Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Notes	Marks
1(a)	$8\cosh^4 x = 8\left(\frac{e^x + e^{-x}}{2}\right)^4 = \frac{8}{16}\left(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x}\right)$ Applies $\cosh x = \frac{e^x + e^{-x}}{2}$ and attempts to expand the bracket to at least 4 different and no more than 5 different terms of the correct form but they may be "uncollected" depending on how they do the expansion. Allow unsimplified terms e.g. $\left(e^x\right)^3 e^{-x}$. May see $8\left(\frac{e^x + e^{-x}}{2}\right)^2\left(\frac{e^x + e^{-x}}{2}\right)^2$ but must attempt to expand as above		M1
	$= \frac{1}{2}\left(e^{4x} + e^{-4x}\right) + 4\left(\frac{e^{2x} + e^{-2x}}{2}\right) + 3 = ...$	Collects appropriate terms and reaches the form $\cosh 4x + p\cosh 2x + q$ or obtains values of p and q.	M1
	$= \cosh 4x + 4\cosh 2x + 3$	Correct expression or values e.g. $p = 4$ and $q = 3$	A1
	No marks are available in (a) if exponentials are not used but note that they may appear in combination with the use of hyperbolic identities e.g.: $8\cosh^4 x = 8\left(\cosh^2 x\right)^2 = 8\left(\frac{\cosh 2x + 1}{2}\right)^2 = 2\left(\frac{e^{2x} + e^{-2x}}{2} + 1\right)^2$ $= 2\left(\frac{e^{4x} + 2 + e^{-4x}}{4} + e^{2x} + e^{-2x} + 1\right) = \frac{e^{4x} + e^{-4x}}{2} + 4\left(\frac{e^{2x} + e^{-2x}}{2}\right) + 2$ $= \cosh 4x + 4\cosh 2x + 3$ **Allow to "meet in the middle" e.g. expands as above and compares with** $\frac{1}{2}\left(e^{4x} + e^{-4x}\right) + p\left(\frac{e^{2x} + e^{-2x}}{2}\right) + q \Rightarrow p = ..., q = ...$ **but to score any marks the expansion must be attempted.**		
			(3)

<table>
<tr><td rowspan="6">(b)
Way 1</td><td colspan="2">$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 8\cosh^4 x - 4\cosh 2x - 3 - 17\cosh 2x + 9 = 0$
$\Rightarrow 8\cosh^4 x - 21\cosh 2x + 6 = 0 \Rightarrow 8\cosh^4 x - 21\left(2\cosh^2 x - 1\right) + 6 = 0$
Uses their result from part (a) and $\cosh 2x = \pm 2\cosh^2 x \pm 1$
to obtain a quadratic equation in $\cosh^2 x$
or
$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\left(2\cosh^2 x - 1\right)^2 - 1 - 17\left(2\cosh^2 x - 1\right) + 9 = 0$
Uses $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ and $\cosh 2x = \pm 2\cosh^2 x \pm 1$
to obtain a quadratic equation in $\cosh^2 x$</td><td>M1</td></tr>
<tr><td>$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$</td><td>Correct 3TQ in $\cosh^2 x$</td><td>A1</td></tr>
<tr><td>$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$
$\Rightarrow \cosh^2 x = \frac{9}{2}\left(, \frac{3}{4}\right)$</td><td>Solves 3TQ in $\cosh^2 x$ (apply usual rules if necessary) to obtain
$\cosh^2 x = k \ \left(k \in \mathbb{R} \text{ and } > 1\right)$. May be implied by their values – check if necessary.</td><td>M1</td></tr>
<tr><td colspan="2">$\cosh^2 x = \frac{9}{2} \Rightarrow \cosh x = \frac{3}{\sqrt{2}} \Rightarrow x = \pm \ln\left(\frac{3}{\sqrt{2}} + \sqrt{\frac{9}{2} - 1}\right)$
or
$\cosh x = \frac{3}{\sqrt{2}} \Rightarrow \frac{e^x + e^{-x}}{2} = \frac{3}{\sqrt{2}} \Rightarrow \sqrt{2}e^{2x} - 6e^x + \sqrt{2} = 0 \Rightarrow e^x = ... \Rightarrow x = ...$
or
$\cosh^2 x = \frac{9}{2} \Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{9}{2} \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = ... \Rightarrow x = ...$
Takes square root to obtain $\cosh x = k \ \left(k > 1\right)$ and applies the correct logarithmic form for arcosh or uses the correct exponential form for $\cosh x$ to obtain at least one value for x
The root(s) must be real to score this mark.</td><td>M1</td></tr>
<tr><td colspan="2">$x = \pm \ln\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{14}}{2}\right)$
Both correct and exact including brackets.
Accept simplified equivalents e.g. $x = \ln\left(\frac{3}{\sqrt{2}} \pm \frac{\sqrt{7}}{\sqrt{2}}\right)$ but withhold this mark if additional answers are given unless they are the same e.g. allow $x = \pm \ln\left(\frac{3\sqrt{2}}{2} \pm \frac{\sqrt{14}}{2}\right)$</td><td>A1</td></tr>
<tr><td></td><td></td><td>(5)</td></tr>
</table>

(b) Way 2	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\cosh^2 2x - 1 - 17\cosh 2x + 9 = 0$ Applies $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ to obtain a quadratic equation in cosh $2x$		M1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$	Correct 3TQ in cosh $2x$	A1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$ $\Rightarrow \cosh 2x = 8\left(, \frac{1}{2}\right)$	Solves 3TQ in cosh $2x$ (apply usual rules if necessary) to obtain $\cosh 2x = k \ \left(k \in \mathbb{R} \text{ and } > 1\right)$	M1
	$\cosh 2x = 8 \Rightarrow 2x = \pm\ln\left(8 + \sqrt{8^2 - 1}\right)$ or $\cosh 2x = 8 \Rightarrow \frac{e^{2x} + e^{-2x}}{2} = 8 \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = \ldots \Rightarrow 2x = \ldots$ Applies the correct logarithmic form for arcosh from $\cosh 2x = k \ \left(k > 1\right)$ or uses the correct exponential form for cosh $2x$ to obtain at least one value for $2x$ The root(s) must be real to score this mark.		M1
	$x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ or e.g. $x = \pm\ln\left(8 + 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \frac{1}{2}\ln\left(8 \pm \sqrt{63}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
(b) Way 3	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow \frac{e^{4x} + e^{-4x}}{2} - \frac{17}{2}\left(e^{2x} + e^{-2x}\right) + 9 = 0$ $\Rightarrow e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ M1: Applies the correct exponential forms and attempts a quartic equation in e^{2x} A1: Correct equation		M1A1
	$e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ $\Rightarrow e^{2x} = 8 \pm 3\sqrt{7}, \ldots$	Solves and proceeds to a value for e^{2x} where $e^{2x} > 1$ and real.	M1
	$\Rightarrow e^{2x} = 8 \pm 3\sqrt{7} \Rightarrow 2x = \ln\left(8 \pm 3\sqrt{7}\right)$	Takes ln's to obtain at least one value for $2x$ The root(s) must be real to score this mark.	M1
	$x = \frac{1}{2}\ln\left(8 \pm 3\sqrt{7}\right)$ or e.g. $x = \ln\left(8 \pm 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
			Total 8

Question Number	Scheme	Notes	Marks
2	$\frac{dx}{d\theta}=\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta$ Correct derivative. Do not condone missing brackets e.g. $\frac{dx}{d\theta}=\frac{1}{\sec\theta+\tan\theta}\times\sec\theta\tan\theta+\sec^2\theta-\cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta-\cos\theta,\ \tan\theta\sin\theta$		B1
	$\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2=\left(\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+\left(-\sin\theta\right)^2$ Attempts $\frac{dy}{d\theta}$ and then $\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2$		M1
	$S=(2\pi)\int\cos\theta\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$ $=(2\pi)\int\cos\theta\sqrt{\left(\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+\left(-\sin\theta\right)^2}\,d\theta$ Applies a correct surface area formula using their $\frac{dx}{d\theta}$ and their $\frac{dy}{d\theta}$ with or without the 2π For reference: $\sqrt{\left(\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta\right)^2+\left(-\sin\theta\right)^2}=\tan\theta$ Allow π in front of the integral but must be an integral		M1
	$(2\pi)\int\sin\theta\ d\theta$	Fully correct simplified integral with or without the 2π	A1
	$=(2\pi)[-\cos\theta](+c)$	Correct integration with or without the 2π	A1
	$(2\pi)[-\cos\theta]_0^{\frac{\pi}{4}}=(2\pi)\left(-\frac{1}{\sqrt{2}}+1\right)$ Applies the limits 0 and $\frac{\pi}{4}$. Must see evidence of both limits if necessary but condone e.g. $(2\pi)\left(-\frac{1}{\sqrt{2}}-1\right)$ **Depends on both previous method marks.**		dM1
	TSA = $2\pi\left(-\frac{1}{\sqrt{2}}+1\right)+\pi\times1^2+\pi\times\left(\frac{1}{\sqrt{2}}\right)^2$	**Correct** expressions for the 2 "ends" and adds these to their curved surface area. **Depends on the previous method mark.**	dM1
	$=\frac{\pi}{2}\left(7-2\sqrt{2}\right)$	Correct answer in the required form or correct values for p and q.	A1
	<u>Note:</u> **The final answer should follow correct work. The final mark should be withheld following e.g. $\frac{dy}{d\theta}$ clearly seen as +sin θ or $\int\sin\theta\ d\theta=+\cos\theta$** **<u>Note:</u>** **Without the "ends" the answer is $\frac{\pi}{2}\left(4-2\sqrt{2}\right)$ (usually scores 6/8)**		
			(8)
			Total 8

Alternative for first 4 marks:

	$\frac{dx}{d\theta}=\frac{\sec\theta\tan\theta+\sec^2\theta}{\sec\theta+\tan\theta}-\cos\theta$ Correct derivative. Do not condone missing brackets e.g. $\frac{dx}{d\theta}=\frac{1}{\sec\theta+\tan\theta}\times\sec\theta\tan\theta+\sec^2\theta-\cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta-\cos\theta,\ \tan\theta\sin\theta$		B1
	$1+\left(\frac{dy}{dx}\right)^2=1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2$ Attempts $1+\left(\frac{dy}{dx}\right)^2$ with $\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$		M1
	$S=(2\pi)\int\cos\theta\sqrt{1+\left(\frac{dy}{dx}\right)^2}\frac{dx}{d\theta}d\theta$ $=(2\pi)\int\cos\theta\sqrt{1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2}(\sec\theta-\cos\theta)d\theta$ Applies a correct surface area formula using their $\frac{dx}{d\theta}$ and their $\frac{dy}{dx}$ with or without the 2π For reference: $\sqrt{1+\left(\frac{-\sin\theta}{\sec\theta-\cos\theta}\right)^2}(\sec\theta-\cos\theta)=\tan\theta$ Allow π in front of the integral but must be an integral		M1
	$(2\pi)\int\sin\theta\ d\theta$	Fully correct simplified integral with or without the 2π	A1

Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2}$ $\Rightarrow \frac{dx}{dy} = -2\,\text{sech}\, y \tanh y$	Takes "sech" of both sides and differentiates to obtain $\frac{dx}{dy} = k\,\text{sech}\, y \tanh y$ or equivalent.	M1
	$\Rightarrow \frac{dx}{dy} = -2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2}$ M1: Replaces sech y with $\frac{x}{2}$ and tanh y with $\sqrt{1-\left(\frac{x}{2}\right)^2}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
			(4)
(a) Way 2	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2}$ $\Rightarrow \cosh y = \frac{2}{x} \Rightarrow \sinh y \frac{dy}{dx} = -\frac{2}{x^2}$	Takes "sech" of both sides, changes to "cosh" and differentiates to obtain $\sinh y \frac{dy}{dx} = \frac{k}{x^2}$ or equivalent.	M1
	$\Rightarrow \frac{dy}{dx} = -\frac{2}{x^2 \sinh y} = -\frac{2}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ M1: Replaces sinh y with $\sqrt{\left(\frac{2}{x}\right)^2 - 1}$ A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) Way 3	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow y = \text{arcosh}\left(\frac{2}{x}\right)$ Changes to "arcosh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{\left(\frac{2}{x}\right)^2 - 1}} \times -\frac{2}{x^2}$ M1: Differentiates to the form $\frac{k}{x^2\sqrt{\left(\frac{2}{x}\right)^2 - 1}}$ oe A1: Correct equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

(a) Way 4	$y=\text{arsech}\left(\frac{x}{2}\right)\Rightarrow \text{sech}\, y=\frac{x}{2}\Rightarrow\left(\frac{x}{2}\right)^2=\text{sech}^2 y\Rightarrow \tanh y=\sqrt{1-\left(\frac{x}{2}\right)^2}$ $\Rightarrow \text{sech}^2 y\frac{\text{d}y}{\text{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ Differentiates to $\text{sech}^2 y\frac{\text{d}y}{\text{d}x}=kx\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ or equivalent		M1
	$\Rightarrow \text{sech}^2 y\frac{\text{d}y}{\text{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}\Rightarrow\frac{x^2}{4}\frac{\text{d}y}{\text{d}x}=-x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}\Rightarrow\frac{\text{d}y}{\text{d}x}=-\frac{4}{x}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ M1: Replaces $\text{sech}^2 y$ with $\left(\frac{2}{x}\right)^2$ A1: Correct equation involving $\frac{\text{d}x}{\text{d}y}$ or $\frac{\text{d}y}{\text{d}x}$ in any form in terms of x only.		M1A1
	$\Rightarrow\frac{\text{d}y}{\text{d}x}=\frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) Way 5	$y=\text{arsech}\left(\frac{x}{2}\right)\Rightarrow \text{sech}\, y=\frac{x}{2}\Rightarrow y=\text{artanh}\left(\sqrt{1-\left(\frac{x}{2}\right)^2}\right)$ Changes to "artanh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow\frac{\text{d}y}{\text{d}x}=\frac{\frac{1}{2}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)}\times-\frac{x}{2}$ M1: Differentiates to the form $\frac{kx\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)}$ oe A1: Correct equation involving $\frac{\text{d}x}{\text{d}y}$ or $\frac{\text{d}y}{\text{d}x}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow\frac{\text{d}y}{\text{d}x}=\frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

There may be other methods used.
If you are in any doubt if the method deserves any marks use Review.

(b)	$\text{f}(x)=\tanh^{-1}(x)+\text{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \text{f}'(x)=\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}$ Correct f ′(*x*) following through their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ Also allow with "made up" *p* and *q* or the letters *p* and *q*.		B1ft
	$\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2(1-x^2)=x\sqrt{4-x^2}\Rightarrow 4(1-x^2)^2=x^2(4-x^2)$ Sets $\frac{dy}{dx}=0$ with their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation		M1
	$5x^4-12x^2+4=0$	Correct quartic	A1
	$5x^4-12x^2+4=0\Rightarrow x^2=2,\ 0.4$ $\Rightarrow x=...$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for *x*. Apply usual rules for solving and check if necessary. Allow complex roots.	M1
	$x=\sqrt{\frac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\frac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x=\pm\sqrt{\frac{2}{5}}$	A1
			(5)
			Total 9

<u>Special case:</u>

It is possible for a correct solution in (b) following a sign error in (a) e.g.

$$\frac{dy}{dx}=\frac{2}{x\sqrt{4-x^2}}$$

$$\text{f}(x)=\tanh^{-1}(x)+\text{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \text{f}'(x)=\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}$$

$$\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2(1-x^2)=-x\sqrt{4-x^2}\Rightarrow 4(1-x^2)^2=x^2(4-x^2)\text{ etc.}$$

This is likely to score M1M1A0A0 in (a) but allow full recovery in (b) if it leads to the correct answer.

Question Number	Scheme	Notes	Marks
4(a)	$\lambda = 3 \Rightarrow \lvert\mathbf{M}-3\mathbf{I}\rvert = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3(8) - k(4k) + 2(-4) = 0$ or e.g. $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert = \begin{vmatrix} 6-\lambda & k & 2 \\ k & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix} = 0$ $\Rightarrow (6-\lambda)(5-\lambda)(7-\lambda) - k(k(7-\lambda)) + 2(0-2(5-\lambda)) = 0 \Rightarrow 24 - k(4k) - 8 = 0$ Correct interpretation of 3 being an eigenvalue leading to the formation of **a quadratic equation in *k* only**. If the method for forming the determinant is not clear then look for at least 2 correct "components". NB rule of Sarrus gives $24 - 8 - 4k^2 = 0$		M1
	$\Rightarrow 4k^2 = 16 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	**d**M1
	$k = \pm 2$	Correct values	A1
			(3)
(a) Way 2	$\begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 6x+ky+2z=3x \\ kx+5y=3y \\ 2x+7z=3z \end{matrix}$ $z = -\frac{1}{2}x,\ y = -\frac{1}{2}kx \Rightarrow 6x - \frac{k^2x}{2} - x = 3x \Rightarrow \frac{k^2}{2} = 2$ Eliminates *z* and *y* and reaches a quadratic equation in *k* only		M1
	$\frac{k^2}{2} = 2 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	**d**M1
	$k = \pm 2$	Correct values	A1
(b)	$k = -2 \Rightarrow \lvert\mathbf{M}-\lambda\mathbf{I}\rvert = \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix}$ $\Rightarrow (6-\lambda)(7-\lambda)(5-\lambda) + 2(2\lambda-14) + 2(2\lambda-10) = 0$ Applies a value of *k* from (a) and a recognisable attempt at the characteristic equation (the "= 0" is not needed here). If the method is not clear then look for at least 2 correct "components".		M1
	$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \Rightarrow \lambda = ...$	Solves cubic. May use λ = 3 as a factor or calculator to solve. **Depends on the first mark.** Allow complex roots.	**d**M1
	$\lambda = 6,\ 9\ (,3)$	Correct values. Allow to come from $k = 2$	A1
			(3)

(c)	$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 6x-2y+2z=3x \\ -2x+5y=3y \\ 2x+7z=3z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ or $\begin{pmatrix} 3 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} 6x-2y+2z=0 \\ -2x+5y=0 \\ 2x+7z=0 \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ Correct strategy for finding the eigenvector using a value of *k* from (a) Note that the cross product of any 2 rows or columns of **M** – 3**I** gives an eigenvector		M1
	$p\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct eigenvector	A1
	$\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct normalised eigenvector	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
5(i)	$x^2-3x+5=\left(x-\frac{3}{2}\right)^2+\frac{11}{4}$	Correct completion of the square	B1
	$\int\frac{1}{\sqrt{x^2-3x+5}}\,\mathrm{d}x=\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{11}{4}}}\,\mathrm{d}x=\sinh^{-1}\frac{2x-3}{\sqrt{11}}(+c)$ M1: Use of $\sinh^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\sinh^{-1}\frac{x-\frac{3}{2}}{\sqrt{\frac{11}{4}}}(+c)$, $\sinh^{-1}\frac{x-\frac{3}{2}}{\frac{\sqrt{11}}{2}}(+c)$ Allow equivalents for $\sinh^{-1}$ e.g. arsinh, arcsinh but **not** arsin or arcsin		M1A1
	You may see logarithmic forms for the answer: e.g. $\ln\left(\frac{2x-3}{\sqrt{11}}+\sqrt{\left(\frac{2x-3}{\sqrt{11}}\right)^2+1}\right)$, $\ln\left(x-\frac{3}{2}+\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{11}{4}}\right)$ but apply isw once a correct answer is seen.		
			(3)
(ii)	$63+4x-4x^2=-4\left(x^2-x-\frac{63}{4}\right)$ $=-4\left(\left(x-\frac{1}{2}\right)^2-\frac{64}{4}\right)$	Obtains $-4\left(\left(x-\frac{1}{2}\right)^2\pm\ldots\right)$ or $-4\left(x-\frac{1}{2}\right)^2\pm\ldots$ or $\ldots-(2x-1)^2$	M1
	$-4\left(\left(x-\frac{1}{2}\right)^2-16\right)$ or $64-4\left(x-\frac{1}{2}\right)^2$ or $64-(2x-1)^2$	Correct completion of the square	A1
	$\int\frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x=\frac{1}{2}\sin^{-1}\left(\frac{2x-1}{8}\right)(+c)$ M1: Use of $\sin^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\frac{1}{2}\sin^{-1}\frac{x-\frac{1}{2}}{4}(+c)$, $-\frac{1}{2}\sin^{-1}\frac{\frac{1}{2}-x}{4}(+c)$ Allow equivalents for $\sin^{-1}$ e.g. arsin, arcsin but not arsinh or arcsinh		M1A1
			(4)
	In (ii) there are no marks for using $\int\frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x=-\int\frac{1}{\sqrt{4x^2-63-4x}}\,\mathrm{d}x$ But if completion of square attempted first allow M1A1 e.g. for $\int\frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x=\int\frac{1}{\sqrt{64-(2x-1)^2}}\,\mathrm{d}x$ but then M0 for $=\int\frac{-1}{\sqrt{(2x-1)^2-64}}\,\mathrm{d}x$		
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\int e^x \sin^n x\,dx = e^x \sin^n x - n\int e^x \sin^{n-1} x\cos x\,dx$ Applies integration by parts to obtain $\pm e^x \sin^n x \pm \alpha\int e^x \sin^{n-1} x\cos x\,dx$		M1
	$= e^x \sin^n x - n\left\{e^x \sin^{n-1} x\cos x - \int e^x\left((n-1)\sin^{n-2} x\cos^2 x - \sin^n x\right)dx\right\}$ M1: Applies integration by parts to $\pm\alpha\int e^x \sin^{n-1} x\cos x\,dx$ to obtain $\pm e^x \sin^{n-1} x\cos x \pm \int e^x\left(\alpha\sin^{n-2} x\cos^2 x - \beta\sin^n x\right)dx$ Or equivalent e.g. $\pm e^x \sin^{n-1} x\cos x \pm \int e^x\left(\alpha\sin^{n-2} x - \beta\sin^n x\right)dx$ (if Pythagoras applied first) A1: Fully correct expression for I_n from parts applied twice.		**d**M1A1
	$= e^x \sin^n x - n\left\{e^x \sin^{n-1} x\cos x - \int e^x\left((n-1)\sin^{n-2} x\left(1-\sin^2 x\right) - \sin^n x\right)dx\right\}$ Applies $\cos^2 x = 1-\sin^2 x$		**d**M1
	$= e^x \sin^n x - n\left\{e^x \sin^{n-1} x\cos x - \int e^x\left((n-1)\sin^{n-2} x - (n-1)\sin^n x - \sin^n x\right)dx\right\}$ $= e^x \sin^n x - n\left\{e^x \sin^{n-1} x\cos x - \int e^x\left((n-1)\sin^{n-2} x - n\sin^n x\right)dx\right\}$ $= e^x \sin^n x - ne^x \sin^{n-1} x\cos x + n(n-1)I_{n-2} - n^2 I_n \Rightarrow I_n = ...$ Completes by introducing I_{n-2} and I_n and makes I_n the subject		**d**M1
	$I_n = \frac{e^x \sin^{n-1} x}{n^2+1}\left(\sin x - n\cos x\right) + \frac{n(n-1)}{n^2+1}I_{n-2}$ * Fully correct proof with no errors but allow e.g. the occasional missing "dx" but any clear errors must be recovered before final answer e.g. missing brackets.		A1*
			(6)

(b)	$$I_4 = \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12}{17} I_2$$ or $$I_2 = \frac{e^x \sin x}{5}(\sin x - 2\cos x) + \frac{2}{5} I_0$$ Applies the reduction formula once	M1
	$$= \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12}{17}\left(\frac{e^x \sin x}{5}(\sin x - 2\cos x) + \frac{2}{5} I_0\right)$$ $$= \frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12e^x \sin x}{85}(\sin x - 2\cos x) + \frac{24}{85} e^x$$ Applies the reduction formula again and uses $I_0 = \int e^x \, dx = e^x$ to obtain an expression in terms of x	M1
	$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx = \left[\frac{e^x \sin^3 x}{17}(\sin x - 4\cos x) + \frac{12e^x \sin x}{85}(\sin x - 2\cos x) + \frac{24}{85} e^x\right]_0^{\frac{\pi}{2}}$$ $$= \frac{e^{\frac{\pi}{2}}}{17} + \frac{12e^{\frac{\pi}{2}}}{85} + \frac{24e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Uses the limits 0 and $\frac{\pi}{2}$ and subtracts**. Depends on both previous marks.**	**d**M1
	$$= \frac{41e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Correct expression or correct values e.g. $A = \ldots$, $B = \ldots$	A1
		(4)
	Note that the limits may be applied as they go e.g.: M1: $I_4 = \frac{e^{\frac{\pi}{2}}}{17}(1-0) + \frac{12}{17} I_2$ $I_2 = \frac{e^{\frac{\pi}{2}}}{5}(1-0) + \frac{2}{5} I_0$ $I_0 = e^{\frac{\pi}{2}} - 1$ M1M1: $I_4 = \frac{e^{\frac{\pi}{2}}}{17} + \frac{12}{17}\left(\frac{e^{\frac{\pi}{2}}}{5} + \frac{2}{5}\left(e^{\frac{\pi}{2}} - 1\right)\right)$ A1: $= \frac{41e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$	
		Total 10

Question Number	Scheme	Notes	Marks
7(a)	$\frac{x-3}{4}=\frac{y-5}{-2}=\frac{z-4}{7}\Rightarrow \mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-2\\7\end{pmatrix}$	Converts to parametric form. "**r** =" is not required	M1
	$2x+4y-z=1$ $\Rightarrow 2(3+4\lambda)+4(5-2\lambda)-4-7\lambda=1$ $\Rightarrow \lambda=...(3)\Rightarrow P$ is ...	Correct strategy for finding *P*. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
			(3)
(a) Way 2	$\frac{x-3}{4}=\frac{y-5}{-2}\Rightarrow x=13-2y$	Uses the Cartesian equation to find *x* in terms of *y*	M1
	$2x+4y-z=1\Rightarrow 26-4y+4y-z=1$ $\Rightarrow z=...,\ x=...,\ y=...$	Correct strategy for finding *P*. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\ -1,\ 25)$	Correct coordinates. Condone if given as a vector.	A1
(b)	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-1\end{pmatrix}=8-8-7=-7$	Applies the scalar product between the direction of l_1 and the normal to the plane	M1
	Examples: $\phi=\cos^{-1}\frac{\pm7}{\sqrt{69}\sqrt{21}}=...\quad \phi=\sin^{-1}\frac{\pm7}{\sqrt{69}\sqrt{21}}=...$ Attempts to find a relevant angle in degrees or radians. **Depends on the first method mark.**		**d**M1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1
			(3)
(b) Way 2	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\times\begin{pmatrix}2\\4\\-1\end{pmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1	M1
	$\sqrt{26^2+18^2+20^2}=\sqrt{21}\sqrt{69}\sin\alpha$ $\sin\alpha=\frac{10\sqrt{46}}{69}\Rightarrow\alpha=...$	Attempts to find a relevant angle. **Depends on the first method mark.**	**d**M1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1

(c)	$\mathbf{a}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 4 & -1\\ 4 & -2 & 7\end{vmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1. If no method is seen expect at least 2 correct components.	M1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 13 & -9 & -10\\ 2 & 4 & -1\end{vmatrix}=\begin{pmatrix}49\\-7\\70\end{pmatrix}$	Attempts vector product of "**a**" with normal to Π to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Depends on both previous M marks Attempts vector equation using their direction vector and their P	ddM1
		Correct equation or any equivalent correct vector equation	A1
			(5)
(c) Way 2	$\lambda=1\Rightarrow(7,\ 3,\ 11)$ lies on l_1 $\mathbf{r}=\begin{pmatrix}7\\3\\11\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(7+2t)+4(3+4t)-11+t=1$ $t=-\frac{2}{3}\Rightarrow\left(\frac{17}{3},\frac{1}{3},\frac{35}{3}\right)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\frac{1}{3}\begin{pmatrix}17\\1\\35\end{pmatrix}=\frac{1}{3}\begin{pmatrix}28\\-4\\40\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}15\\-1\\25\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r}=$ and not e.g. $l_2=\ldots$	A1
(c) Way 3	Normal to plane from l_1 $\mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}+t\begin{pmatrix}2\\4\\-1\end{pmatrix}$ $\Rightarrow 2(3+2t)+4(5+4t)-(4-t)=1$ $t=-1\Rightarrow(1,1,5)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix}15\\-1\\25\end{pmatrix}-\begin{pmatrix}1\\1\\5\end{pmatrix}=\begin{pmatrix}14\\-2\\20\end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+\mu\begin{pmatrix}7\\-1\\10\end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r}=$ and not e.g. $l_2=\ldots$	A1
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 4 = 9\left(1-e^2\right) \Rightarrow e = ...$ or e.g. $e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = ...$	Uses a correct formula with *a* and *b* correctly placed to find a value for *e*	M1
	$e = \frac{\sqrt{5}}{3}$	Correct value (or equivalent) $e = \pm\frac{\sqrt{5}}{3}$ scores A0	A1
			(2)
(b)(i)	$\left(\pm ae,\ 0\right) = \left(\pm\sqrt{5},\ 0\right)$ or $\left(\pm 3\frac{\sqrt{5}}{3},\ 0\right)$ Correct foci. Must be coordinates but allow unsimplified and isw if necessary. Follow through their *e* so allow for (± 3×their *e*, 0)		B1ft
(ii)	$x = \pm\frac{a}{e} = \pm\frac{9}{\sqrt{5}}$ or $x = \pm\frac{3}{\frac{\sqrt{5}}{3}}$ Correct directrices. Must be equations but allow unsimplified and isw if necessary. Follow through their *e* so allow for $x = \pm 3$/their *e*		B1ft
			(2)
	Special case: Use of a^2 for *a* and b^2 for *b* **consistently** scores M0A0 in (a) and B1ft B1ft in (b). This gives $e = \frac{\sqrt{65}}{9}$, $\left(\pm\sqrt{65},\ 0\right)$, $x = \pm\frac{81}{\sqrt{65}}$		
(c)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = -3\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta} = 2\cos\theta$ or $\frac{2x}{9} + \frac{2y}{4}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ or $y = \left(4-\frac{4x^2}{9}\right)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4x}{9}\left(4-\frac{4x^2}{9}\right)^{-\frac{1}{2}}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = ...\left(= \frac{2\cos\theta}{-3\sin\theta}\right)$	Correct strategy for the gradient of *l* in terms of *θ*. Allow $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\cos\theta}{-3\sin\theta}$ to be stated.	M1
	$y - 2\sin\theta = \frac{2\cos\theta}{-3\sin\theta}\left(x - 3\cos\theta\right)$	Correct straight line method (any complete method). Finding the equation of the normal is M0.	M1
	$-3y\sin\theta + 6\sin^2\theta = 2x\cos\theta - 6\cos^2\theta$ $2x\cos\theta + 3y\sin\theta = 6$*	Cso with at least one intermediate line of working	A1*
			(3)

(d)	$l_2: \quad y = \dfrac{3\sin\theta}{2\cos\theta}x$	Correct equation for l_2	B1
	$2x\cos\theta + 3y\sin\theta = 6,\ y = \dfrac{3\sin\theta}{2\cos\theta}x$ $\Rightarrow x = ...,\ y = ...$	Complete method for Q	M1
	$Q: \left(\dfrac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta}, \dfrac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta}\right)$ Correct coordinates. Allow as $x = ..., y = ...$ and allow equivalent correct expressions as long as they are single fractions e.g. $x = \dfrac{12\cos\theta}{4 + 5\sin^2\theta} \quad y = \dfrac{18\sin\theta}{4 + 5\sin^2\theta}, \quad x = \dfrac{12\cos\theta}{9 - 5\cos^2\theta} \quad y = \dfrac{18\sin\theta}{9 - 5\cos^2\theta}$		A1
			(3)

(e)	At Q, $\frac{y}{x}=\frac{3}{2}\tan\theta$	Uses their coordinates of Q to attempt an equation connecting x, y and θ or states or uses the equation found in (d)	M1
	$x=\frac{12\cos\theta}{4\cos^2\theta+9\sin^2\theta}=\frac{12\sec\theta}{4+9\tan^2\theta}\Rightarrow x^2=\frac{144\sec^2\theta}{\left(4+9\tan^2\theta\right)^2}=\frac{144\left(1+\frac{4y^2}{9x^2}\right)}{\left(4+9\times\frac{4y^2}{9x^2}\right)^2}$ or $y=\frac{18\sin\theta}{4\cos^2\theta+9\sin^2\theta}=\frac{12\sec\theta\tan\theta}{4+9\tan^2\theta}$ $\Rightarrow y^2=\frac{324\sec^2\theta\tan^2\theta}{\left(4+9\tan^2\theta\right)^2}=\frac{324\left(1+\frac{4y^2}{9x^2}\right)\frac{4y^2}{9x^2}}{\left(4+9\times\frac{4y^2}{9x^2}\right)^2}$ Eliminates θ **Depends on the first mark.**		**d**M1
	$\Rightarrow x^2=\frac{x^2\left(9x^2+4y^2\right)}{\left(x^2+y^2\right)^2}\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ or $\Rightarrow 9\times16x^2y^2\left(1+\frac{y^2}{x^2}\right)^2=4\times18^2\left(1+\frac{4y^2}{9x^2}\right)\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ Correct equation or correct values for α and β.		A1
			(3)
(e) Way 2	$x=\frac{12\cos\theta}{4+5\sin^2\theta}\quad y=\frac{18\sin\theta}{4+5\sin^2\theta}\Rightarrow\left(x^2+y^2\right)^2=\left(\frac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2$ Uses their Q to obtain an expression for $\left(x^2+y^2\right)^2$ in terms of θ		M1
	$\left(\frac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\frac{144+180\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\frac{36\left(4+5\sin^2\theta\right)}{\left(4+5\sin^2\theta\right)^2}\right)^2=\frac{1296}{\left(4+5\sin^2\theta\right)^2}$ $\frac{1296}{\left(4+5\sin^2\theta\right)^2}=\alpha x^2+\beta y^2=\alpha\frac{144\cos^2\theta}{\left(4+5\sin^2\theta\right)^2}+\beta\frac{324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\Rightarrow\alpha=...,\beta=...$ Substitutes into the given answer and solves for α and β **Depends on the first mark.**		**d**M1
	$\left(x^2+y^2\right)^2=9x^2+4y^2$	Correct expression or correct values for α and β.	A1
			Total 13