Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

The table shows the unit costs of transporting goods from supply points to demand points. In each case:

a use the north-west corner method to find initial solution,

b verify that, for each solution, the number of occupied cells = number of supply points + number of demand points − 1.

c determine the cost of each initial solution.

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	

Solution:

a

	P	Q	R	Supply
A	28	4		32
B		41	3	44
C			34	34
Demand	28	45	37	110

b Supply points = 3, demand points = 3, occupied cells = 5.
$3+3-1=5=$ number of occupied cells. Yes formula holds.

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	110

c $\text{Cost} = 28\times150+4\times213+41\times204+3\times218+34\times246=22\,434$

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 2

Question:

The table shows the unit costs of transporting goods from supply points to demand points. In each case:

a use the north-west corner method to find initial solution,

b verify that, for each solution, the number of occupied cells = number of supply points + number of demand points − 1.

c determine the cost of each initial solution.

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	

Solution:

a

	P	Q	R	S	Supply
A	21	32	1		54
B			50	17	67
C				29	29
Demand	21	32	51	46	150

b Supply points $= 3$, demand points $= 4$, occupied cells $= 6$.

$3 + 4 - 1 = 6 =$ number of occupied cells. Yes formula holds.

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	150

c $\text{Cost} = 21 \times 27 + 32 \times 33 + 1 \times 34 + 50 \times 37 + 17 \times 30 + 29 \times 35 = 5032$

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 3

Question:

The table shows the unit costs of transporting goods from supply points to demand points. In each case:

a use the north-west corner method to find initial solution,

b verify that, for each solution, the number of occupied cells = number of supply points + number of demand points − 1.

c determine the cost of each initial solution.

	P	Q	R	Supply
A	17	24	19	123
B	15	21	25	143
C	19	22	18	84
D	20	27	16	150
Demand	200	100	200	500

Solution:

a

	P	Q	R	Supply
A	123			123
B	77	66		143
C		34	50	84
D			150	150
Demand	200	100	200	500

b Supply points $= 4$, demand points $= 3$. Occupied cells $= 6$.

$4 + 3 - 1 = 6 =$ number of occupied cells. Yes formula holds.

	P	Q	R	Supply
A	17	24	19	123
B	15	21	25	143
C	19	22	18	84
D	20	27	16	150
Demand	200	100	200	500

c Cost $= 123\times17 + 77\times15 + 66\times21 + 34\times22 + 50\times18 + 150\times16 = 8680$

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 4

Question:

The table shows the unit costs of transporting goods from supply points to demand points. In each case:

a use the north-west corner method to find initial solution,

b verify that, for each solution, the number of occupied cells = number of supply points + number of demand points − 1.

c determine the cost of each initial solution.

	P	**Q**	**R**	**S**	**Supply**
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	700

Solution:

a

	P	Q	R	S	Supply
A	134				134
B	41	162			203
C		13	163		176
D			12	175	187
Demand	175	175	175	175	700

b Supply points $= 4$, demand points $= 4$, occupied cells $= 7$.

$4 + 4 - 1 = 7 =$ number of occupied cells. Yes formula holds.

	P	Q	R	S	Supply
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	700

c $\text{Cost} = 134 \times 56 + 41 \times 59 + 162 \times 76 + 13 \times 70 + 163 \times 57 + 12 \times 75 + 175 \times 71$

$= 45\,761$

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 5

Question:

	A	B	C	D	Supply
X	27	33	34	41	60
Y	31	29	37	30	60
Z	40	32	28	35	80
Demand	40	70	50	20	

Four sandwich shops A, B, C and D can be supplied with bread from three bakeries, X, Y, and Z. The table shows the cost, in pence, of transporting one tray of bread from each supplier to each shop, the number of trays of bread required by each shop and the number of trays of bread that can be supplied by each bakery.

a Explain why it is necessary to add a dummy demand point in order to solve this problem, and what this dummy point means in practical terms.

b Use the north-west corner method to determine an initial solution to this problem and the cost of this solution.

Solution:

a The total supply is 200, but the total demand is 180. A dummy is needed to absorb this excess, so that total supply equals total demand.

b

	A	B	C	D	Supply
X	27	33	34	41	60
Y	31	29	37	30	60
Z	40	32	28	35	80
Demand	40	70	50	20	

Becomes

	A	B	C	D	Dummy	Supply
X	27	33	34	41	0	60
Y	31	29	37	30	0	60
Z	40	32	28	35	0	80
Demand	40	70	50	20	20	200

North-west corner solution is

	A	B	C	D	Dummy	Supply
X	40	20				60
Y		50	10			60
Z			40	20	20	80
Demand	40	70	50	20	20	200

$\text{Cost} = 40\times 27+20\times 33+50\times 29+10\times 37+40\times 28+20\times 35+20\times 0=5\,380$

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 6

Question:

	K	L	M	N	Supply
A	35	46	62	80	20
B	24	53	73	52	15
C	67	61	50	65	20
D	92	81	41	42	20
Demand	25	10	18	22	

A company needs to supply ready-mixed concrete from four depots A, B, C and D to four work sites K, L, M and N. The number of loads that can be supplied from each depot and the number of loads required at each site are shown in the table above, as well as the transportation cost per load from each depot to each work site.

a Explain what is meant by a degenerate solution.

b Demonstrate that the north-west corner method gives a degenerate solution.

c Adapt your solution to give a non-degenerate initial solution.

Solution:

a A degenerate solution occurs when the number of cells used in a solution is fewer than the number of rows + number of columns − 1. It will happen when an entry, other than the last, completes both the supply requirement of the row and the demand requirement of the column.

b

	K	L	M	N	Stock
A	20				20
B	5	10			15
C			18	2	20
D				20	20
Demand	25	10	18	22	

c

	K	L	M	N	Stock
A	20				20
B	5	10			15
C		0	18	2	20
D				20	20
Demand	25	10	18	22	

or

	K	L	M	N	Stock
A	20				20
B	5	10	0		15
C			18	2	20
D				20	20
Demand	25	10	18	22	

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 7

Question:

	L	M	N	Supply
P	3	5	9	22
Q	4	3	7	a
R	6	4	8	11
S	8	2	5	b
Demand	15	17	20	

The table shows a balanced transportation problem. The initial solution, given by the north-west corner method, is degenerate.

a Use this information to determine the values of a and b.

b Hence write down the initial, degenerate solution given by the north west-corner method.

Solution:

	L	M	N	Supply
P	3	5	9	22
Q	4	3	7	a
R	6	4	8	11
S	8	2	5	b
Demand	15	17	20	

a Since the problem is **balanced** the total supply = the total demand, giving

$a+b+22+11=15+17+20$

hence $a+b=\mathbf{19}$ (1)

Since the initial north-west corner solution is **degenerate** we know that the supply and demand are both met before the final entry.

(Since $22+a+11>15+17$ we know that this must occur before row 3)

Hence $22+a=15+17$, giving $a=10$

Using equation (1) we get $b=9$

So the values are: $a=10$ and $b=9$

b

	L	M	N	Supply
P	15	7		22
Q		10		10
R			11	11
S			9	9
Demand	15	17	20	

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 1

Question:

Start with the initial, north-west corner, solutions found in question 1 of exercise 1A.
In each case use the initial solution, and the original cost matrix, shown below,
to find

a the shadow costs,
b the improvement indices
c the entering cell, if appropriate.

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	

Solution:

a

Shadow costs		150	213	227	
		P	Q	R	Supply
0	A	150	213	222	32
−9	B	175	204	218	44
19	C	188	198	246	34
	Demand	28	45	37	110

b Improvement indices for cells:

$$BP = 175 + 9 - 150 = 34$$
$$CP = 188 - 19 - 150 = 19$$
$$CQ = 198 - 19 - 213 = -34$$
$$AR = 222 - 0 - 227 = -5$$

c Entering cell is CQ, since it has the most negative improvement index.

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

Start with the initial, north-west corner, solutions found in question 2 of exercise 1A.
In each case use the initial solution, and the original cost matrix, shown below, to find

a the shadow costs,
b the improvement indices
c the entering cell, if appropriate.

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	

Solution:

a

Shadow costs		27	33	34	27	
		P	Q	R	S	Supply
0	A	27	33	34	41	54
3	B	31	29	37	30	67
8	C	40	32	28	35	29
	Demand	21	32	51	46	

b Improvement indices for cells:

$BP = 31 - 3 - 27 = 1$

$CP = 40 - 8 - 27 = 5$

$BQ = 29 - 3 - 33 = 7$

$CQ = 32 - 8 - 33 = -9$

$CR = 28 - 8 - 34 = -14$

$AS = 41 - 0 - 27 = 14$

c Entering cell is CR, since it has the most negative improvement index.

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 3

Question:

Start with the initial, north-west corner, solutions found in question 3 of exercise 1A.
In each case use the initial solution, and the original cost matrix, shown below, to find

a the shadow costs,
b the improvement indices
c the entering cell, if appropriate.

	P	Q	R	Supply
A	17	24	19	123
B	15	21	25	143
C	19	22	18	84
D	20	27	16	150
Demand	200	100	200	

Solution:

a

Shadow costs		17	23	19	
		P	Q	R	Supply
0	A	17	24	19	123
−2	B	15	21	25	143
−1	C	19	22	18	84
−3	D	20	27	16	150
	Demand	200	100	200	

b Improvement indices for cells:

$CP = 19+1-17=3$

$DP = 20+3-17=6$

$AQ = 24-0-23=1$

$DQ = 27+3-23=7$

$AR = 19-0-19=0$

$BR = 25+2-19=8$

c There are no negative improvement indices, so the solution is optimal.

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise B, Question 4

Question:

Start with the initial, north-west corner, solutions found in question 4 of exercise 1A.
In each case use the initial solution, and the original cost matrix, shown below,
to find
a the shadow costs,
b the improvement indices
c the entering cell, if appropriate.

	P	Q	R	S	Supply
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	

Solution:

a

Shadow costs		56	73	60	56	
		P	Q	R	S	Supply
0	A	56	86	80	61	134
3	B	59	76	78	65	203
-3	C	62	70	57	67	176
15	D	60	68	75	71	187
	Demand	175	175	175	175	

b Improvement indices for cells:

$CP = 62+3-56=9$

$DP = 60-15-56=-11$

$AQ = 86-0-73=13$

$DQ = 68-15-73=-20$

$AR = 80-0-60=20$

$BR = 78-3-60=15$

$AS = 61-0-56=5$

$BS = 65-3-56=6$

$CS = 67+3-56=14$

c Entering cell is DQ, since it has the most negative improvement index.

Solutionbank D2

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

Complete your solutions to the transportation problems from question 1 in exercise 1A. You should demonstrate that your solution is optimal.

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	

Solution:

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	110

Our current solution is

	P	Q	R	Supply
A	28	4		32
B		41	3	44
C			34	34
Demand	28	45	37	110

We established in question 1 of exercise 1B that the entering cell was CQ, so we enter θ into cell CQ and get the following stepping stone route

	P	Q	R	Supply
A	28	4		32
B		$41-\theta$	$3+\theta$	44
C		θ	$34-\theta$	34
Demand	28	45	37	110

The largest possible value of θ is 34, making CR the exiting cell, and giving the improved solution.

	P	Q	R	Supply
A	28	4		32
B		7	37	44
C		34		34
Demand	28	45	37	110

This gives the following shadow costs:

Shadow costs		150	213	227	
		P	Q	R	Supply
0	A	150	213	222	32
−9	B	175	204	218	44
−15	C	188	198	246	34
	Demand	28	45	37	110

Improvement indices for cells:

$BP = 175 + 9 - 150 = 34$

$CP = 188 + 15 - 150 = 53$

$AR = 222 - 0 - 227 = -5$

$CR = 246 + 15 - 227 = 34$

The solution is not optimal, since we have a negative improvement index and the new entering cell is AR

We insert θ into cell AR and set the following stepping stone route

	P	Q	R	Supply
A	28	$4-\theta$	θ	32
B		$7+\theta$	$37-\theta$	44
C		34		34
Demand	28	45	37	50

The maximum value of θ is 4 making AQ the exiting cell
Improved solution

	P	Q	R	Supply
A	28		4	32
B		11	33	44
C		34		34
Demand	28	45	37	150

Shadow costs

		150	208	222	
		P	Q	R	Supply
0	A	(150)	213	(222)	32
−4	B	175	(204)	(218)	44
−10	C	188	(198)	246	34
	Demand	28	45	37	150

Improvement indices

$AQ = 213 - 0 - 208 = 5$

$BP = 175 + 4 - 150 = 29$

$CP = 188 + 10 - 150 = 48$

$CR = 246 + 10 - 222 = 34$

Since all improvement indices are non-negative we have the optimal solution

	P	Q	R	Supply
A	28		4	32
B		11	33	44
C		34		34
Demand	28	45	37	150

28 units A to P
4 units A to R
11 units B to Q
33 units B to R
34 units C to Q cost 21 258

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

Complete your solutions to the transportation problems from question 2 in exercise 1A. You should demonstrate that your solution is optimal.

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	

Solution:

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	150

Our current solution from question 2 of exercise 1A is

	P	Q	R	S	Supply
A	21	32	1		54
B			50	17	67
C				29	29
Demand	21	32	51	46	150

We established in question 2 of exercise 1B that the entering cell was CR, so we enter θ into cell CR and get the following stepping stone route

	P	Q	R	S	Supply
A	21	32	1		54
B			$50-\theta$	$17+\theta$	67
C			θ	$29-\theta$	29
Demand	21	32	51	46	150

The largest possible value of θ is 29, making CS the exiting cell and giving the improved solution

	P	Q	R	S	Supply
A	21	32	1		54
B			21	46	67
C			29		29
Demand	21	32	51	46	150

This gives the following shadow costs:

Shadow costs		27	33	34	27	
		P	Q	R	S	Supply
0	A	27	33	34	41	54
3	B	31	29	37	30	67
−6	C	40	32	28	35	29
	Demand	21	32	51	46	150

Improvement indices

$AS = 41 - 0 - 27 = 14$

$BP = 31 - 3 - 27 = 1$

$BQ = 29 - 3 - 33 = -7$

$CP = 40 + 6 - 27 = 19$

$CQ = 32 + 6 - 33 = 5$

$CS = 35 + 6 - 27 = 14$

The entering cell must be BQ, giving the following stepping stone route

	P	Q	R	S	Supply
A	21	$32-\theta$	$1+\theta$		54
B		θ	$21-\theta$	46	67
C			29		29
Demand	21	32	51	46	150

The largest possible value of θ is 21, making BR the exiting cell and giving the following improved solution.

	P	Q	R	S	Supply
A	21	11	22		54
B		21		46	67
C			29		29
Demand	21	32	51	46	150

This gives the following shadow costs

Shadow costs		27	33	34	34	
		P	Q	R	S	Supply
0	A	(27)	(33)	(34)	41	54
−4	B	31	(29)	37	(30)	67
−6	C	40	32	(28)	35	29
	Demand	21	32	51	46	150

Improvement indices:

$AS = 41 - 0 - 34 = 7$

$BP = 31 + 4 - 27 = 8$

$BR = 37 + 4 - 34 = 7$

$CP = 40 + 6 - 27 = 19$

$CQ = 32 + 6 - 33 = 5$

$CS = 35 + 6 - 34 = 7$

All improvement indices are non-negative so our solution is optimal

Optimal solution
21 units A to P
11 units A to Q
22 units A to R
21 units B to Q
46 units B to S
29 units C to R Cost 4479

Solutionbank D2
Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

Complete your solutions to the transportation problems from question 4 in exercise 1B. You should demonstrate that your solution is optimal.

	P	Q	R	S	Supply
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	

The solution to question **3** requires a number of iterations, plus the optimality check – you will certainly get lots of practise in implementing the algorithms!

Solution:

	P	Q	R	S	Supply
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	700

Initial solution (from question 4 of exercise 1A)

	P	Q	R	S	Supply
A	134				134
B	41	162			203
C		13	163		176
D			12	175	187
Demand	175	175	175	175	700

The entering cell is DQ (from question 4 of exercise 1B)

Stepping stone route

	P	Q	R	S	Supply
A	134				134
B	41	162			203
C		$13-\theta$	$163+\theta$		176
D		θ	$12-\theta$	175	187
Demand	175	175	175	175	700

$\theta = 12$ Exiting cell is CS
improved solution

	P	Q	R	S	Supply
A	134				134
B	41	162			203
C		1	175		176
D		12		175	187
Demand	175	175	175	175	700

Shadow costs		56	73	60	76	
		P	Q	R	S	Supply
0	A	(56)	86	80	61	134
3	B	(59)	(76)	78	65	203
−3	C	62	(70)	(57)	67	176
−5	D	60	(68)	75	(71)	187
	Demand	175	175	175	175	700

Improvement indices: $AQ = 86 - 0 - 73 = 13$

$AR = 80 - 0 - 60 = 20$

$AS = 61 - 0 - 76 = -15$

$BR = 78 - 3 - 60 = 15$

$BS = 65 - 3 - 76 = -14$

$CP = 62 + 3 - 56 = 9$

$CS = 67 + 3 - 76 = -6$

$DP = 60 + 5 - 56 = 9$

$DR = 75 + 5 - 60 = 20$

Entering cell AS
stepping stone route

	P	Q	R	S	Supply
A	$134-\theta$			θ	134
B	$41+\theta$	$162-\theta$			203
C		1	175		176
D		$12+\theta$		$175-\theta$	187
Demand	175	175	175	175	700

Greatest value of θ is 134
Exiting cell is AP.

	P	Q	R	S	Supply
A				134	134
B	175	28			203
C		1	175		176
D		146		41	187
Demand	175	175	175	175	700

Shadow costs		41	58	45	61	
		P	Q	R	S	Supply
0	A	56	86	80	(61)	134
18	B	(59)	(76)	78	65	203
12	C	62	(70)	(57)	67	176
10	D	60	(68)	75	(71)	187
	Demand	175	175	175	175	700

Improvement indices: AP $= 56 - 0 - 41 = 15$

AQ $= 86 - 0 - 58 = 28$

AR $= 80 - 0 - 45 = 35$

BR $= 78 - 18 - 45 = 15$

BS $= 65 - 18 - 61 = -14$

CP $= 62 - 12 - 41 = 9$

CS $= 67 - 12 - 61 = -6$

DP $= 60 - 10 - 41 = 9$

DR $= 75 - 10 - 45 = 20$

Entering cell is BS

	P	Q	R	S	Supply
A				134	134
B	175	$28 - \theta$		θ	203
C		1	175		176
D		$146 + \theta$		$41 - \theta$	187
Demand	175	175	175	175	700

$\theta = 28$ Exiting cell is BQ

	P	Q	R	S	Supply
A				134	134
B	175			28	203
C		1	175		176
D		174		13	187
Demand	175	175	175	175	700

Shadow costs		55	58	45	61	
		P	Q	R	S	Supply
0	A	56	86	80	(61)	134
4	B	(59)	76	78	(65)	203
12	C	62	(70)	(57)	67	176
10	D	60	(68)	75	(71)	187
	Demand	175	175	175	175	700

Entering cell is CS

	P	Q	R	S	Supply
A				134	134
B	175			28	203
C		$1-\theta$	175	θ	176
D		$174+\theta$		$13-\theta$	187
Demand	175	175	175	175	700

$\theta = 1$ Exiting cell CQ

	P	Q	R	S	Supply
A				134	134
B	175			28	203
C			175	1	176
D		175		12	187
Demand	175	175	175	175	700

Shadow costs		55	58	51	61	
		P	Q	R	S	Supply
0	A	56	86	80	(61)	134
4	B	(59)	76	78	(65)	203
6	C	62	70	(57)	(67)	176
10	D	60	(68)	75	(71)	187
	Demand	175	175	175	175	700

Improvement indices:

$AP = 56-0-55=1$

$AQ = 86-0-58=28$

$AR = 80-0-51=29$

$BQ = 76-4-58=14$

$BR = 78-4-51=23$

$CP = 62-6-55=1$

$CQ = 70-6-58=6$

$DP = 60-10-55=-5$

$DR = 75-10-51=14$

Entering cell is DP.

	P	Q	R	S	Supply
A				134	134
B	$175-\theta$			$28+\theta$	203
C			175	1	176
D	θ	175		$12-\theta$	187
Demand	175	175	175	175	700

$\theta = 12$ Exiting cell is DS

	P	Q	R	S	Supply
A				134	134
B	163			40	203
C			175	1	176
D	12	175			187
Demand	175	175	175	175	700

Shadow costs		55	63	51	61	
		P	Q	R	S	Supply
0	A	56	86	80	61	134
4	B	59	76	78	65	203
6	C	62	70	57	67	176
5	D	60	68	75	71	187
	Demand	175	175	175	175	700

Improvement indices: $AP = 56 - 0 - 55 = 1$

$AQ = 86 - 0 - 63 = 23$

$AR = 80 - 0 - 51 = 29$

$BQ = 76 - 4 - 63 = 9$

$BR = 78 - 4 - 51 = 23$

$CP = 62 - 6 - 55 = 1$

$CQ = 70 - 6 - 63 = 1$

$DR = 75 - 5 - 51 = 19$

$DS = 71 - 5 - 61 = 5$

No negative improvement indices so our solution is optimal.

134 units A to S
163 units B to P
40 units B to S
175 units C to R
1 unit C to S
12 units D to P
175 units D to Q Cost 43 053

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Exercise C, Question 4

Question:

	P	Q	Stock
A	2	6	3
B	2	7	5
C	6	9	2
Demand	6	4	

The table shows the unit cost, in pounds, of transporting goods from each of three warehouses A, B and C to each of two supermarkets P and Q. It also shows the stock at each warehouse and the demand at each supermarket.
Solve the transportation problem shown in the table. Use the north-west corner method to obtain an initial solution. You must state your shadow costs, improvement indices, stepping-stone routes, θ values, entering cells and exiting cells. You must state the initial cost and the improved cost after each iteration.

Solution:

	P	Q	Stock
A	2	6	3
B	2	7	5
C	6	9	2
Demand	6	4	10

Initial solution

	P	Q	Stock
A	3		3
B	3	2	5
C		2	2
Demand	6	4	10

Shadow costs		2	7	
		P	Q	Stock
0	A	2	6	3
0	B	2	7	5
2	C	6	9	2
	Demand	6	4	10

Improvement indices

$AQ = 6 - 0 - 7 = -1$

$CP = 6 - 2 - 2 = 2$

AQ is the entering cell

	P	Q	Stock
A	$3-\theta$	θ	3
B	$3+\theta$	$2-\theta$	5
C		2	2
Demand	6	4	10

$\theta = 2$ Exiting cell BQ.

	P	Q	Stock
A	1	2	3
B	5		5
C		2	2
Demand	6	4	

Shadow costs		2	6	
		P	Q	Stock
0	A	(2)	(6)	3
0	B	(2)	7	5
3	C	6	(9)	2
	Demand	6	4	

Improvement indices

$BQ = 7 - 0 - 6 = 1$
$CP = 6 - 3 - 2 = 1$

There are no negative improvement indices so our solution is optimal

1 unit from A to P
2 units from A to Q
5 units from B to P
2 units from C to Q Cost 42

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Exercise D, Question 1

Question:

Formulate the following transportation problem as a linear programming problem.

	P	Q	R	Supply
A	150	213	222	32
B	175	204	218	44
C	188	198	246	34
Demand	28	45	37	

Solution:

Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C\}$

$j \in \{P, Q, R\}$

$x_{ij} \geq 0$

$$\text{Minimise } C = 150x_{11} + 213x_{12} + 222x_{13} + 175x_{21} + 204x_{22} + 218x_{23} + 188x_{31} + 198x_{32} + 246x_{33}$$

Subject to

$$x_{11} + x_{12} + x_{13} \leq 32$$
$$x_{21} + x_{22} + x_{23} \leq 44$$
$$x_{31} + x_{32} + x_{33} \leq 34$$
$$x_{11} + x_{21} + x_{31} \leq 28$$
$$x_{12} + x_{22} + x_{32} \leq 45$$
$$x_{13} + x_{23} + x_{33} \leq 37$$

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Exercise D, Question 2

Question:

Formulate the following transportation problem as a linear programming problem.

	P	Q	R	S	Supply
A	27	33	34	41	54
B	31	29	37	30	67
C	40	32	28	35	29
Demand	21	32	51	46	

Solution:

Let x_{ij} be the number of units transported from i to j where

$i \in \{A, B, C\}$

$j \in \{P, Q, R, S\}$

$x_{ij} \geq 0$

$$\text{Minimise } C = 27x_{11} + 33x_{12} + 34x_{13} + 41x_{14}$$
$$31x_{21} + 29x_{22} + 37x_{23} + 30x_{24}$$
$$40x_{31} + 32x_{32} + 28x_{33} + 35x_{34}$$

Subject to
$$x_{11} + x_{12} + x_{13} + x_{14} \leq 54$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 67$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 29$$
$$x_{11} + x_{21} + x_{31} \leq 21$$
$$x_{12} + x_{22} + x_{32} \leq 32$$
$$x_{13} + x_{23} + x_{33} \leq 51$$
$$x_{14} + x_{24} + x_{34} \leq 46$$

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Exercise D, Question 3

Question:

Formulate the following transportation problem as a linear programming problem.

	P	Q	R	Supply
A	17	24	19	123
B	15	21	25	143
C	19	22	18	84
D	20	27	16	150
Demand	200	100	200	

Solution:

Let x_{ij} be the number of units transported from i to j where $i \in \{A, B, C, D\}$
and $j \in \{P, Q, R\}$

$x_{ij} \geq 0$

$$\text{Minimise } C = 17x_{11} + 24x_{12} + 19x_{13} + 15x_{21} + 21x_{22} + 25x_{23} + 19x_{31} + 22x_{32} + 18x_{33} + 20x_{41} + 27x_{42} + 16x_{43}$$

Subject to

$$x_{11} + x_{12} + x_{13} \leq 123$$
$$x_{21} + x_{22} + x_{23} \leq 143$$
$$x_{31} + x_{32} + x_{33} \leq 84$$
$$x_{41} + x_{42} + x_{43} \leq 150$$
$$x_{11} + x_{21} + x_{31} + x_{41} \leq 200$$
$$x_{12} + x_{22} + x_{32} + x_{42} \leq 100$$
$$x_{13} + x_{23} + x_{33} + x_{43} \leq 200$$

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Exercise D, Question 4

Question:

Formulate the following transportation problem as a linear programming problem.

	P	Q	R	S	Supply
A	56	86	80	61	134
B	59	76	78	65	203
C	62	70	57	67	176
D	60	68	75	71	187
Demand	175	175	175	175	

Solution:

Let x_{ij} be the number of units transported from i to j where $i \in \{A, B, C, D\}$ and $j \in \{P, Q, R, S\}$

$$\begin{aligned}\text{Minimise } C = &\ 56x_{11} + 86x_{12} + 80x_{13} + 61x_{14}\\ &+59x_{21} + 76x_{22} + 78x_{23} + 65x_{24}\\ &+62x_{31} + 70x_{32} + 57x_{33} + 67x_{34}\\ &+60x_{41} + 68x_{42} + 75x_{43} + 71x_{44}\end{aligned}$$

$$\begin{aligned}\text{Subject to }\ & x_{11} + x_{12} + x_{13} + x_{14} \le 134\\ & x_{21} + x_{22} + x_{23} + x_{24} \le 203\\ & x_{31} + x_{32} + x_{33} + x_{34} \le 176\\ & x_{41} + x_{42} + x_{43} + x_{44} \le 187\\ & x_{11} + x_{21} + x_{31} + x_{41} \le 175\\ & x_{12} + x_{22} + x_{32} + x_{42} \le 175\\ & x_{13} + x_{23} + x_{33} + x_{43} \le 175\\ & x_{14} + x_{24} + x_{34} + x_{44} \le 175\end{aligned}$$

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Exercise E, Question 1

Question:

	L	M	Supply
A	20	70	15
B	40	30	5
C	60	90	8
Demand	16	12	

The table shows the cost, in pounds, of transporting a car from each of three factories A, B and C to each of two showrooms L and M. It also shows the number of cars available for delivery at each factory and the number required at each showroom.

a Use the north-west corner method to find an initial solution.
b Solve the transportation problem, stating shadow costs, improvement indices, entering cells, stepping-stone routes, θ values and exiting cells.
c Demonstrate that your solution is optimal and find the cost of your optimal solution.
d Formulate this problem as a linear programming problem, making your decision variables, objective function and constraints clear.
e Verify that your optimal solution lies in the feasible region of the linear programming problem.

Solution:

a

	L	M	Stock
A	15		15
B	1	4	5
C		8	8
Demand	16	12	28

b

Shadow costs		20	10	
		L	M	Stock
0	A	(20)	70	15
20	B	(40)	(30)	5
80	C	60	(90)	8
		16	12	28

Improvement indices.

$AM = 70 - 0 - 10 = 60$

$CL = 60 - 80 - 20 = -40$

Entering cell CL $\theta = 1$

Exiting cell BL

	L	M	Stock
A	15		15
B	$1-\theta$	$4+\theta$	5
C	θ	$8-\theta$	8
Demand	16	12	28

	L	M	Stock
A	15		15
B		5	5
C	1	7	8
Demand	16	12	28

Shadow costs		20	50	
		L	M	Stock
0	A	(20)	70	15
−20	B	40	(30)	5
40	C	(60)	(90)	8
	Demand	16	12	28

c Improvement indices:

AM = 70 − 0 − 50 = 20

BL = 40 + 20 − 20 = 40

No negative improvement indices, so optimal solution.

15 units from A to L
5 units from B to M Cost: 1 140
1 unit from C to L
7 units from C to M

d Let x_{ij} be the number of units transported from *i* to *j* where $i \in \{A, B, C\}$ and $j \in \{L, M\}$

$x_{ij} \geq 0$

Minimise $C = 20x_{11} + 70x_{12}$
$+40x_{21} + 30x_{22}$
$+60x_{31} + 90x_{32}$

Subject to
$x_{11} + x_{12} \leq 15$
$x_{21} + x_{22} \leq 5$
$x_{31} + x_{32} \leq 8$
$x_{11} + x_{21} + x_{31} \leq 16$
$x_{12} + x_{22} + x_{32} \leq 12$

e In our solution

$x_{11} = 15 \quad x_{12} = 0$
$x_{21} = 0 \quad x_{22} = 5$
$x_{31} = 1 \quad x_{32} = 7$
$x_{11} + x_{12} \leq 15 \quad 15 + 0 \leq 15$
$x_{21} + x_{22} \leq 5 \quad 0 + 5 \leq 5$
$x_{31} + x_{32} \leq 8 \quad 1 + 7 \leq 8$
$x_{11} + x_{21} + x_{31} \leq 16 \quad 15 + 0 + 1 \leq 16$
$x_{12} + x_{22} + x_{32} \leq 12 \quad 0 + 5 + 7 \leq 12$

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Exercise E, Question 2

Question:

	P	Q	R	Supply
F	23	21	22	15
G	21	23	24	35
H	22	21	23	10
Demand	10	30	20	

The table shows the cost of transporting one unit of stock from each of three supply points F, G and H to each of three sales points P, Q and R. It also shows the stock held at each supply point and the amount required at each sales point.

a Use the north-west corner method to obtain an initial solution.
b Taking the most negative improvement index to indicate the entering square, perform two complete iterations of the stepping-stone method. You must state your shadow costs, improvement indices, stepping-stone routes and exiting cells.
c Explain how you can tell that your current solution is optimal.
d State the cost of your optimal solution.
e Taking the zero improvement index to indicate the entering square, perform one further iteration to obtain a second optimal solution.

Solution:

a

	P	Q	R	Stock
F	10	5		15
G		25	10	35
H			10	10
Demand	10	30	20	60

b

Shadow cost		23	21	22	
		P	Q	R	Stock
0	F	(23)	(21)	22	15
2	G	21	(23)	(24)	35
1	H	22	21	(23)	10
	Demand	10	30	20	60

Improvement indices:

$FR = 22 - 0 - 22 = 0$

$GP = 21 - 2 - 23 = -4$

$HP = 22 - 1 - 23 = -2$

$HQ = 21 - 1 - 21 = -1$

Entering cell is GP.

	P	Q	R	Stock
F	$10-\theta$	$5+\theta$		15
G	θ	$25-\theta$	10	35
H			10	10
Demand	10	30	20	60

$\theta = 10$ exiting cell FP
improved solution:

	P	Q	R	Stock
F		15		15
G	10	15	10	35
H			10	10
Demand	10	30	20	60

Shadow cost		19	21	22	
		P	Q	R	Stock
0	F	23	(21)	22	15
2	G	(21)	(23)	(24)	35
1	H	22	21	(23)	10
	Demand	10	30	20	60

Improvement indices:

$FP = 23 - 0 - 19 = 4$
$FR = 22 - 0 - 22 = 0$
$HP = 22 - 1 - 19 = 2$
$HQ = 21 - 1 - 21 = -1$

Entering cell is HQ

	P	Q	R	Stock
F		15		15
G	10	$15-\theta$	$10+\theta$	35
H		θ	$10-\theta$	10
Demand	10	30	20	60

$\theta = 10$

Exiting cell is HR
Improved solution

	P	Q	R	Stock
F		15		15
G	10	5	20	35
H		10		10
Demand	10	30	20	60

Shadow costs		19	21	22	
		P	Q	R	Stock
0	F	23	(21)	22	15
2	G	(21)	(23)	(24)	35
0	H	22	(21)	23	10
	Demand	10	30	20	60

Improvement indices:

$FP = 23 - 0 - 19 = 4$

$FR = 22 - 0 - 22 = 0$

$HP = 22 - 0 - 19 = 3$

$HR = 23 - 0 - 22 = 1$

c No negative improvement indices, so optimal solution

d 15 units F to Q

10 units G to P
5 units G to Q
20 units G to R
10 units H to Q

Cost: 1 330

e entering cell FR.

	P	Q	R	Stock
F		$15-\theta$	θ	15
G	10	$5+\theta$	$20-\theta$	35
H		10		10
Demand	10	30	20	60

$\theta = 15$

exiting cell FQ

second solution

	P	Q	R	Stock
F			15	15
G	10	20	5	35
H		10		10
Demand	10	30	20	60

Cost also 1330

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Exercise E, Question 3

Question:

	X	Y	Z	Supply
J	8	5	7	30
K	5	5	9	40
L	7	2	10	50
M	6	3	15	50
Demand	25	45	100	

The transportation problem represented by the table above is to be solved. A possible north-west corner solution is

	X	Y	Z	Supply
J	25	5		30
K		40		40
L		0	50	50
M			50	50
Demand	25	45	100	

a Explain why it is was necessary to add a zero entry (in cell LY) to the solution.
b State the cost of this initial solution.
c Choosing cell MX as the entering cell, perform one iteration of the stepping-stone method to obtain an improved solution. You must make your route clear, state your exiting cell and the cost of the improved solution.
d Determine whether your current solution is optimal. Give a reason for your answer.

After two more iterations the following solution was found.

	X	Y	Z	Supply
J			30	30
K		20	20	40
L			50	50
M	25	25		50
Demand	25	45	100	

e Taking the most negative improvement index to indicate the entering square, perform one further complete iteration of the stepping-stone method. You must state your shadow costs, improvement indices, stepping-stone route and exiting cell.

Solution:

a Otherwise the solution would be degenerate.

b 1675

c

	X	Y	Z	Supply
J	$25-\theta$	$5+\theta$		30
K		40		40
L		$0-\theta$	$50+\theta$	50
M	θ		$50-\theta$	50
Demand	25	45	100	170

The exiting cell is LY $\theta = 0$

	X	Y	Z	Supply
J	25	5		30
K		40		40
L			50	50
M	0		50	50
Demand	25	45	100	170

The cost is unchanged at 1675

d

Shadow costs		8	5	17	
		X	Y	Z	Supply
0	J	(8)	(5)	7	30
0	K	5	(5)	9	40
−7	L	7	2	(10)	50
−2	M	(6)	3	(15)	50
	Demand	25	45	100	170

Improvement indices:

JZ $= 7-0-17 = -10$

KX $= 5-0-8 = -3$

KZ $= 9-0-17 = -8$

LX $= 7+7-8 = 6$

LY $= 2+7-5 = 4$

MY $= 3+2-5 = 0$

This solution is not optimal since there are negative improvement indices.

e

	X	Y	Z	Supply
J			30	30
K		20	20	40
L			50	50
M	25	25		50
Demand	25	45	100	170

Shadow costs		6	3	7	
		X	Y	Z	Supply
0	J	8	5	(7)	30
2	K	5	(5)	(9)	40
3	L	7	2	(10)	50
0	M	(6)	(3)	15	50
	Demand	25	45	100	170

Improvement indices:

$JX = 8 - 0 - 6 = 2$

$JY = 5 - 0 - 3 = 2$

$KX = 5 - 2 - 6 = -3$

$LX = 7 - 3 - 6 = -2$

$LY = 2 - 3 - 3 = -4$

$MZ = 15 - 0 - 7 = 8$

Entering square is LY

	X	Y	Z	Supply
J			30	30
K		$20 - \theta$	$20 + \theta$	40
L		θ	$50 - \theta$	50
M	25	25		50
Demand	25	45	100	170

$\theta = 20$ Exiting square is KY.
Improved solution

	X	Y	Z	Supply
J			30	30
K			40	40
L		20	30	50
M	25	25		50
Demand	25	45	100	170

Shadow costs		2	−1	7	
		X	Y	Z	Supply
0	J	8	5	(7)	30
2	K	5	5	(9)	40
3	L	7	(2)	(10)	50
4	M	(6)	(3)	15	50
	Demand	25	45	100	170

Improvement indices:

$JX = 8 - 0 - 2 = 6$

$JY = 5 - 0 + 1 = 6$

$KX = 5 - 2 - 2 = 1$

$KY = 5 - 2 + 1 = 4$

$LX = 7 - 3 - 2 = 2$

$MZ = 15 - 4 - 7 = 4$

All improvement indices are non-negative, so we have an optimal solution.

25 units M to X
25 units M to Y
20 units L to Y   Cost 1135
30 units L to Z
40 units K to Z
30 units J to Z

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Exercise E, Question 4

Question:

	S	T	U	Supply
A	6	10	7	50
B	7	5	8	70
C	6	7	7	50
Demand	100	30	20	

a Explain why a dummy demand point might be needed when solving a transportation problem.

The table shows the cost, in pounds, of transporting one van load of fruit tree seedlings from each of three greenhouses A, B and C to three garden centres S, T and U. It also shows the stock held at each greenhouse and the amount required at each garden centre.

b Use the north-west corner method to obtain an initial solution.

c Taking the most negative improvement index in each case to indicate the entering square, use the stepping-stone method to obtain an optimal solution. You must state your shadow costs, improvement indices, stepping-stone routes, entering squares and exiting cells.

d State the cost of your optimal solution.

e Formulate this problem as a linear programming problem. Make your decision variables, objective function and constraints clear.

Solution:

a The total demand is 150, the total stock is 170 so demand $<$ stock
We need a dummy demand to absorb the surplus stock.

	S	T	U	Dummy	Stock
A	6	10	7	0	50
B	7	5	8	0	70
C	6	7	7	0	50
Demand	100	30	20	20	170

b Initial solution

	S	T	U	Dummy	Stock
A	50				50
B	50	20			70
C		10	20	20	50
Demand	100	30	20	20	170

c

Shadow costs		6	4	4	−3	
		S	T	U	Dummy	Stock
0	A	6	10	7	0	50
1	B	7	5	8	0	70
3	C	6	7	7	0	50
	Demand	100	30	20	20	170

Improvement indices:

$$\text{AT} = 10 - 0 - 4 = 6$$
$$\text{AU} = 7 - 0 - 4 = 3$$
$$\text{A Dummy} = 0 - 0 + 3 = 3$$
$$\text{BU} = 8 - 1 - 4 = 3$$
$$\text{B Dummy} = 0 - 1 + 3 = 2$$
$$\text{CS} = 6 - 3 - 6 = -3$$

	S	T	U	Dummy	Stock
A	50				50
B	$50-\theta$	$20+\theta$			70
C	θ	$10-\theta$	20	20	50
Demand	100	30	20	20	170

Entering square CS $\theta = 10$
Exiting square CT

Improved solution

	S	T	U	Dummy	Stock
A	50				50
B	40	30			70
C	10		20	20	50
Demand	100	30	20	20	170

Shadow costs		6	4	7	0	
		S	T	U	Dummy	Stock
0	A	(6)	10	7	0	50
1	B	(7)	(5)	8	0	70
0	C	(6)	7	(7)	(0)	50
	Demand	100	30	20	20	170

Improvement indices:

$$\begin{aligned} AT &= 10-0-4=6 \\ AU &= 7-0-7=0 \\ A\ \text{Dummy} &= 0-0+0=0 \\ BU &= 8-1-7=0 \\ B\ \text{Dummy} &= 0-1-0=-1 \\ CT &= 7-0-4=3 \end{aligned}$$

	S	T	U	Dummy	Stock
A	50				50
B	$40-\theta$	30		θ	70
C	$10+\theta$		20	$20-\theta$	50
Demand	100	30	20	20	170

Entering square B Dummy
$\theta = 20$

Exiting square C Dummy.

Improved solution

	S	T	U	Dummy	Stock
A	50				50
B	20	30		20	70
C	30		20		50
Demand	100	30	20	20	170

Shadow costs		6	4	7	−1	
		S	T	U	Dummy	Stock
0	A	(6)	10	7	0	50
1	B	(7)	(5)	8	(0)	70
0	C	(6)	7	(7)	0	50
	Demand	100	30	20	20	170

Improvement indices:

$$\text{AT} = 10 - 0 - 4 = 6$$
$$\text{AU} = 7 - 0 - 7 = 0$$
$$\text{A Dummy} = 0 - 0 + 1 = 1$$
$$\text{BU} = 8 - 1 - 7 = 0$$
$$\text{CT} = 7 - 0 - 4 = 3$$
$$\text{C Dummy} = 0 - 0 + 1 = 1$$

The two zero improvement indices indicate that there are two further optimal solutions.

Using AU as an entering square, we get

	S	T	U	Dummy
A	$50-\theta$		θ	
B	20	30		20
C	$30+\theta$		$20-\theta$	

→

	S	T	U	Dummy	Stock
A	30		20		50
B	20	30		20	70
C	50				50
Demand	100	30	20	20	170

Using BU as an entering square, we get

	S	T	U	Dummy
A	50			
B	$20-\theta$	30	θ	20
C	$30+\theta$		$20-\theta$	

→

	S	T	U	Dummy	Stock
A	50				50
B		30	20	20	70
C	50		0		50
Demand	100	30	20	20	170

↘

	S	T	U	Dummy	Stock
A	50				50
B	0	30	20	20	70
C	50				50
Demand	100	30	20	20	170

d Cost 910

e Let x_{ij} be the number of units transported from i to j where $i \in \{A, B, C\}$ and $j \in \{S, T, U, \text{dummy}\}$

$x_{ij} \geq 0$

$$\text{Minimize } C = 6x_{11} + 10x_{12} + 7x_{13} + 7x_{21} + 5x_{22} + 8x_{23} + 6x_{31} + 7x_{32} + 7x_{33}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 50$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 70$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 50$$
$$x_{11} + x_{21} + x_{31} \leq 100$$
$$x_{12} + x_{22} + x_{32} \leq 30$$
$$x_{13} + x_{23} + x_{33} \leq 20$$
$$x_{14} + x_{24} + x_{34} \leq 20$$