angle.

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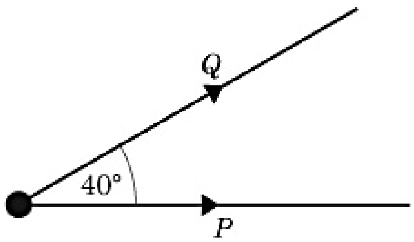
2 Review Exercise
Exercise A, Question 35

Question:



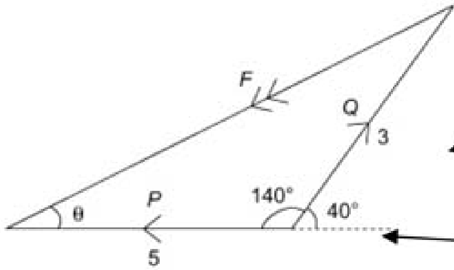
Two forces **P** and **Q**, act on a particle. The force **P** has magnitude 5 N and the force **Q** has magnitude 3 N. The angle between the directions of **P** and **Q** is 40°, as shown in the diagram. The resultant of **P** and **Q** is **F**.

a Find, to three significant figures, the magnitude of **F**.

b Find, in degrees to one decimal place, the angle between the directions of **F** and **P.**

Solution:

a Draw a vector triangle:-



Draw a triangle to illustrate $\mathbf{P}+\mathbf{Q}=\mathbf{F}$.

The angle in the triangle $=180^\circ-40^\circ$
$=140^\circ$

Use the cosine rule:

$$|\mathbf{F}|^2 = 5^2+3^2-2\times5\times3\times\cos140^\circ$$
$$\therefore|\mathbf{F}| = 7.55\ (3\text{ s.f.})$$

b Use the sine rule:

$$\frac{\sin\theta}{3} = \frac{\sin140^\circ}{|\mathbf{F}|}$$
$$\therefore\sin\theta = 3\times\frac{\sin140^\circ}{7.55}$$
$$= 0.255$$
$$\therefore\theta = 14.8^\circ\ (1\text{ d.p.})$$

Rearrange to make $\sin\theta$ the subject of the formula.

$\theta=\arcsin 0.255$

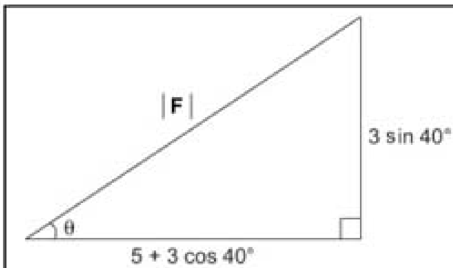
Alternative method

a $\mathbf{P}=\begin{pmatrix}5\\0\end{pmatrix}$ and $\theta=\begin{pmatrix}3\cos40^\circ\\3\sin40^\circ\end{pmatrix}$

Express the forces in component form.

$$\therefore\mathbf{P}+\mathbf{Q} = \begin{pmatrix}5+3\cos40^\circ\\3\sin40^\circ\end{pmatrix}$$
$$\therefore|\mathbf{F}|^2 = (5+3\cos40^\circ)^2+(3\sin40^\circ)^2$$
$$\therefore|\mathbf{F}| = 7.55\ (3\text{ s.f.})$$

Use $\mathbf{F}=\mathbf{P}+\mathbf{Q}$ to give the components of $\mathbf{F}$.



Use Pythagoras' Theorem to find $|\mathbf{F}|$

b

$$\tan\theta = \frac{3\sin40^\circ}{5+3\cos40^\circ}$$
$$\therefore\theta = 14.8^\circ\ (1\text{ d.p.})$$

Use trigonometry of the triangle to find θ.

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2 Review Exercise
Exercise A, Question 36

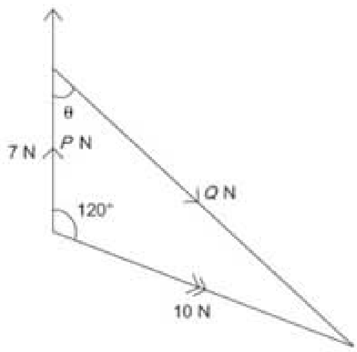
Question:

Two forces **P** and **Q** act on a particle. The force **P** has magnitude 7 N and acts due north. The resultant of **P** and **Q** is a force of magnitude 10 N acting in a direction with bearing 120°. Find

a the magnitude of **Q**,

b the direction of **Q**, giving your answer as a bearing.

Solution:



Draw a triangle to illustrate **P** and **Q** and their resultant.

a Use the cosine rule to find **Q**

$$\mathbf{Q}^2 = 7^2 + 10^2 - 2\times 7\times 10\times \cos 120^\circ$$
$$= 219$$
$$\therefore \mathbf{Q} = 14.8$$

Find **Q** by using the cosine rule.

b

$$\frac{\sin\theta}{10} = \frac{\sin 120^\circ}{Q}$$
$$\therefore \sin\theta = 10\times\frac{\sin 120^\circ}{14.8}$$
$$= 0.585$$
$$\therefore \theta = 35.8^\circ$$

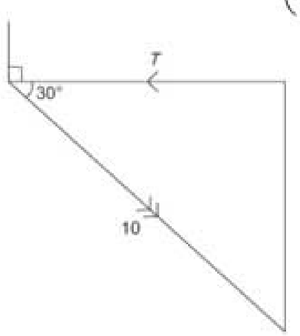
Find θ by using the sine rule, then calculate $180-\theta$, the

$\therefore$ The direction of **Q** is $180^\circ - 35.8^\circ = 144.2^\circ$.

Alternative method

a Resultant $\mathbf{P}+\mathbf{Q} = \begin{pmatrix} 10\cos 30^\circ \\ -10\sin 30^\circ \end{pmatrix}$

Express the forces in component form.



Express the resultant force and the force **P** in component form.

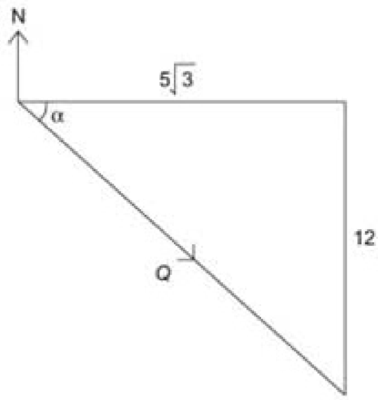
Also $\mathbf{P} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

$$\therefore \mathbf{Q} = \begin{pmatrix} 10\cos 30^\circ \\ -10\sin 30^\circ \end{pmatrix} - \begin{pmatrix} 0 \\ 7 \end{pmatrix} = \begin{pmatrix} 5\sqrt{3} \\ -12 \end{pmatrix}$$

Find **Q** in resultant form.

Magnitude $= \sqrt{(5\sqrt{3})^2 + 12^2} = 14.8$ (3 s.f.)

b



Use a sketch to find the required bearing.

In the diagram $\tan\alpha = \dfrac{12}{5\sqrt{3}}$

$\therefore \alpha = 54.2^\circ$

$$\therefore \text{bearing} = 90^\circ + 54.2^\circ$$
$$= 144^\circ (3 \text{ s.f.})$$

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2 Review Exercise
Exercise A, Question 37

Question:

Two forces $F_1 = (2i + 3j)$ N and $F_2 = (\lambda i + \mu j)$ N , where λ and μ are scalars, act on a particle. The resultant of the two forces is **R**, where **R** is parallel to the vector $i + 2j$.

a Find, to the nearest degree, the acute angle between the line of action of **R** and the vector **i**.

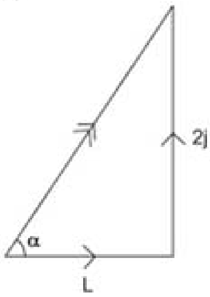
b Show that $2\lambda - \mu + 1 = 0$.

Given that the direction of F_2 is parallel to **j**,

c find, to three significant figures, the magnitude of **R**.

Solution:

a



Draw a diagram to show the vector $\mathbf{i}+2j$.

Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63.4^\circ$

As $\mathbf{R}$ is parallel to $\mathbf{i}+2\mathbf{j}$, $\mathbf{R}$ also makes an angle α with $\mathbf{i}$.

b As $\mathbf{F}_1+\mathbf{F}_2=\mathbf{R}$
$(2\mathbf{i}+3\mathbf{j})+(\lambda\mathbf{i}+\mu\mathbf{j})=k(\mathbf{i}+2\mathbf{j})$

Let $\mathbf{R}=k(\mathbf{i}+2\mathbf{j})$ where k is constant.

where k is a constant.
$\therefore 2+\lambda = k$ and $3+\mu = 2k$ *

Put $\mathbf{i}$ components equal.

Put $\mathbf{j}$ components equal.

Eliminate k from these two equations.
Then

$$2(2+\lambda) = 3+\mu$$
$$\therefore 2\lambda-\mu+1 = 0 \quad *$$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda = 0$.
Substituting $\lambda = 0$ into * gives

$\mu = 1$ and $k = 2$

Substitute $k=2$ into $\mathbf{R}=k(\mathbf{i}+2\mathbf{j})$

$$\therefore \mathbf{R} = 2\mathbf{i}+4\mathbf{j}$$
$$\therefore |\mathbf{R}| = \sqrt{2^2+4^2}$$
$$= \sqrt{20}$$
$$= 4.47 \text{ (3 s.f.)}$$

The magnitude of $x\mathbf{i}+y\mathbf{j}=\sqrt{x^2+y^2}$ from Pythagoras Theorem.

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2 Review Exercise
Exercise A, Question 38

Question:

A force **R** acts on a particle, where $R = (7i + 16j)$ N .

Calculate

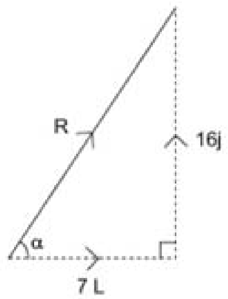
a the magnitude of **R**, giving your answers to one decimal place,

b the angle between the line of action of **R** and **i**, giving your answer to the nearest degree.

The force **R** is the resultant of two forces **P** and **Q**. The line of action of P is parallel to the vector $(i + 4j)$ and the line of action of **Q** is parallel to the vector $(i + j)$.

c Determine the forces **P** and **Q** expressing each in terms of **i** and **j**.

Solution:



Use Pythagoras to find the magnitude.

a The magnitude of

$$\mathbf{R} = \sqrt{7^2 + 16^2}$$
$$= 17.5 \text{ (1 d.p.)}$$

b

$$\tan\alpha = \frac{16}{7}$$
$$\therefore \alpha = \arctan\left(\frac{16}{7}\right)$$
$$= 66^\circ \text{(nearest degree)}$$

Use trigonometry to find the angle and give it to the nearest degree.

c Let

$$\mathbf{P} = \lambda(\mathbf{i}+4\mathbf{j}) \text{ and}$$
$$\mathbf{Q} = \mu(\mathbf{i}+\mathbf{j})$$

Express **P** and **Q** as multiples of the vectors $\mathbf{i}+4\mathbf{j}$ and $\mathbf{i}+\mathbf{j}$ respectively.

As $\mathbf{P}+\mathbf{Q}=\mathbf{R}$

$\therefore \lambda(\mathbf{i}+4\mathbf{j})+\mu(\mathbf{i}+\mathbf{j})=(7\mathbf{i}+16\mathbf{j})$

$\mathbf{P}+\mathbf{Q}=\mathbf{R}$ is the vector addition law.

Equating **i** components

$\lambda+\mu=7$ (1)

Equating **j** components

$4\lambda+\mu=16$ (2)

Subtract $(2)-(1)$

$$3\lambda = 9$$
$$\therefore \lambda = 3$$

Solve the simultaneous equations to find λ and μ.

Substitute into equation (1)

$$\therefore 3+\mu = 7$$
$$\therefore \mu = 4$$

$$\therefore \mathbf{P} = 3(\mathbf{i}+4\mathbf{j}) = 3\mathbf{i}+12\mathbf{j} \text{ and}$$
$$\mathbf{Q} = 4(\mathbf{i}+\mathbf{j}) = 4\mathbf{i}+4\mathbf{j}$$

Substitute λ and μ to find **P** and **Q** in terms of **i** and **j**.

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2 Review Exercise
Exercise A, Question 39

Question:

A particle *P* moves in a straight line with constant velocity. Initially *P* is at the point A with position vector $(2i - j)$ m relative to a fixed origin *O*, and 2s later it is at the point *B* with position vector $(6i + j)$ m.

a Find the velocity of *P*.

b Find, in degrees to one decimal place, the size of the angle between the direction of motion of *P* and the vector **i**.

Three second after it passes *B* the particle *P* reaches the point *C*.

c Find, in metres to one decimal place, the distance *OC*.

Solution:

a

$$\text{velocity} = \frac{\text{difference in displacement}}{\text{time}}$$
$$= \frac{(6\mathbf{i}+\mathbf{j})-(2\mathbf{i}-\mathbf{j})}{2}$$
$$= \frac{4\mathbf{i}+2\mathbf{j}}{2}$$
$$= 2\mathbf{i}+\mathbf{j}$$

Use the definition for constant velocity.

b

Let the angle between v and $\mathbf{i}$ be α.

This angle gives the direction of motion of P which is asked for in the question.

Then $\tan\alpha = \frac{1}{2}$

$$\therefore \alpha = \arctan\left(\frac{1}{2}\right)$$
$$= 26.6^\circ \text{ (1 d.p.)}$$

c

$$\text{Displacement} = \text{velocity} \times \text{time}$$
$$\text{In 3 s displacement} = 3(2\mathbf{i}+\mathbf{j})$$
$$\therefore \overrightarrow{BC} = 6\mathbf{i}+3\mathbf{j}$$

But

$$\overrightarrow{OB} = 6\mathbf{i}+\mathbf{j}$$
$$\therefore \overrightarrow{OC} = \overrightarrow{OB}+\overrightarrow{BC}$$
$$= 12\mathbf{i}+4\mathbf{j}$$

Use the vector law of addition to find $\overrightarrow{OC}$.

The distance

$$OC = \sqrt{12^2+4^2}$$
$$= \sqrt{144+16}$$
$$= \sqrt{160}$$
$$= 12.6 \text{ m (1 d.p.)}$$

Use Pythagoras' Theorem to find the distance OC.

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2 Review Exercise
Exercise A, Question 40

Question:

A particle P is moving with constant velocity $(5i - 3j)$ m s^{-1}. At time $t = 0$, its position vector, with respect to a fixed origin O, is $(-2i + j)$ m. Find, to three significant figures.

a the speed of P,

b the distance of P from O when $t = 2s$.

Solution:

a The velocity $u = 5\mathbf{i} - 3\mathbf{j}$
The speed

$$|u| = \sqrt{5^2 + (-3)^2}$$
$$|u| = \sqrt{34}$$
$$= 5.83 \text{ M s}^{-1}$$

Use the formula for magnitude of a vector.

b The displacement from $t = 0$ to $t = 2$ is $(5\mathbf{i} - 3\mathbf{j}) \times 2$

Find the displacement between $t = 0$ and $t = 2$, using velocity $\times$ time.

The position vector of P at $t = 2$ is $(-2\mathbf{i} + \mathbf{j}) + (10\mathbf{i} - 6\mathbf{j})$

Use the vector law of addition.

$= 8\mathbf{i} - 5\mathbf{j}$
$\therefore$ The distance of P from O is

$$\sqrt{8^2 + (-5)^2}$$
$$= \sqrt{64 + 25}$$
$$= \sqrt{89}$$
$$= 9.43$$

Use the formula for the magnitude of a vector.

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2 Review Exercise
Exercise A, Question 41

Question:

A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3i - 4j)$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8i + 11j)$ km.

a Find the velocity of B, giving your answer in the form $pi + qj$.

At time t hours after noon, the position vector of B is **b** km.

b Find, in terms of t, an expression for **b**.

Another boat C is also moving with constant velocity. The position vector of C, **c** km, at time t hours after noon, is given by

$c = (-9i + 20j) + t(6i + \ -j)$, where λ is a constant.

Given that C intercepts B,

c find the value of λ,

d show that, before C intercepts B, the boats are moving with the same speed.

Solution:

a Displacement between noon and 14.30 is

$$(8\mathbf{i}+11\mathbf{j}) - (3\mathbf{i}-4\mathbf{j}) = (5\mathbf{i}+15\mathbf{j})$$
$$\therefore \text{velocity} = (5\mathbf{i}+15\mathbf{j}) \div 2\frac{1}{2} \text{ km/h}$$
$$= (2\mathbf{i}+6\mathbf{j}) \text{ km/h.}$$

Use velocity $= \dfrac{\text{displacement}}{\text{time}}$

b The position vector t hours after noon is

$\mathbf{b} = (3\mathbf{i}-4\mathbf{j}) + (2\mathbf{i}+6\mathbf{j})t$ for boat B.

Use vector law of addition.

c For boat C

$\mathbf{c} = (-9\mathbf{i}+20\mathbf{j}) + t(6\mathbf{i}+\lambda\mathbf{j})$

When C intercepts B

$$\begin{pmatrix} 3+2t \\ -4+6t \end{pmatrix} = \begin{pmatrix} -9+6t \\ 20+\lambda t \end{pmatrix}$$

Equate the position vectors of B and of C.

$$\therefore 3+2t = -9+6t$$
$$\Rightarrow 12 = 4t$$
$$\therefore t = 3$$

Equate the **i** components and solve to give the value for t.

Also

$$-4+6t = 20+\lambda t \text{ at } t=3$$
$$\therefore -4+18 = 20+3\lambda$$
$$\therefore -6 = 3\lambda$$
$$\therefore \lambda = -2$$

Equate the **j** components and use your value for t to give λ.

d

The velocity of boat B = $(2\mathbf{i}+6\mathbf{j})$ kmh^{-1}
The velocity of boat C = $(6\mathbf{i}-2\mathbf{j})$ kmh^{-1}

The magnitude of the velocities are $\sqrt{2^2+6^2}$ and $\sqrt{6^2+(-2)^2}$ which are equal.

speed of boat B = speed of boat C

$$= \sqrt{36+4} = \sqrt{40} \text{ kmh}^{-1}$$

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2 Review Exercise
Exercise A, Question 42

Question:

A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}. Find

a the acceleration of P in terms of **i** and **j**,

b the magnitude of **F**,

c the velocity of P at time $t = 6$ s.

Solution:

a As the force is constant, the acceleration is constant.
Given $u = 3\mathbf{i} + 2\mathbf{j}, v = 15\mathbf{i} - 4\mathbf{j}$ and $t = 4$, to find a use

$$v = u + at \text{ or } v - u = at$$
$$\therefore 15\mathbf{i} - 4\mathbf{j} - (3\mathbf{i} + 2\mathbf{j}) = 4a$$
$$\text{i.e. } 12\mathbf{i} - 6\mathbf{j} = 4a$$
$$\therefore a = 3\mathbf{i} - 1.5\mathbf{j}$$

You may use constant acceleration formulae.

Choose this as it connects u, v, a and t.

b Use

$$\mathbf{F} = ma$$
$$\mathbf{F} = 2(3\mathbf{i} - 1.5\mathbf{j})$$
$$= 6\mathbf{i} - 3\mathbf{j}$$

Use Newton's Second Law.

$$\therefore |\mathbf{F}| = \sqrt{6^2 + (-3)^2}$$
$$= \sqrt{45}$$
$$= 6.71$$

Use the formula for magnitude of a vector.

c Use

$$v = u + at \text{ with } t = 6$$
$$\therefore v = 3\mathbf{i} + 2\mathbf{j} + 6(3\mathbf{i} - 1.5\mathbf{j})$$
$$= 21\mathbf{i} - 7\mathbf{j}$$

Use the value for a found in part **a**.

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2 Review Exercise
Exercise A, Question 43

Question:

The horizontal unit vectors **i** *and* **j** *are due east and due north respectively.*

A ship *S* is moving with constant velocity $(-2.5\mathbf{i} + 6\mathbf{j})\text{km h}^{-1}$. At time 1200, the position vector of *S* relative to a fixed origin *O* is $(16\mathbf{i} + 5\mathbf{j})\text{km}$. Find

a the speed of *S*,

b the bearing on which *S* is moving.

The ship is heading directly towards a submerged rock *R*. A radar tracking station calculates that, if *S* continues on the same course with the same speed, it will hit *R* at the time 1500.

c Find the position vector of *R*.

The tracking station warns the ship's captain of the situation. The captain maintains *S* on its course with the same speed until the time is 1400. He then changes course so that *S* moves due north at a constant speed of 5 km h^{-1}. Assuming that *S* continues to move with this new constant vector, find

d an expression for the position vector of the ship *t* hours after 1400.

e The time when *S* will be due east of *R*.

f The distance of *S* from *R* at the time 1500.

Solution:

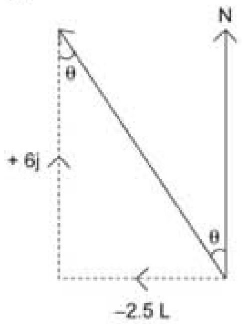
a $v = -2.5\mathbf{i} + 6\mathbf{j}$

$$\therefore \text{speed} = |v| = \sqrt{(-2.5)^2 + 6^2} = \sqrt{42.25} = 6.5 \text{ kmh}^{-1}$$

You know that speed is the magnitude of velocity.

Use the formula for magnitude of a vector.

b



The velocity vector indicates the direction of motion, so draw a diagram showing this vector.

$$\tan\theta = \frac{2.5}{6}$$

$$\therefore \theta = 22.6^\circ$$

The bearing on which S moves is 337°.

Use trigonometry to find θ.

Use geometry $360^\circ - 22.6^\circ$ to find the bearing (measured clockwise from North).

c At 15.00 the ship has moved for 3 hours since noon.

$$\text{Its displacement} = (-2.5\mathbf{i} + 6\mathbf{j}) \times 3 = -7.5\mathbf{i} + 18\mathbf{j}$$

Use velocity × time = displacement.

It started at $16\mathbf{i} + 5\mathbf{j}$ so finishes at $8.5\mathbf{i} + 23\mathbf{j}$.

Use vector addition $16\mathbf{i} + 5\mathbf{j} - 7.5\mathbf{i} + 18\mathbf{j}$

This is position vector of R.

d At 14.00 the ship has moved for 2 hours since noon to $(-2.5\mathbf{i} + 6\mathbf{j}) \times 2$

$= -5\mathbf{i} + 12\mathbf{j}$ from noon position.

Use velocity × time.

$\therefore$ At 14.00 ship is at $16\mathbf{i} + 5\mathbf{j} - 5\mathbf{i} + 12\mathbf{j}$

$= 11\mathbf{i} + 17\mathbf{j}$

Use vector addition.

It then moves North at 5 kmh^{-1} for t hours.

Its displacement is $5t\mathbf{j}$

$\therefore$ t hours after 14.00 ship is at

$11\mathbf{i}+(17+5t)\mathbf{j}$

Use vector addition.

e Ship is due east of R when

Equate **j** components and find t.

$$23\mathbf{j} = (17+5t)\mathbf{j}$$

$$\text{i.e. } t = \frac{6}{5}\text{ hrs}$$

$$= 1\text{ hr } 12\text{ min}$$

The time at which ship is due east of R is 15.12

Use $t=\frac{6}{5}$ hrs after 14.00 to give the time.

f At 16.00 the ship is at

$11\mathbf{i}+(17+10)\mathbf{j}=11\mathbf{i}+27\mathbf{j}$

Substitute t=2 into the position vector for S.

The distance of S from R is

$$\sqrt{\left((11-8.5)^2+(27-23)^2\right)}$$

$$= \sqrt{\left((2.5)^2+4^2\right)}$$

$$= \sqrt{22.25}$$

$$= 4.72\text{ km (3 s.f.)}$$

Use the formula, based on Pythagoras' Theorem, for the distance between two points.

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2 Review Exercise
Exercise A, Question 44

Question:

The horizontal unit vectors **i** *and* **j** *are due east and due north respectively.*

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})\,\text{m}\,\text{s}^{-1}$. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})\,\text{m}$. Find

a the speed of A,

b the direction in which A is moving, giving your answer as a bearing.

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})\,\text{m}$.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$,

c show that A and B will collide at a point P and find the position vector of P.

Given instead that B has speed $8\,\text{m}\,\text{s}^{-1}$ and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$.

d find the distance of B from P when $t = 7s$.

Solution:

a

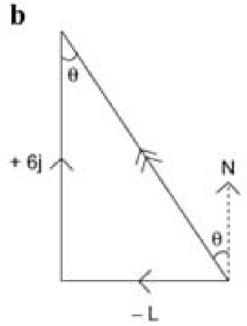
Velocity of A is $-\mathbf{i}+6\mathbf{j}$

$\therefore$ Speed of A $=\sqrt{(-1)^2+6^2} = \sqrt{37}$ m s^{-1}

$= 6.08$ m s^{-1} (3 s.f.)

Use the definition that speed is the magnitude of velocity and use the formula for magnitude of a vector.

b



Draw a diagram showing the velocity vector.

From the triangle:

$\tan\theta = \frac{1}{6}$

$\therefore \theta = 9.46^\circ$

Use trigonometry to find θ, then calculate the bearing (angle measured clockwise from North).

$\therefore$ Bearing $= 360^\circ - 9.46^\circ$

$= 351^\circ$ (3 s.f.)

c At time t:
position vector of A is $\mathbf{r} = 2\mathbf{i}-10\mathbf{j}+t(-\mathbf{i}+6\mathbf{j})$
position vector of B is $\mathbf{r} = -26\mathbf{i}+4\mathbf{j}+t(3\mathbf{i}+4\mathbf{j})$

Use displacement = velocity × time .

If A and B collide for some value of t,

$(2-t)\mathbf{i}+(-10+6t)\mathbf{j} = (-26+3t)\mathbf{i}+(4+4t)\mathbf{j}$
Equate **i** and **j** components.

Equate components of position vectors and solve equations to find t.

$2 - t = -26 + 3t$ and $-10 + 6t = 4 + 4t$

In both cases $t = 7$

$\therefore$ They do collide at

$$2\mathbf{i} - 10\mathbf{j} + 7(-\mathbf{i} + 6\mathbf{j}) = -5\mathbf{i} + 32\mathbf{j}$$

Check that position vectors of A and of B are the same when $t = 7$.

The position vector of P is $-5\mathbf{i} + 32\mathbf{j}$.

d The new velocity of B is $\frac{8}{5}(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}

The new velocity is $\frac{8}{5}$ the previous velocity as the previous speed was 5 m s^{-1}(3, 4, 5Δ).

Position vector of B at $t = 7$ is

$$-26\mathbf{i} + 4\mathbf{j} + \frac{8}{5}(3\mathbf{i} + 4\mathbf{j}) \times 7 = 7.6\mathbf{i} + 48.8\mathbf{j}$$

$$\therefore \overrightarrow{PB} = \mathbf{b} - \mathbf{p} = 12.6\mathbf{i} + 16.8\mathbf{j}$$

$$|\overrightarrow{PB}| = \text{distance} = \sqrt{12.6^2 + 16.8^2} = 21 \text{ m.}$$

From the vector triangle law $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$.

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2 Review Exercise
Exercise A, Question 45

Question:

The horizontal unit vectors **i** *and* **j** *are due east and due north respectively.*

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

a the speed of the ball,

b the position vector of the ball after t seconds.

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

c Find the time when the ball is due north of B.

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

d find the value of v.

e State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic.

Solution:

a

velocity $= 5\mathbf{i}+8\mathbf{j}$ m s^{-1}
$\therefore$ speed $= \sqrt{5^2+8^2}$ m s^{-1}
$= \sqrt{89}$ m s^{-1} $= 9.43$ m s^{-1} (3 s.f.)

Use speed = magnitude of velocity.
Use formula for magnitude of vector.

b After t seconds, position vector is

$\mathbf{r} = 2\mathbf{i}+\mathbf{j}+t(5\mathbf{i}+8\mathbf{j})$

Use displacement = velocity × time.

c When the ball is due north of $10\mathbf{i}+7\mathbf{j}$

$2+5t = 10$
$\therefore 5t = 8 \Rightarrow t = 1.6$ s

Equate the **i** component of the ball to 10.

d At $t = 1.6$ ball is at $10\mathbf{i}+13.8\mathbf{j}$
The second player moves from
$10\mathbf{i}+7\mathbf{j}$ to $10\mathbf{i}+13.8\mathbf{j}$ in 1.6 s.
His velocity is $6.8\mathbf{j} \div 1.6$
His speed is $6.8 \div 1.6 = 4.25$ m s^{-1}.

Find the displacement of the second player.
Use velocity = displacement ÷ time.

e Friction on the field – so velocity of ball not constant
or
vertical component of ball's motion
or
time for player to accelerate.

Any of these answers would be valid.

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2 Review Exercise
Exercise A, Question 46

Question:

A destroyer is moving due west at a constant speed of 10 km h $^{-1}$. It has radar on board which, at time $t = 0$, identifies a cruiser, 50 km due west and moving due north with a constant speed of 20 km h $^{-1}$. The unit vectors **i** and **j** are directed due east and north respectively, and the origin *O* is taken to be the initial position of the destroyer. Each vessel maintains its constant velocity.

a Write down the velocity of each vessel in vector form.

b Find the position vector of each vessel at time *t* hours.

c Show that the distance *d* km between the vessels at time t hours is given by

$d^2 = 500t^2 - 1000t + 2500$

The radar on the cruiser detects vessels only up to a distance of 40 km. By finding the minimum value of d^2, or otherwise,

d determine whether the destroyer will be detected by the cruiser's radar.

Solution:

a

$\mathbf{v}_{\text{Destroyer}} = -10\mathbf{i}\ \text{kmh}^{-1}$

$\mathbf{v}_{\text{cruiser}} = 20\mathbf{j}\ \text{kmh}^{-1}$

As it moves *west* its direction is parallel to $-\mathbf{i}$.

The cruiser is moving due North so is a positive multiple of $\mathbf{j}$.

b

$\mathbf{r}_{\text{destroyer}} = -10t\,\mathbf{i} = \mathbf{d}$

$\mathbf{r}_{\text{cruiser}} = -50\mathbf{i} + 20t\,\mathbf{j} = \mathbf{c}$

Using $\mathbf{r}_{\text{new}} = \mathbf{r}_{\text{old}} + \mathbf{v}t$

c The vector $\overrightarrow{CD} = \mathbf{d} - \mathbf{c}$

$$\begin{aligned} &= -10t\mathbf{i} - (-50\mathbf{i} + 20t\mathbf{j}) \\ &= (50 - 10t)\mathbf{i} - 20t\mathbf{j} \\ \therefore |\overrightarrow{CD}| &= \sqrt{(50-10t)^2 + (-20t)^2} \\ &= \sqrt{2500 - 1000t + 100t^2 + 400t^2} \\ \therefore d^2 &= 500t^2 - 1000t + 2500 \end{aligned}$$

Using the triangle law, or vector subtraction.

Use Pythagoras' Theorem to find the magnitude of $\overrightarrow{CD}$.

d

$$\begin{aligned} d^2 &= 500(t^2 - 2t + 5) \\ &= 500((t-1)^2 + 4) \end{aligned}$$

The minimum value of d^2 is when $t = 1$ and

$$\begin{aligned} d^2 &= 500 \times 4 \\ &= 2000 \\ \therefore d &= \sqrt{2000} \\ &= 44.72 \end{aligned}$$

The minimum value of d^2 can be found by completion of the square.

$\therefore$ As $44.72 > 40$ cruiser will not be able to detect the destroyer.

An alternative method would be to attempt to solve $d^2 = 40^2$.
This gives a quadratic with no real solutions.

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2 Review Exercise
Exercise A, Question 47

Question:

In this question the vectors **i** and **j** are horizontal unit vectors in the directions due east and due north respectively. Two boats A and B are moving with constant velocities. Boat A moves with velocity $9j\ \text{km h}^{-1}$. Boat B moves with velocity $(3i + 5j)\ \text{km h}^{-1}$.

a Find the bearings on which B is moving.

At noon A is at the point O and B is 10 km due west of O. At time t hours after noon, the position vectors of A and B relative to O are **a** km and **b** km respectively.

b Find expressions for **a** and **b** in terms of t, giving your answer in the form

$pi + qj$

c Find the time when B is due south of A.

At time time t hours after noon, the distance between A and B is d km. By finding an expression for AB,

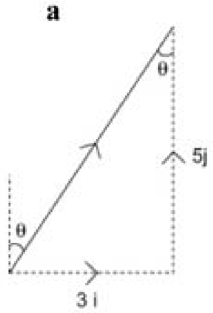
d show that $d^2 = 25t^2 - 60t + 100$.

At noon the boats are 10 km apart.

e Find the time after noon at which the boats are again 10 km apart.

Solution:

a



Let the bearing on which B is moving be α.

Then $\tan\alpha = \frac{3}{5}$

$$\therefore \alpha = \arctan(0.6) = 30.96^\circ$$

Use trigonometry to find α. The bearing is the angle measured clockwise from the North direction.

$\therefore$ bearing is 031° (nearest degree)

b

$$\mathbf{a} = 0 + 9t\,\mathbf{j}$$
$$\mathbf{b} = -10\mathbf{i} + (3\mathbf{i} + 5\mathbf{j})t = (3t - 10)\mathbf{i} + 5t\mathbf{j}$$

using $\mathbf{r}_{\text{new}} = \mathbf{r}_{\text{old}} + \mathbf{v}t$.

c When B is due south of A

$$3t - 10 = 0$$
$$\therefore t = \frac{10}{3} = 3\text{ h }20\text{ min}$$

As A has **i** component zero.

i.e. the time is 15.20

d

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = (3t - 10)\mathbf{i} - 4t\mathbf{j}$$
$$\therefore |\overrightarrow{AB}| = \sqrt{(3t-10)^2 + (-4t)^2} = \sqrt{(9t^2 - 60t + 100 + 16t^2)}$$
$$\therefore d^2 = 25t^2 - 60t + 100$$

Using the 'triangle law' or vector subtraction.

Use Pythagoras to find the magnitude of $\overrightarrow{AB}$.

e When $d = 10$

$100 = 25t^2 - 60t + 100$

Form and solve a quadratic equation in t.

$$\therefore 25t^2 - 60t = 0$$
$$\therefore 5t(5t - 12) = 0$$
$$\therefore t = 0 \text{ or } t = \frac{12}{5} = 2.4\text{ h}$$

Express your answer as a time in hours and minutes.

So the boats are 10 km apart at 14.24.