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Review Exercise 2

Exercise A, Question 1

Question:

In a game theory explain what is meant by

a zero-sum game,

b saddle point. ***E***

Solution:

a A game in which the gain to one player is equal to the loss of the other

b If there is a *stable solution*(s) a_{ij} in a game, the *location* of this stable solution is called the saddle point.
It is the point(s) where row maximin = column minimax

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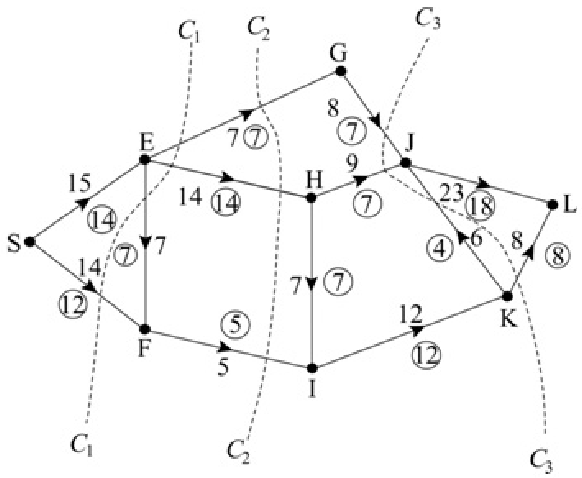
Review Exercise 2
Exercise A, Question 2

Question:



The diagram shows a network of roads represented by arcs. The capacity of the road represented by that arc is shown on each arc. The numbers in circles represent a possible flow of 26 from B to L.

Three cuts C_1, C_2 and C_3 are shown.

a Find the capacity of each of the three cuts.
b Verify that the flow of 26 is maximal.
The government aims to maximise the possible flow from S to L by using one of two options.
Option 1: Build a new road from E to J with capacity 5.
or
Option 2: Build a new road from F to H with capacity 3.
c By considering **both** options, explain which one meets the government's aim. ***E***

Solution:

a $C_1 = 7+14+0+14 = 35$
$C_2 = 7+14+5 = 26$
$C_3 = 8+9+6+8 = 31$

b Either Min cut = Max flow and we have a flow of 26 and a cut of 26
or C2 is through saturated arcs

c Using EJ (capacity 5) e.g. – will increase flow by 1 – i.e. increase it to 27 since only one more unit can leave E. – BEJL – 1
Using FH (capacity 3) e.g. – will increase flow by 2 – i.e. increase it to 28 since only two more units can leave F. – BFHJL – 2

Thus choose option 2 add FH capacity 3.

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Review Exercise 2
Exercise A, Question 3

Question:

A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays I	B plays II	B plays III
A plays I	−3	2	5
A plays II	4	−1	−4

a Write down the pay off matrix for player B.
b Formulate the game as a linear programming problem for player B, writing the constraints as equalities and stating your variables clearly. ***E***

Solution:

a

	A(I)	A(II)
B(I)	3	−4
B(II)	−2	1
B(III)	−5	4

b Add 6 to each element to make all terms positive

	A(I)	A(II)
B(I)	9	2
B(II)	4	7
B(III)	1	10

Let q_1 be the probability that B plays row 1
Let q_2 be the probability that B plays row 2
Let q_3 be the probability that B plays row 3
Let value of the game be v and let $V = v + 6$
where $q_1, q_2, q_3 \geq 0$
e.g. maximise $P = V$
Subject to $V - 9q_1 - 4q_2 - q_3 + r = 0$
$V - 2q_1 - 7q_2 - 10q_3 + s = 0$
$q_1 + q_2 + q_3 + t = 1$

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Review Exercise 2
Exercise A, Question 4

Question:

An engineering firm makes motors. They can make up to five in any one month, but if they make more than four they have to hire additional premises at a cost of £500 per month. They can store up to two motors for £100 per motor per month. The overhead costs are £200 in any month in which work is done.

Motors are delivered to buyers at the end of each month. There are no motors in stock at the beginning of May and there should be none in stock after the September delivery.

The order book for motors is:

Month	May	June	July	Aug.	Sept.
Number of motors	3	3	7	5	4

Use dynamic programming to determine the production schedule that minimises the costs, showing your working in a table. ***E***

Solution:

e.g.

Stage	**State**	**Action**	**Dest**	**Value**
1 (Sept)	2	2	0	$200+200=400$ *
	1	3	0	$200+100=300$ *
	0	4	0	$200=200$ *
2 (Aug)	2	5	2	$200+200+500+400=1300$
		4	1	$200+200+300=700$
		3	0	$200+200+200=600$ *
	1	5	1	$200+100+500+300=1100$
		4	0	$200+100+200=500$ *
	0	5	0	$200+500+200=900$ *
3 (July)	2	5	0	$200+200+500+900=1800$ *
4 (June)	2	3	2	$200+200+1800=2200$ *
	1	4	2	$200+100+1800=2100$ *
	0	5	2	$200+500+1800=2500$ *
5 (May)	0	5	2	$200+500+2200=2900$
		4	1	$200+2100=2300$ *
		3	0	$200+2500=2700$

Month	May	June	July	August	September
Production schedule	4	4	5	5	4

Cost £2300

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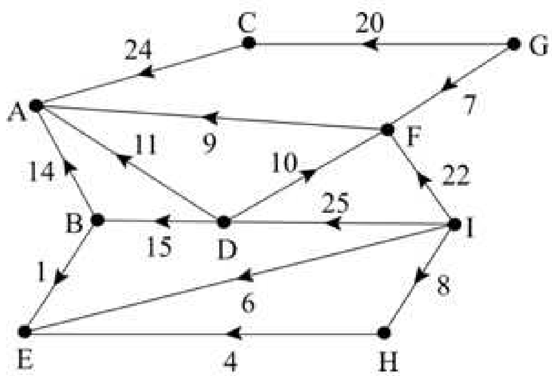
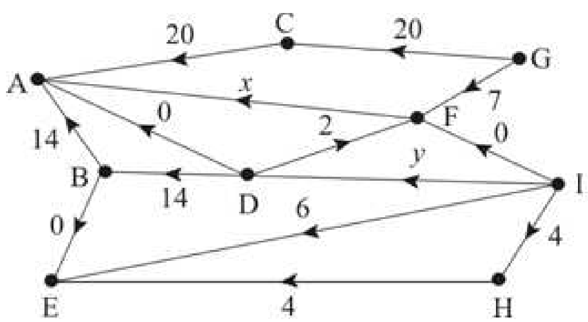
Review Exercise 2
Exercise A, Question 5

Question:



The diagram shows a capacitated directed network. The number on each arc is its capacity.



This shows a feasible initial flow through the same network.

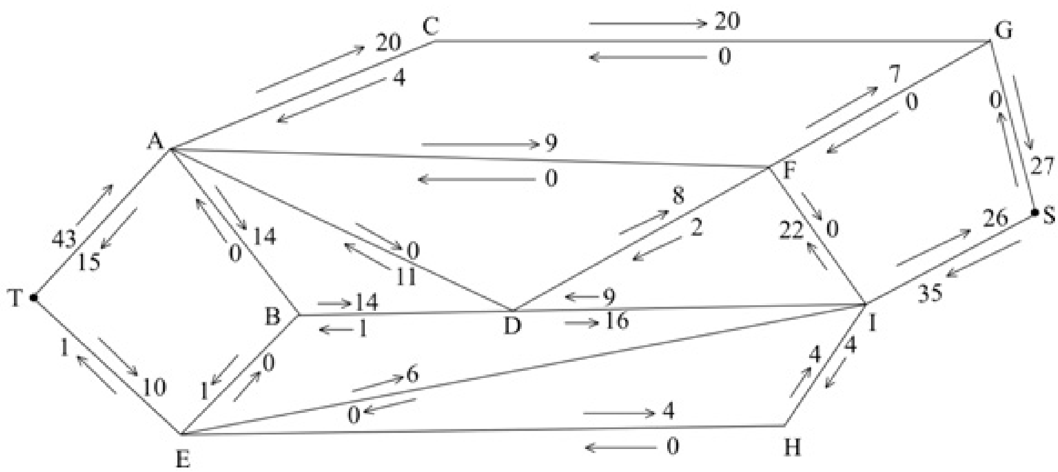
a Write down the values of the flow x and the flow y.
b Obtain the value of the initial flow through the network, and explain how you know it is not maximal.
c Use this initial flow and the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.
d Show your maximal flow pattern.
e Prove that your flow is maximal. ***E***

Solution:

a $x = 9, y = 16$

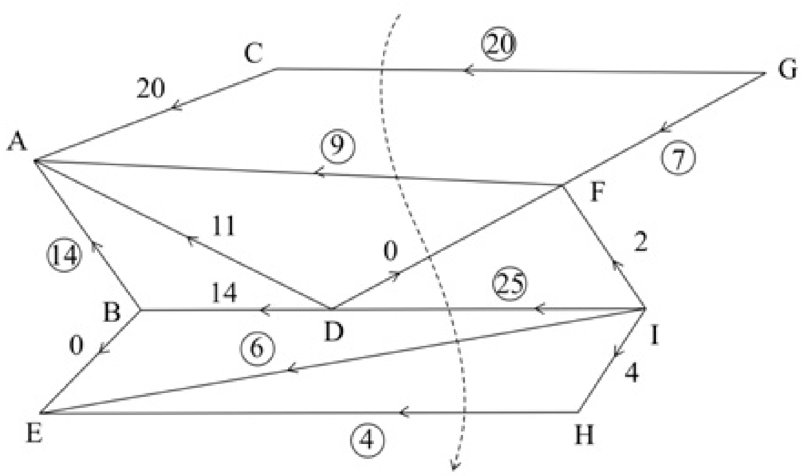
b Initial flow = 53 – either finds a flow-augmenting route or demonstrates not enough saturated arcs for a minimum cut

c



e.g. IDA – 9
IFDA – 24
max flow – 64

d



e Max flow – min cut
Finds a cut GC, AF, DF, DI, EI, EH value 64
Note: must not use supersource or supersink arcs.

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Review Exercise 2
Exercise A, Question 6

Question:

A two-person zero-sum game is represented by the following pay-off matrix for player A.

		B I	B II	B III	B IV
	I	−4	−5	−2	4
A	**II**	−1	1	−1	2
	III	0	5	−2	−4
	IV	−1	3	−1	1

a Determine the play safe strategy for each player.
b Verify that there is a stable solution and determine the saddle points.
c State the value of the game to B. ***E***

Solution:

a Row minima $-5, -1, -4, -1$ max is -1
Column maxima $0, 5, -1, 4$ min is -1
Play safe is A plays II or IV and B plays III

b Since $(-1) - (-1) \equiv (-1) + 1 = 0$ there is a stable solution
Saddle points (II, III) and (IV, III)

c Value of game to B is $-(-1) = 1$

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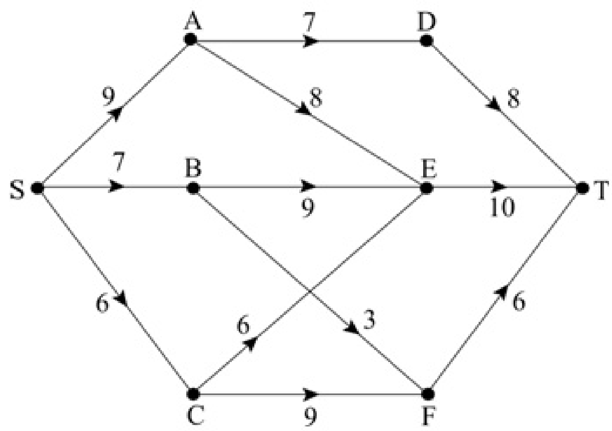
Review Exercise 2
Exercise A, Question 7

Question:



The network shows possible routes that an aircraft can take from S to T. The numbers on the directed arcs give the amount of fuel used on that part of the route, in appropriate units. The airline wishes to choose the route for which the maximum amount of fuel used on any part of the route is as small as possible. This is the minimax route.

a Complete a table to show the information.
b Hence obtain the minimax route from S to T and state the maximum amount of fuel used on any part of this route. ***E***

Solution:

a

Stage	Initial state	Action	Destination	Value
1	D	DT	T	8*
	E	ET	T	10*
	F	FT	T	6*
2	A	AD	D	$\max(7,8)=8$*
		AE	E	$\max(8,10)=10$
	B	BE	E	$\max(9,10)=10$
		BF	F	$\max(3,6)=6$*
	C	CE	E	$\max(6,10)=10$
		CF	F	$\max(9,6)=9$*
3		SA	A	$\max(9,8)=9$
	S	SB	B	$\max(7,6)=7$*
		SC	C	$\max(6,9)=9$

b Minimax route is S B F T
Maximum amount of fuel used is 7 units

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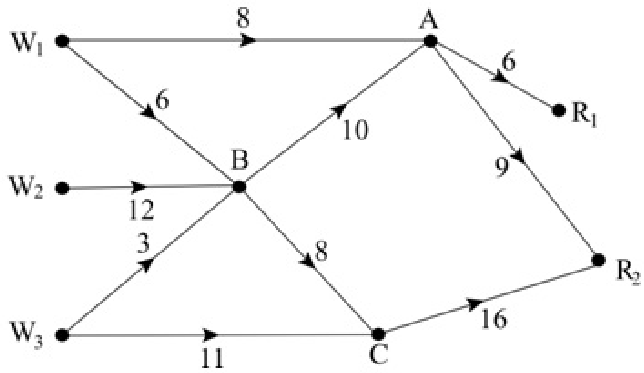
Review Exercise 2
Exercise A, Question 8

Question:



A company has three warehouses W_1, W_2 and W_3. It needs to transport the goods stored there to two retail outlets R_1 and R_2. The capacities of the possible routes, in van loads per day, are shown. Warehouses W_1, W_2 and W_3 have 14, 12 and 14 van loads respectively available per day and retail outlets R_1 and R_2 can accept 6 and 25 van loads respectively per day.

a On a copy of the diagram add a supersource W, a supersink R and the appropriate directed arcs to obtain a single-source, single-sink capacitated network. State the minimum capacity of each arc you have added.

b State the maximum flow along

i $W\ W_1\ A\ R_1\ R$,

ii $W\ W_3\ C\ R_2\ R$.

c Taking your answers to part **b** as the initial flow pattern, use the labelling procedure to obtain a maximum flow through the network from W to R. Show your working. List each flow-augmenting route you use, together with its flow.

d From your final flow pattern, determine the number of van loads passing through B each day.

The company has the opportunity to increase the number of van loads from one of the warehouses W_1, W_2, W_3 to A, B or C.

e Determine how the company should use this opportunity so that it achieves a maximum flow. ***E***

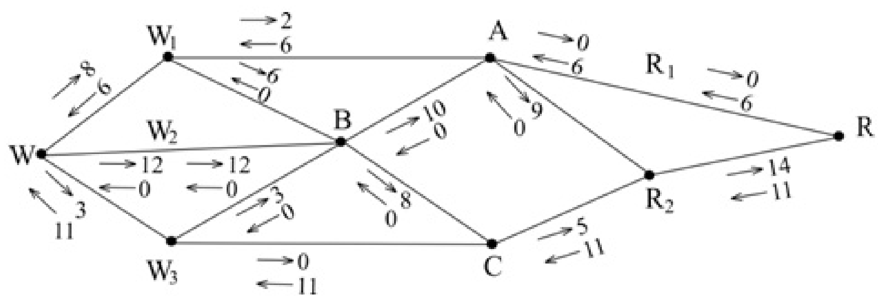
Solution:

a

b **i** $WW_1 A R_1 R - 6$

ii $WW_3 C R_2 R - 11$

c



e.g.

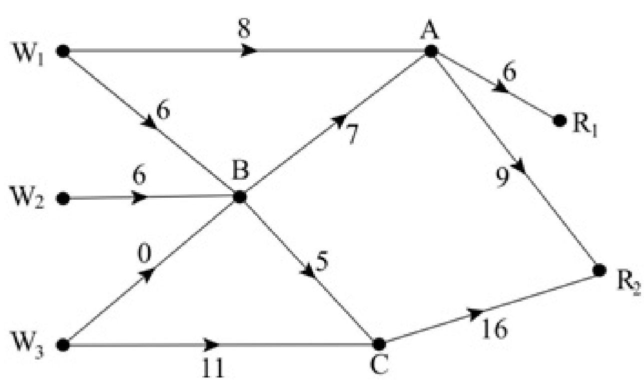
$W W_1 B A R_2 R - 6$

$W W_1 A R_2 R - 2$

$W W_2 B C R_2 R - 5$

$W W_2 B A R_2 R - 1$

Max flow 31



d 12 for this network (but may be different for other solutions)

e No use.
All arcs out of A and C are saturated, so the total flow cannot be increased unless the number of van loads from A or C to R_1 or R_2 is increased

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Review Exercise 2
Exercise A, Question 9

Question:

Emma and Freddie play a zero-sum game. This game is represented by the following pay-off matrix for Emma.

$$\begin{pmatrix} -4 & -1 & 3 \\ 2 & 1 & -2 \end{pmatrix}$$

a Show that there is no stable solution.
b Find the best strategy for Emma and the value of the game to her.
c Write down the value of the game to Freddie and his pay-off matrix. ***E***

Solution:

a

$$\text{Col max}\begin{pmatrix}-4 & -1 & 3\\ 2 & 1 & -2\end{pmatrix}\begin{matrix}\text{row min}\\ -4\\ -2\end{matrix}\quad \leftarrow \text{max}$$

$$\begin{matrix}2 & 1 & 3\\ & \uparrow & \\ & \text{min} & \end{matrix}$$

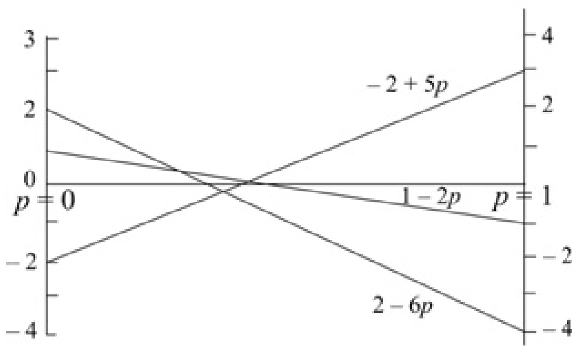
$-2 \neq 1 \;\therefore$ not stable

b Let Emma play R_1 with probability p

If Freddie plays C_1 Emma's winnings are $-4p+2(1-p)=2-6p$

If Freddie plays C_2 Emma's winnings are $-p+1(1-p)=1-2p$

If Freddie plays C_3 Emma's winnings are $3p-2(1-p)=-2+5p$



need intersection of $2-6p$ and $-2+5p$

$2-6p=-2+5p$

$4=11p$

$p=\frac{4}{11}$

So Emma should play R_1 with probability $\frac{4}{11}$

R_2 with probability $\frac{7}{11}$

The value of the game is $\frac{-2}{11}$ to Emma

c Value to Freddie $\frac{2}{11}$, matrix $\begin{pmatrix}4 & -2\\ 1 & -1\\ -3 & 2\end{pmatrix}$

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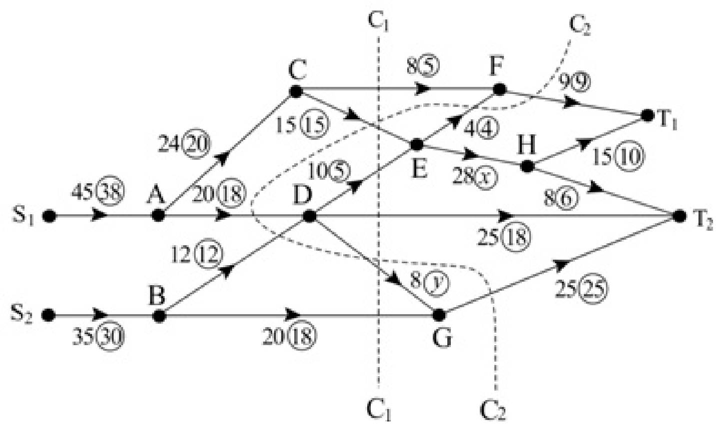
Review Exercise 2
Exercise A, Question 10

Question:



The diagram shows a capacitated, directed network. The unbracketed number on each arc indicates the capacity of that arc, and the numbers in circles show a feasible flow of value 68 through the network.

a Add a supersource and a supersink, and arcs of appropriate capacity, to a copy of the diagram.

b Find the values of x and y, explaining your method briefly.

c Find the value of cuts C_1 and C_2.

Starting with the given feasible flow of 68,

d use the labelling procedure to find a maximum flow through this network. List each flow-augmenting route you use, together with its flow.

e Show your maximum flow and state its value.

f Prove that your flow is maximal. ***E***

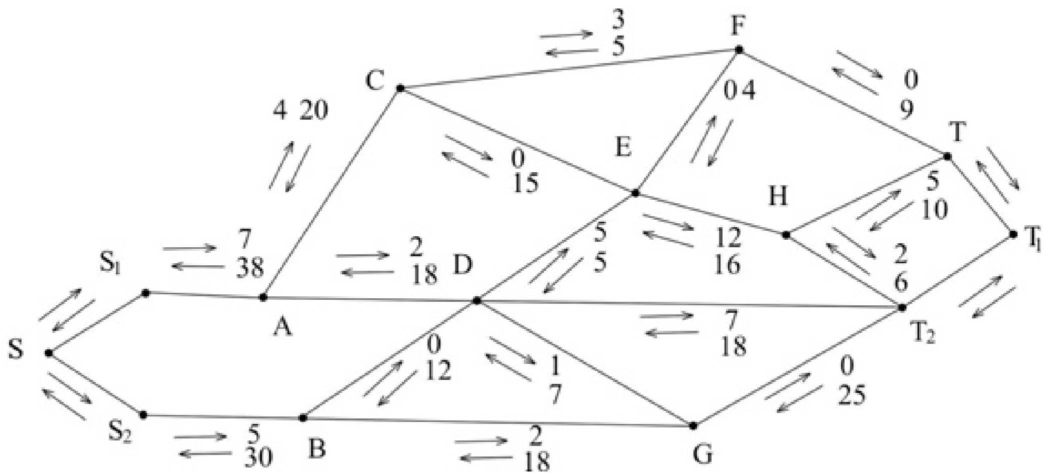
Solution:

a Adds S and T and arcs
$SS_1 \geq 45, SS_2 \geq 35, T_1T \geq 24, T_2T \geq 58$

b Using conservation of flow through vertices $x = 16$ and $y = 7$

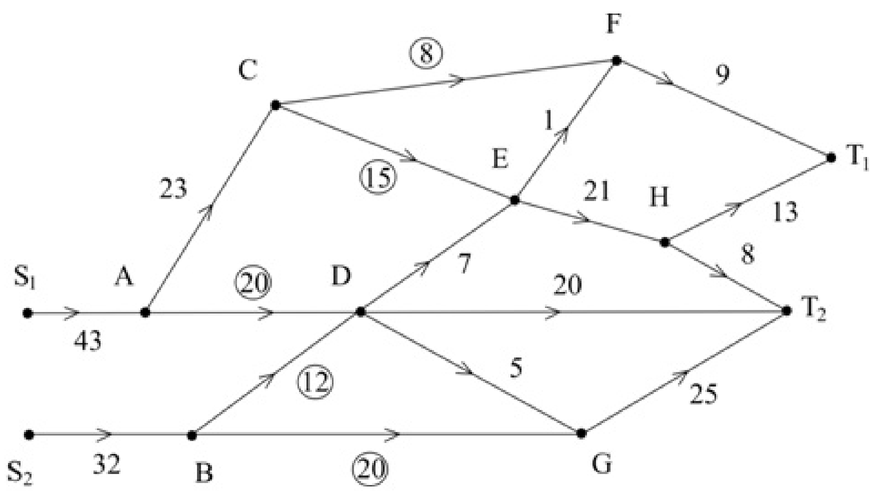
c $C_1 = 86, C_2 = 81$

d



SS_1 A D E H T_2 T − 2
e.g. SS_1 A C F E H T_1T − 3
SS_2 B G D T_2T − 2

e For example:



Flow 75

f Max flow – min cut theorem cut through CF, CE, AD, BD, BG (value 75)

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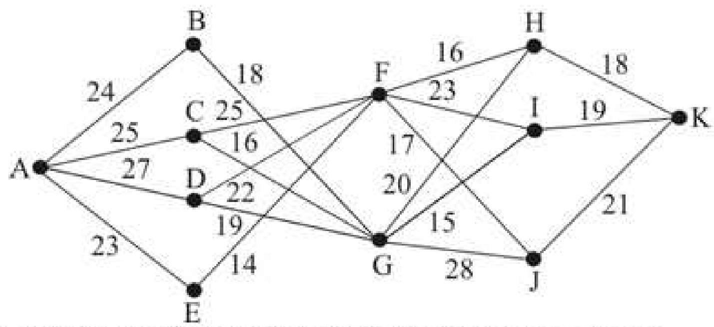
Review Exercise 2
Exercise A, Question 11

Question:

a Explain what is meant by a maximin route in dynamic programming, and give an example of a situation that would require a maximin solution.



A maximin route is to be found through the network shown.

b Complete the table on the worksheet, and hence find a maximin route.

c List **all** other maximin routes through the network. *E*

Solution:

a The route from start to finish in which the arc of minimum length is as large as possible.
Example must be practical, involve choice of route, have arc 'costs'.

b e.g. A company is planning its strategy for the next 4 years.
The number on each arc represents the expected profit resulting from each action.
The company wishes to ensure the minimum yearly profit is as large as possible.

Stage	State	Action	Value
1	H	HK	18*
	I	IK	19*
	J	JK	21*
2	F	FH	min(16,18) = 16
		FI	min(23,19) = 19*
		FJ	min(17,21) = 17
	G	GH	min(20,18) = 18
		GI	min(15,19) = 15
		GJ	min(28,21) = 21*
3	B	BG	min(18,21) = 18*
	C	CF	min(25,19) = 19*
		CG	min(16,21) = 16
	D	DF	min(22,19) = 19*
		DG	min(19,21) = 19*
	E	EF	min(14,19) = 14*
4	A	AB	min(24,18) = 18
		AC	min(25,19) = 19*
		AD	min(27,19) = 19*
		AE	min(23,14) = 14

c Routes: ACFIK, ADFIK, ADGJK

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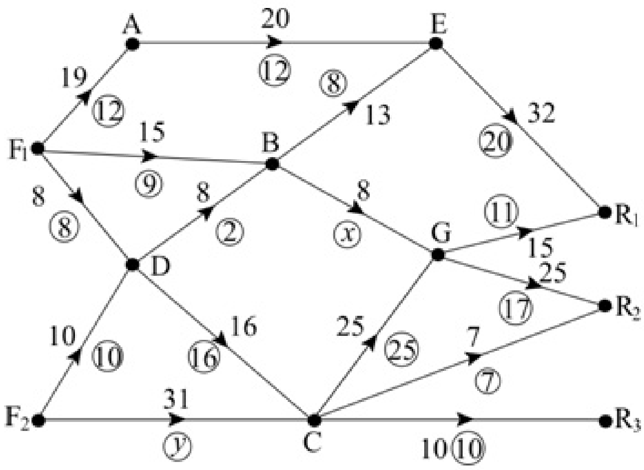
Review Exercise 2
Exercise A, Question 12

Question:



The diagram shows a capacitated, directed network of pipes flowing from two oil fields F_1 and F_2 to three refineries R_1, R_2 and R_3. The number on each arc represents the capacity of the pipe and the numbers in the circles represent a possible flow of 65.

a Find the value of x and the value of y.

b On the worksheet, add a supersource and a supersink, and arcs showing their minimum capacities.

c Taking the given flow of 65 as the initial flow pattern, use the labelling procedure to find the maximum flow. State clearly your flow augmenting routes.

d Show the maximum flow and write down its value.

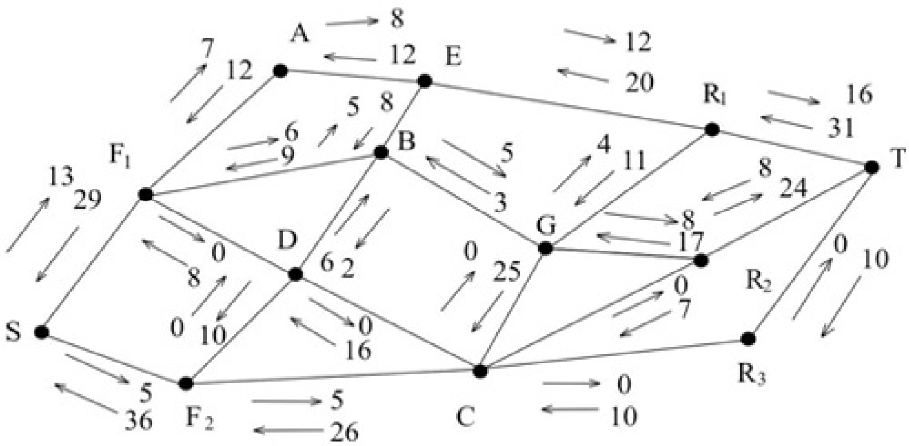
e Verify that this is the maximum flow by finding a cut equal to the flow. *E*

Solution:

a $x = 3, y = 26$

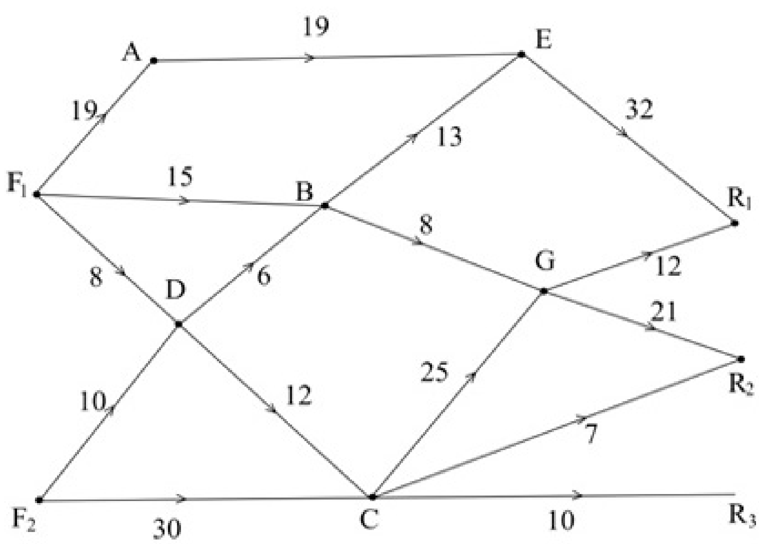
b

c



e.g.
$SF_1AER_1T - 7$
$SF_1BER_1T - 5$
$SF_1BGR_1T - 1$
$SF_2CDBGR_2T - 4$

d e.g.



Max Flow 82

e e.g.
$F_1A, BE, BG, CG, CR_2, CR_3 (= 82)$
or $ER_1, BG, CG, CR_2, CR_3 (= 82)$

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Review Exercise 2
Exercise A, Question 13

Question:

A two person zero-sum game is represented by the following pay-off matrix for player A.

	B plays I	B plays II	B plays III
A plays I	2	−1	3
A plays II	1	3	0
A plays III	0	1	−3

a Identify the play safe strategies for each player.
b Verify that there is no stable solution to this game.
c Explain why the pay-off matrix above may be reduced to

	B plays I	B plays II	B plays III
A plays I	2	−1	3
A plays II	1	3	0

d Find the best strategy for player A, and the value of the game. ***E***

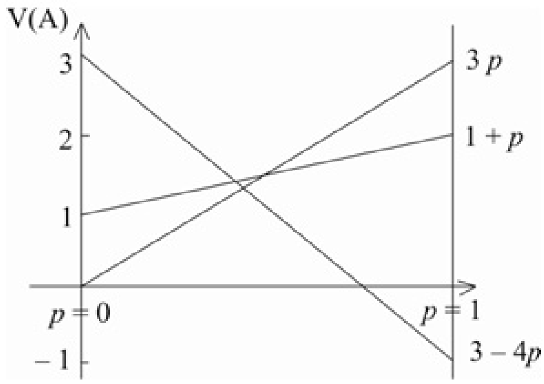
Solution:

a Player A: Row minima are $-1, 0, -3$ so maximin choice is play II
Player B: column maxima are 2, 3, 3 so minimax choice is play I

b Since A's maximin (0) $\neq$ B's minimax (2) no stable solution

c For player A row II dominates row III, (so A will never play III), since
$1 > 0 \quad 3 > 1 \quad 0 > -3$

d Let A play I with probability p and II with probability $(1-p)$
If B plays I A's expected winnings are $2p + (1-p) = 1 + p$
If B plays II A's expected winnings are $-p + 3(1-p) = 3 - 4p$
If B plays III A's expected winnings are $3p$



$3 - 4p = 3p \Rightarrow p = \frac{3}{7}$

A should play I with probability $\frac{3}{7}$

II with probability $\frac{4}{7}$

and never play III

The value of the game is $\frac{9}{7}$ to A

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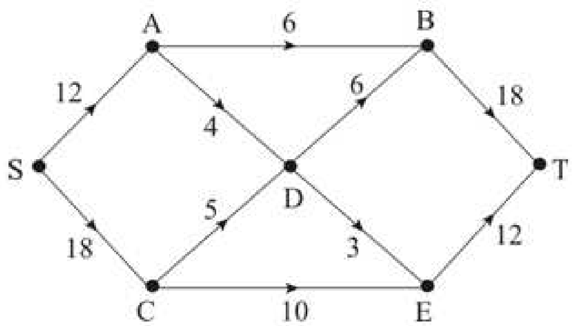
Review Exercise 2
Exercise A, Question 14

Question:



The diagram shows a capacitated network. The numbers on each arc indicate the capacity of that arc in appropriate units.

a Explain why it is not possible to achieve a flow of 30 through the network from S to T.

b State the maximum flow along
i SABT,
ii SCET.

c Show these flows on the worksheet.

d Taking your answer to part **c** as the initial flow pattern, use the labelling procedure to find a maximum flow from S to T. Show your working. List each flow-augmenting path you use together with its flow.

e Indicate a maximum flow.

f Prove that your flow is maximal. ***E***

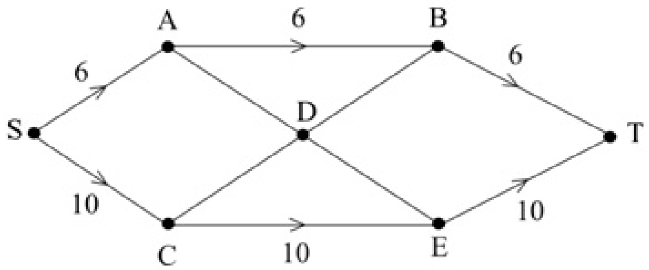
Solution:

a Finds a cut less than 30 giving its value.
e.g. cut through AB, AD, CD, CE (– 25) or AB, BD, ET (– 24)
or a consideration of flow input / flow output through A and C.

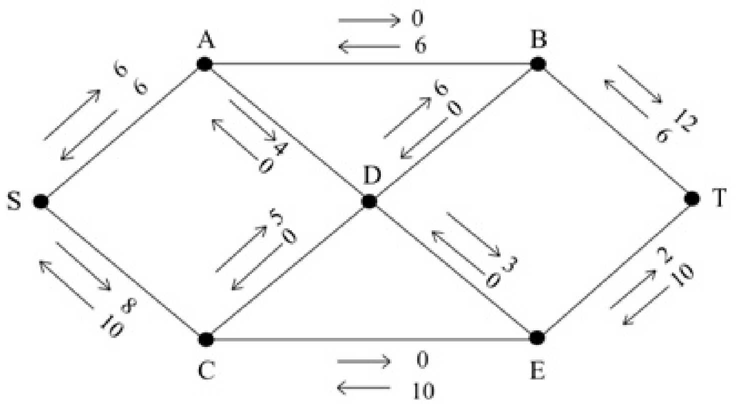
b **i** SABT (– 6)

ii SCET (10)

c

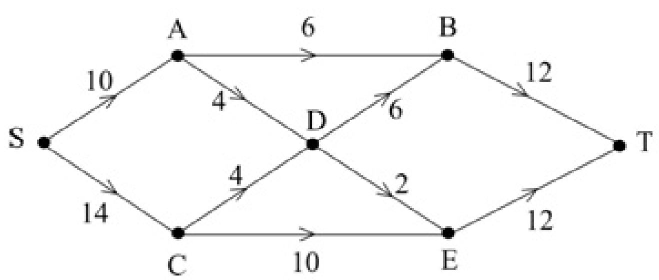


d



e.g. SADBT – 4
SCDBT – 2
SCDET – 2, max flow 24

e e.g.



f Refers to max flow – min cut theorem and the cut through AB, BD, ET of value 24.

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Review Exercise 2
Exercise A, Question 15

Question:

Andrew (A) and Barbara (B) play a zero-sum game. This game is represented by the following pay-off matrix for Andrew.

$$\begin{pmatrix} 3 & 5 & 4 \\ 1 & 4 & 2 \\ 6 & 3 & 7 \end{pmatrix}$$

a Explain why this matrix may be reduced to

$$\begin{pmatrix} 3 & 5 \\ 6 & 3 \end{pmatrix}.$$

b Hence find the best strategy for each player and the value of the game. ***E***

Solution:

a Row 1 dominates row 2 so A will never choose R2
Column 1 dominates column 3 so B will never choose C3
Thus Row 2 and column 3 may be deleted.

b Let A play row 1 with probability p and hence row 2 with probability $(1-p)$
If B plays 1 A's expected gain is $3p+6(1-p)=6-3p$
If B plays 2 A's expected gain is $5p+3(1-p)=2p+3$

Optimal when $6-3p=2p+3$

$$5p=3$$

$$p=\frac{3}{5}$$

Hence A should play row 1 with probability $\frac{3}{5}$ and row 3 with probability $\frac{2}{5}$ and row 2 never

Similarly, let B play column 1 with probability q

$3q+5(1-q)=6q+3(1-q) \Rightarrow 5-2q=3q+3$

$$5q=2$$

$$q=\frac{2}{5}$$

So B should play column 1 with probability $\frac{2}{5}$ and column 2 with probability $\frac{3}{5}$ and column 3 never

Value of game is $4\frac{1}{5}$ to A

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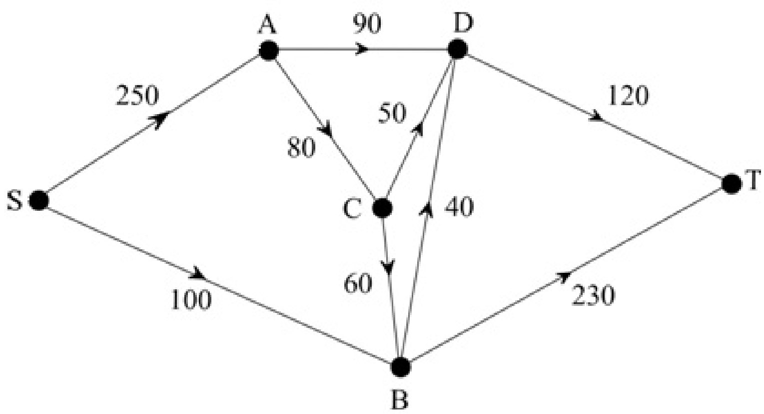
Review Exercise 2
Exercise A, Question 16

Question:



Natural gas is produced at S and is transported to a refinery at T by a network of underwater pipelines. The capacity of each pipeline, in appropriate units, is given in the diagram which shows the network of pipelines

a State the maximum flow along
  i SACDT,
  ii SBT.

b Show these two maximum flows on Diagram 1 of the worksheet.

c Taking your answer to part **b** as the initial flow pattern, use the labelling procedure to find a maximum flow from S to T showing your working on Diagram 2. List each flow augmenting route you find and state its flow.

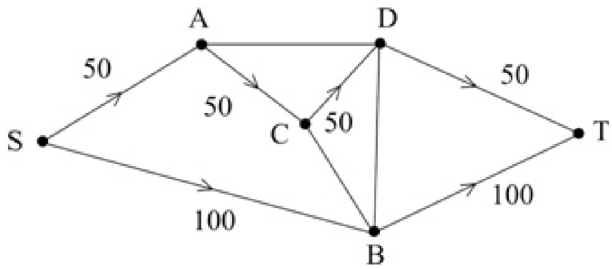
d Show your maximum flow pattern on Diagram 3.

e Prove that your flow is maximal.

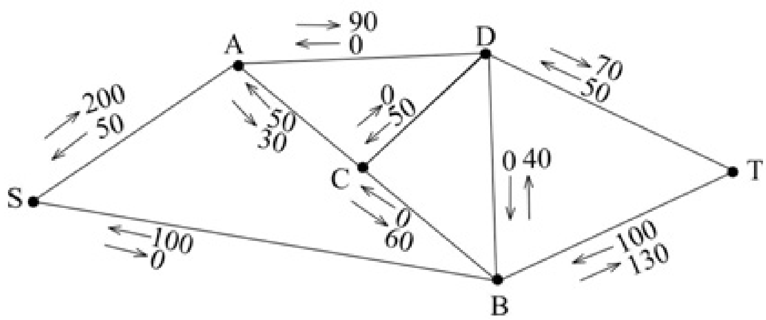
Solution:

a **i** max flow along S A C D T = 50
ii max flow along S B T = 100

b

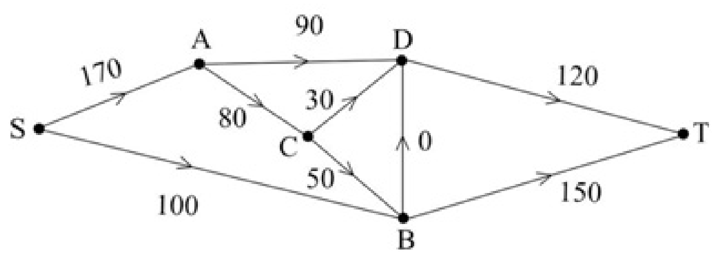


c



S A D T − 70
S A C B T − 30
S A D C B T − 20
Maximum flow 270

d



e Use max flow − min cut theorem
Cut though AD, AC and SB = 270 which equals flow
∴ maximal

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Review Exercise 2
Exercise A, Question 17

Question:

Kris produces custom made racing cycles. She can produce up to four cycles each month, but if she wishes to produce more than three in any one month she has to hire additional help at a cost of £350 for that month. In any month when cycles are produced, the overhead costs are £200. A maximum of three cycles can be held in stock in any one month, at a cost of £40 per cycle per month. Cycles must be delivered at the end of the month. The order book for cycles is

Month	Aug.	Sept.	Oct.	Nov.
Number of cycles required	3	3	5	2

Disregarding the cost of parts and Kris' time,

a determine the total cost of storing two cycles and producing four cycles in a given month, making your calculations clear.

There is no stock at the beginning of August and Kris plans to have no stock after the November delivery.

b Use dynamic programming to determine the production schedule which minimises the costs, showing your working in a table.

The fixed cost of parts is £600 per cycle and of Kris' time is £500 per month. She sells the cycles for £2000 each.

c Determine her total profit for the four-month period. ***E***

Solution:

a total cost $= 2\times40+350+200 = £630$

b

Stage	Demand	State	Action	Destination	Value
(2) Oct	(5)	(1)	(4)	(0)	$(590+200=790)*$
		(2)	(3)	(0)	$280+200=480*$
			(4)	(1)	$630+240=870$
		(3)	(2)	0	$320+200=520*$
			3	1	$320+240=560$
			4	2	$670+80=750$
3 Sept	3	0	4	1	$550+790=1340*$
		1	3	1	$240+790=1030*$
			4	2	$590+480=1070$
4 Aug	3	0	3	0	$200+1340=1540*$
			4	1	$550+1030=1580$

Month	August	September	October	November
Make	3	4	4	2

cost $= £1540$

c Profit per cycle $= 13\times1400$ cost of Kris' time $= £2000$

$= 18\,200$ cost of production $= £1540$

$\therefore$ total profit $= 18\,200 - 3540$

$= £14\,660$

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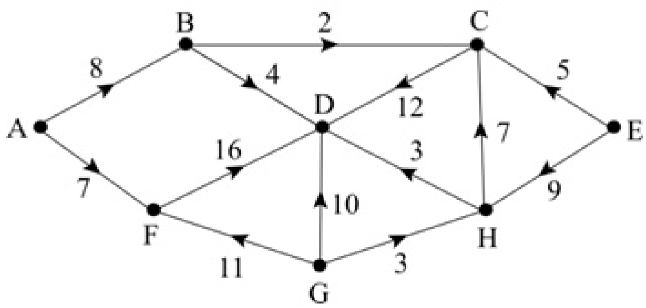
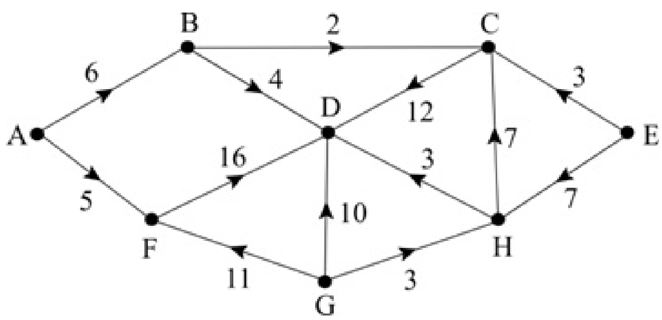
Review Exercise 2
Exercise A, Question 18

Question:



The network above models a drainage system. The number on each arc indicates the capacity of that arc, in litres per second.

a Write down the source vertices.



This network shows a feasible flow through the same network.

b State the value of the feasible flow shown.

Taking the flow shown as your initial flow pattern,

c use the labelling procedure to find a maximum flow through this network. You should list each flow-augmenting route you use, together with its flow.

d Show the maximum flow and state its value.

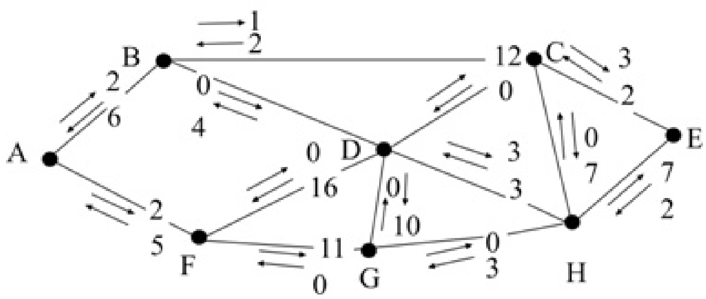
e Prove that your flow is maximal. ***E***

Solution:

a A. E and G

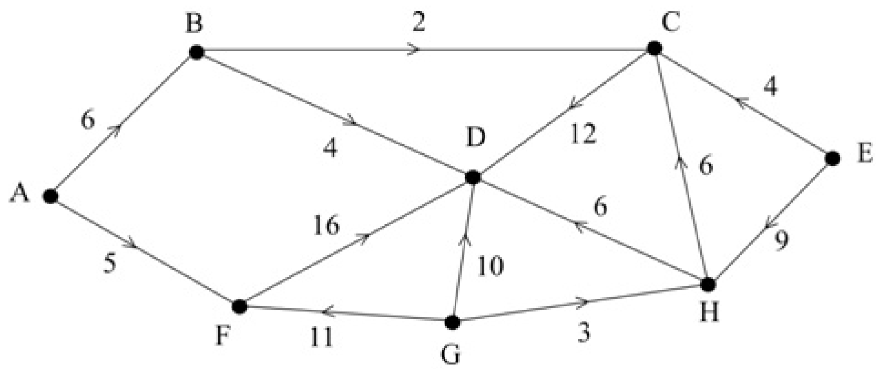
b 45

c



e.g $\begin{matrix} \text{EHD} - 2 \\ \text{ECHD} - 1 \end{matrix}$

d



Maximum flow 48

e Max flow – Min cut theorem

Cut through DB, DC, DH, DG, DF

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Review Exercise 2
Exercise A, Question 19

Question:

A two-person zero-sum game is represented by the following pay-off matrix for player A.

	B plays 1	B plays 2	B plays 3	B plays 4
A plays 1	−2	1	3	−1
A plays 2	−1	3	2	1
A plays 3	−4	2	0	−1
A plays 4	1	−2	−1	3

a Verify that there is no stable solution to this game.
b Explain why the 4×4 game above may be reduced to the following 3×3 game.

−2	1	3
−1	3	2
1	−2	−1

c Formulate the 3×3 game as a linear programming problem for player A. Write the constraints as inequalities. Define your variables clearly. ***E***

Solution:

a Row minimums $\{-2, -1, -4, -2\}$ row maximin $= -1$
Column maximums $\{1, 3, 3, 3\}$ column minimax $= 1$
Since $1 \neq -1$ not stable

b Row 2 dominates Row 3
column 1 dominates column 4

c Let A play row R, with probability P_1, R_2 with probability P_2 and "R_3" with probability P_3

$$\begin{pmatrix} -2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & -2 & -1 \end{pmatrix} \quad \underset{+3}{\overset{\text{e.g.}}{\rightarrow}} \quad \begin{pmatrix} 1 & 4 & 6 \\ 2 & 6 & 5 \\ 4 & 1 & 2 \end{pmatrix}$$

e.g. maximise $P = V$

subject to
$$V - P_1 - 2P_2 - 4P_3 \leq 0$$
$$V - 4P_1 - 6P_2 - P_3 \leq 0$$
$$V - 6P_1 - 5P_2 - 2P_3 \leq 0$$
$$P_1 + P_2 + P_3 \leq 1$$
$$V_1\ P_1\ P_2\ P_3 \geq 0$$

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Review Exercise 2
Exercise A, Question 20

Question:

a Explain briefly what is meant by a zero-sum game.
A two person zero-sum game is represented by the following pay-off matrix for player A.

	I	II	III
I	5	2	3
II	3	5	4

b Verify that there is no stable solution to this game.
c Find the best strategy for player A and the value of the game to her.
d Formulate the game as a linear programming problem for player B. Write the constraints as inequalities and define your variables clearly. ***E***

Solution:

a A zero-sum game is one in which the sum of the gains for all players is zero.

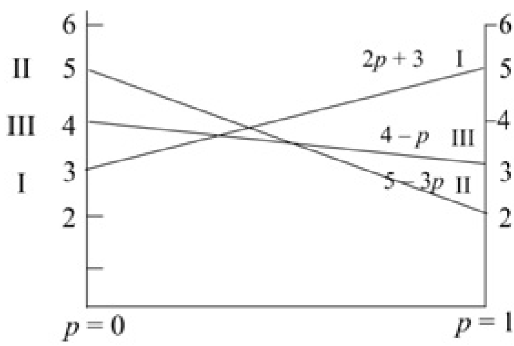
b

	I	II	III	
I	5	2	3	min 2
II	3	5	4	min 3 ← max
	max 5	5	4	
			↑	
			min	

Since $3 \neq 4$ not stable

c Let A play I with probability p
Let A play II with probability $(1-p)$

If B play I A's gains are $5p+3(1-p)=2p+3$
If B plays II A's gains are $2p+5(1-p)=5-3p$
If B plays III A's gains are $3p+4(1-p)=4-p$



Intersection of $2p+3$ and $4-p \Rightarrow p=\frac{1}{3}$

$\therefore$ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) $=3\frac{2}{3}$

d Let B play I with probability q_1, II with probability q_2 and III with probability q_3

e.g. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$

maximise $P=V$

$V-q_1-4q_2-3q_3 \le 0$

Subject to $V-3q_1-q_2-2q_3 \le 0 \quad q_1+q_2+q_3 \le 1$

$v_1, q_1, q_2, q_3 \ge 0 \quad \text{or} = 1$

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Review Exercise 2
Exercise A, Question 21

Question:

Joan sells ice cream. She needs to decide which three shows to visit over a three-week period in the summer. She starts the three-week period at home and finishes at home. She will spend one week at each of the three shows she chooses, travelling directly from one show to the next.

Table 1 gives the week in which each show is held. Table 2 gives the expected profits from visiting each show. Table 3 gives the cost of travel between shows.

Table 1

Week	1	2	3
Shows	A, B, C	D, E	F, G, H

Table 2

Show	A	B	C	D
Expected profit (£)	900	800	1000	1500
Show	E	F	G	H
Expected profit (£)	1300	500	700	600

Table 3

Travel costs (£)	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Home	70	80	150			80	90	70
A				180	150			
B				140	120			
C				200	210			
D						200	160	120
E						170	100	110

It is decided to use dynamic programming to find a schedule that maximises the total expected profit, taking into account the travel costs.

a Define suitable stage, state and action variables.

b Determine the schedule that maximises the total profit. Show your working in a table.

c Advise Joan on the shows that she should visit and state her total expected profit. ***E***

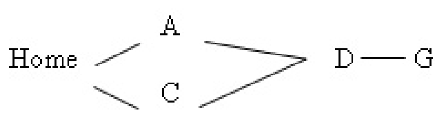
Solution:

a Stage – Number of weeks to finish
State – Show being attended
Action – Next journey to undertake

b

Stage	State	Action	Value
1	F	F – Home	$500-80=420*$
	G	G – Home	$700-90=610*$
	H	H – Home	$600-70=530*$
2	D	DF	$1500-200+420=1720$
		DG	$1500-160+610=1950*$
		DH	$1500-120+530=1910$
	E	EF	$1300-170+420=1550$
		EG	$1300-100+610=1810*$
		EH	$1300-110+530=1720$
3	A	AD	$900-180+1950=2670*$
		AE	$900-150+1810=2560$
	B	BD	$800-140+1950=2610*$
		BE	$800-120+1810=2490$
	C	CD	$1000-200+1950=2750*$
		CE	$1000-210+1810=2600$
4	Home	Home – A	$-70+2670=2600*$
		Home – B	$-80+2610=2530$
		Home – C	$-150+2750=2600*$

c



Total profit £2600