P2 Revision Notes

Algebra

The order of the polynomial is the highest power. The number of turning points is one less than the power.

$f(x)$	
$f(x) + 2$	Shifted 2 units up the y-axis
$f(x) - 2$	Shifted 2 units down the y-axis
$2f(x)$	Stretched vertically, factor of 2
$f(2x)$	Stretched horizontally, factor of ½ (compressed)
$f(nx)$	Stretched horizontally, factor of $^1/_n$
$f(x + 2)$	Shifted 2 units LEFT (x-axis)
$f(x - 2)$	Shifted 2 units RIGHT (x-axis)
$f(-x)$	Reflection in y-axis
$-f(x)$	Reflection in x-axis
$-f(-x)$	Rotation about origin

Interval Notation

$(1,2)$: $1 < x < 2$
$[1,2]$: $1 \leq x \leq 2$
$[1,2)$: $1 \leq x < 2$
$(1,2]$: $1 < x \leq 2$

Functions

A Mapping is a relationship between objects in one set and objects in another. In a mapping the domain is the input and the range is the output.
There are several kinds of mapping:
Many to Many
Many to One
One to Many
One to One
A function is a many to one or one to one mapping. Each input yields one specific output.
A function is fully defined by stating the mapping and the domain (set of inputs).
= the integers
= the rational numbers
= the real numbers
= the +ve real numbers excluding 0
A function is <u>even</u> if $f(-x) = f(x)$. (Self reflection in y-axis. E.G. $y=x^2$)
A function is <u>odd</u> if $f(x) = -f(-x)$. (Rotational symmetry about origin. E.G. $y=x^3$)

Composite Functions

$fg(x) = f(g(x))$
$fgh(x) = f(g(h(x)))$
Substitute the solution for the previous function (or the function itself) as the x value in the next outward function in the nesting.

Inverse Functions

Notation is $f^{-1}(x)$
Only one to one functions have inverse functions. The range of the function is the domain of the inverse function, and vice versa.

To find the inverse function (make $f(x)$ be y) swap the x and y terms around and rearrange to find x.
E.G. $y = 3x - 1$ $x = 3y - 1$ $y = {}^{1}/_{3}(x - 1)$

Modulus Function
$|x|$ (modulus function negates the +ve or –ve sign)
A | | function is a many to one mapping. It is a reflection of –ve values about the x-axis.
E.G. $|3x + 5|$ becomes $3x + 5$ for some values of x, and $-3x - 5$ for other values. This ensures that the result is always +ve.

Sequences and Series

If there are n objects and n boxes then the number of arrangements are n!
If there are n objects and only r are chosen to arrange then the number of permutations are $^{n}P_{r} = \frac{n!}{(n-r)!}$
If some objects are the same, divide the number of permutations (n!) by the number of identical objects of one type (A!). If there are multiple objects that are duplicated then multiply them together (A!B!)
In general, when there are r objects selected from n, the no. of combinations are:
$^{n}C_{r} = \frac{n!}{(n-r)!r!}$

Binomial Series
Pascal's Triangle
$(a+b)n \equiv \sum {}^{n}C_{r}a^{n-r}b^{r}$ (from r=0 to n)

E.G. To find the term independent of x in the expansion of $(4x^2 + {}^{3}/_{x})^{12}$
General term (r^{th}).
$^{12}C_{r}(4x^2)^{r}({}^{3}/_{x})^{12-r}$
Power of x is $\frac{x^{2r}}{x^{12-r}} = x^{3r-12}$
So r = 4.

Trigonometry

$Cot\theta = Tan^{-1}\theta$
$Sec\theta = Cos^{-1}\theta$
$Cosec\theta = Sin^{-1}\theta$
$Sin(A+B) = SinACosB + CosASinB$
$Sin(A-B) = SinACosB - CosASinB$
$Cos(A+B) = CosACosB - SinASinB$
$Cos(A-B) = CosACosB + SinASinB$
$Tan(A+B) = \frac{TanA + TanB}{1 - TanATanB}$
$Tan(A-B) = \frac{TanA - TanB}{1 + TanATanB}$
$Cot(A+B) = \frac{CotACotB - 1}{CotB + CotA}$
$Sin2A = 2SinACosA$
$Cos2A = Cos^2A - Sin^2A$
Tan2A = <u>2TanA</u>

$1 - Tan^2A$
$aCos\theta \pm bSin\theta = RSin(\theta \pm \alpha)$
$ArcSinX = Sin^{-1}X = \theta$

Exponential and Logarithms

2^x 3^x and 2.5^x are all exponentials.
The number is the "Base" and the power is the "Index or Exponent"

E.G. y^x (y is the base and x is the power/index/exponent/logarithm)

All exponential graphs to the power something x cross at (0,1), and the gradient is always positive and increasing.
The gradient divided by the y value ($^{dy}/_{dx} \div y$) is constant in exponential graphs. For e (2.72) this value is 1.
2^x is an exponential function but e^x is THE exponential function.

$\log_a b = c \Leftrightarrow a^c = b$
$\log_a b + \log_a c = \log_a bc$
$\log_a b - \log_a c = \log_a {}^b/_c$
$\log_a x^n = n\log_a x$
$\log_a 1 = 0$
$\log_a b = \dfrac{\log_c b}{\log_c a}$
$\log X = \log_{10} X$
$\ln X = \log_e X$

Differentiation

$^d/_{dx}\ ke^x = ke^x$
$^d/_{dx}\ \ln|x| = {}^1/_x$

Integration

$\int ke^x = ke^x$
$\int {}^1/_x = \ln|x|$
Area against y-axis so rearrange to get x in terms of y.
E.G. $y = \ln|x|$
$x = e^y$
Then integrate with respect to y.
$\int x \cdot dy = \int e^y \cdot dy$

Volumes of Integration
About x-axis: $Vol = \pi \int y^2 \cdot dx$
About y-axis: $Vol = \pi \int x^2 \cdot dy$

Numerical Methods

Iteration
Rearrainge to make x = something in terms of x. Then say x_{n+1} = something in terms of x_n, and if it converges it will converge on a root.