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Principal Examiner Feedback

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General

This paper proved to be a fair test of student knowledge and understanding. It provided the higher ability students with plenty of challenge but there were also many accessible marks available to all students who were suitably proficient in topic areas such as hyperbolic functions, conic sections, matrices, vectors and use of calculus.

Reports on Individual Questions

Question 1

The opening question was on the use of the exponential definitions of hyperbolic functions and the solving of a related equation. Most students scored well here although the final mark was not widely awarded.

In part (a) almost all used the correct exponential expression for cosh x. Most chose sensibly to expand $\cosh^4 x$ using the binomial theorem but there were other methods, primarily the use of $(\cosh^2 x)(\cosh^2 x)$. Errors in these expansions were not widely seen and most went on to group the resulting terms appropriately to arrive at an answer for p and q. Two slips were common: $\frac{4e^{2x} + 4e^{-2x}}{2}$ was sometimes replaced with $2\cosh 2x$ instead of $4\cosh 2x$ and the need to halve the constant 6 was overlooked on occasion.

A few students used identities before going on to apply exponentials which was acceptable. However, there were a significant number of responses that failed to use any exponential definitions at any stage. A small number of students used a “meet in the middle” approach but this could only score marks if it was clear that the required expansion had been attempted.
There were a range of routes available in part (b). Most students did opt to follow the “Hence...” and use their result from part (a) although there were occasional errors in substituting their expression for $\cosh 4x$ into the given equation. A few now thought they had a solvable equation and did not proceed to replace the term in $\cosh 2x$. Those who did obtain a quadratic in $\cosh^2 x$ generally used the correct $\cosh^2 x = 2\cosh^2 x – 1$, although there were sign and other errors seen with this identity. A small number chose not to use part (a) and instead produced a quadratic equation in $\cosh^2 x$ by replacing both $\cosh 4x$ and $\cosh 2x$ which required a little extra effort. Most solved their quadratic correctly although a few thought their solution to the equation was a solution for $\cosh x$ instead of $\cosh^2 x$. A largely successful alternative was to apply $\cosh 4x = 2\cosh^2 2x – 1$ to obtain a quadratic equation in $\cosh 2x$.

Attempts to find x were a mixture of using the logarithmic form of arcosh and using exponentials to produce quadratics. It was rare to see errors with these approaches and very few students attempted either of these methods with any undefined values for $\cosh x$ or $\cosh 2x$. A further option was to work in exponentials from the start. Most were able to produce a quartic in e^{2x} although some attempts stopped at this point.

The use of exponentials often led to more success in scoring the last mark by finding the two required answers. Using the logarithmic formula of arcosh is more straightforward but students need to remember the additional negative value.

Question 2

This question on finding the arc length of a curve saw a reasonable amount of good scoring.

Most were able to use the chain rule to obtain the correct $\frac{dx}{d\theta}$ although a few neglected to differentiate the $-\sin\theta$. However, those who did not simplify the fraction by spotting that the numerator could be written as $\sec\theta(\sec\theta + \tan\theta)$ then proceeded to square and got into difficulties handling the resulting expression, sometimes combined with poor squaring. Some converted their derivative into one in terms of $\sin\theta$ and $\cos\theta$ but not simplifying it before squaring made reaching the required integrand unlikely.

The first three marks were widely scored since the surface area formula was generally applied correctly (although the "y" was occasionally missing or not replaced). A small number tried to proceed using the non-parametric form of the formula but this usually led to the failure to make a complete substitution into the integral. It was very rare to see incorrect approaches such as the volume of revolution formula or attempts at $\int y \, dx$.

Those who achieved the correct integrand invariably scored the next three marks. It was rare to see a sign slip with the integration or any error applying the limits. However, despite the requirement for the **total** surface area, with the word "total" emphasised in bold type on the question paper, it was unfortunate to see only a small number of students proceed to add the areas of the two circles to their result.

Question 3

Performance was mixed on this differentiation question although many fully correct solutions to both parts were seen. A wide range of methods were viable in part (a) and all were used. There were some students who clearly did not recognise "arsech" and this led to little or no meaningful progress.

Way 1 was the most popular route and most obtained the correct form when differentiating $\text{sech}\, y$ although omitting the minus sign was a common slip. Some converted the equation to one involving $\cosh y$ and tended to be less prone to any sign error. Changing the equation to one in $\tanh y$ was much rarer. A few students were able to write $\text{arsech}\left(\frac{x}{2}\right)$ as an equivalent expression as an arcosh or artanh followed by direct differentiation. The error of replacing $\text{arsech}\left(\frac{x}{2}\right)$ with $\frac{1}{\text{arcosh}\left(\frac{x}{2}\right)}$ was occasionally seen. Using the logarithmic form of $\text{arcosh}\left(\frac{2}{x}\right)$ was not a good choice here and rarely successful. A very small number knew and used the correct derivative for arsech. Also rare, but generally successful, were attempts that used exponential definitions.

Solutions that required replacements of terms in y generally saw the correct identities being used although there were some errors, usually in signs. Most students who obtained a correct equation in terms of x involving either $\frac{dy}{dx}$ or $\frac{dx}{dy}$ usually used the appropriate algebraic processing to obtain the correct values of p and q.

Occasional missing "h"s in functions such as $\text{sech}\, y$ were generally condoned here but teachers and students are reminded of the need for clarity when working with hyperbolic functions and their inverses.

Students who had made little progress in (a) often did not attempt part (b). However, there were up to three of the five marks available to those who had shown some initiative in making up values for p and q or using the letters p and q. Most correctly differentiated artanh(x) and the first follow through mark was widely scored. The majority went on to achieve a quartic equation although a small but significant minority could not execute a strategy to deal with the fractions and square root.

Those who obtained a quartic usually obtained a consistent value for x although a small number who solved it as a quadratic in x^2 failed to then square root. The correct exact answer was widely seen although on occasion one or more of the invalid answers of $-\sqrt{\frac{2}{5}}$ or $\pm\sqrt{2}$ were also seen and not rejected.

Question 4

This question on eigenvalues and eigenvectors was a good source of marks for most students. It was unusual to see an incorrect approach taken to multiplying out the relevant determinants. Some used the noughts in the matrix to make the algebra easier.

In part (a), a suitable method was usually evident, although det $\mathbf{M} = 0$ was seen on occasion. Those who replaced λ with 3 before forming $|\mathbf{M} - \lambda\mathbf{I}|$ tended to proceed to the correct quadratic equation in k and the correct values. Substituting into the characteristic equation in terms of λ and k was more prone to error. Using $\mathbf{M}\boldsymbol{x} = 3\boldsymbol{x}$ to form and solve three simultaneous equations in x, y, z and k was much less common and had mixed outcomes.

Although the question stated that $k < 0$, working with any found value of k was condoned in part (b) and it was surprisingly common to see $k = 2$ used instead of -2. Full marks were available in this part for this error although this made the accuracy marks in part(c) unobtainable.

As with part (a), some students worked in both λ and k with the characteristic equation (often using their work in part (a)) and this did again lead to some algebraic errors. However, the correct cubic was widely seen and usually solved correctly. Although it was acceptable for a calculator to be used for solving the cubic some students did use long division by $\lambda - 3$ to factorise it. A small number of students were able to avoid multiplying out via some impressive algebra.

The method for obtaining eigenvectors was not quite as well-known. Some students gave no response to part (c) or incorrectly used $(\mathbf{M} - 3\mathbf{I})\boldsymbol{x} = 3\boldsymbol{x}$ rather than $\mathbf{M}\boldsymbol{x} = 3\boldsymbol{x}$ or $(\mathbf{M} - 3\mathbf{I})\boldsymbol{x} = \mathbf{0}$. Taking a vector product of two rows (or columns with this symmetric matrix) was a very rare alternative. Some slips with simultaneous equations were seen but a correct eigenvector was commonly obtained. A small number forgot to normalise and a few normalised incorrectly (usually the result of having fractions as components and then multiplying the vector by the magnitude instead of dividing by it).

Question 5

Many students confidently scored all the available marks with these two integrals and it was rare to be awarding no marks, although a very small number did not appreciate the need to complete the square on the quadratics. Although many used simple substitutions after completing the square, such as $u = 2x - 1$ in (ii), attempts using more complicated ones such as $2x = \sqrt{11}\sinh u + 3$ were sensibly avoided on the whole – there was no requirement stated in the question for this approach.

In part (i), completing the square was almost always performed correctly and most went on to achieve an arsinh expression, although some were careless in their use of the formula book, for example, forgetting the square root which led to an answer of arctan... Forgetting the "h" in arsinh or $\sinh^{-1}$ was not condoned here. Other slips were rare although there were a couple of incidences where the required denominator of $\sqrt{\frac{11}{4}}$ was immediately written as $\frac{\sqrt{11}}{4}$. A small number of students went straight to logarithmic forms, usually correctly.

Part (ii) was more demanding. Some students erroneously "extracted" a minus sign from the square root and proceeded to complete the square on $4x^2 - 4x - 63$. There were also slips seen when working on the correct $-4x^2 + 4x + 63$, usually sign errors leading to, for example, $62 - (2x - 1)^2$ instead of $64 - (2x - 1)^2$. Most who completed the square correctly went on to obtain an expression in arcsin with the correct argument, but those working with $2x$ often lost the required multiplier of $\frac{1}{2}$ in their final answer.

Question 6

This reduction formula question proved to be challenging for most although there were a creditable number of fully correct proofs seen in part(a). Part (b) was a decent source of marks for many students although the last mark was fairly elusive.

Part (a) was certainly demanding and there were quite a few who made no attempt or offered short-lived attempts. However, many were able to correctly assign u and v' and apply parts correctly to score at least the first mark. Incorrect differentiation of $\sin^{n-1}x$ was a fairly common error. Some made the correct assignment for u and v' for the second application of parts but did not seem prepared for the differentiation of a product that was required. As in Question 2, those who simplified the derivative at this point were more likely to be completely successful. Disorganised presentation of work, particularly with regard to bracketing, often led to sign errors. Most who scored the first two or three marks were generally in a good position to obtain the final two method marks. There were many attempts to "fudge" the given answer including largely unsubstantial approaches which tried to work backwards to meet the first application of parts. Recovery was only permitted for occasional notational slips and this had to be seen before the given answer for students to score the final accuracy mark.

There were other possible ways through this proof by splitting $\sin^{n-1}x$ – however, these were more difficult routes and correct solutions via these alternatives were very uncommon, with many attempts abandoned at a very early stage.

Part (b) was reasonably well done on the whole. The first mark for a completed attempt at one use of the reduction formula was widely awarded. Most were able to find an expression for I_4 although the integration of e^x for I_0 was occasionally wrong (often as a result of using the reduction formula a third time). Those who had obtained an expression for I_4 in terms of x tended to use the limits appropriately although some believed that the zero limit gave a total of 0 as a result of calculating e^0 as 0 instead of 1. Slips in substitution and errors using the nested brackets meant the last mark was not widely scored. Students who applied limits as they went along from the "bottom up" tended to be quite successful.

Question 7

This vector question saw good scoring in parts (a) and (b) but part (c) proved to be very discriminating.

In part (a), a correct parametric form was widely seen and the method to find P was well known. Some did succumb to sign or copying errors with the vectors and occasionally some substituted their parametric components into the plane $2x + 4y - z = 0$ rather than $2x + 4y - z = 1$. The correct coordinates were achieved by many and condoned if given as the vector $\overrightarrow{OP}$. There were a small number of attempts via Way 2 where expressions for two variables in terms of the other were obtained from the Cartesian equation and substituted into the plane equation. These approaches were usually correctly completed.

Part (b) was slightly more challenging but those who chose the correct vectors to work with tended to score at least two of the three marks here. A few thought that any vector in the plane through P could be used. Most opted for the scalar product approach and obtained a relevant angle with few slips with the product or magnitude calculations. Many were unclear what angle they had actually found however and so failed to modify it as necessary or changed a correct answer into a wrong final one.

It was rare to be awarding marks in part (c) although some completely correct solutions were seen. Many omitted this part or tried to form a scalar product equation with a direction vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ for l_2 and could not see any way of solving this. Some scored full marks via Way 1, often with impressive explanations and/or diagrams regarding the geometrical situation. A small number of others were able to at least obtain a relevant vector product but were unable to progress further. The similar approaches of Ways 2 and 3 were also used. Again, those who had sketched a realistic diagram were often more able to understand what was required. A handful of students found the equation of a further plane which contained l_1, the normal to the given plane and point P, then proceeding to find the line of intersection of the two planes. It was unfortunate to see final answers which were otherwise correct given as $l_2 =$... rather than $\mathbf{r} =$...

Question 8

The final question on an ellipse and an associated locus saw fairly good scoring. As usual for the last question, some rushed responses were seen from students who had not managed their time effectively.

In part (a) a correct eccentricity formula was regularly seen although there were the usual errors with the values of a and b and their placement. Attempts to use the eccentricity formula for the hyperbola was very rarely seen. Those with a correct equation in e usually obtained the correct value although some left it as $\pm$ or rejected the positive value on the basis that $e < 1$ - unaware that $+\frac{\sqrt{5}}{3} = 0.745... < 1$.

The two marks in part (b) were widely scored. Correct formulae for both the foci and directrices were almost always used and it was rare to withhold marks for foci not being given as coordinates or directrices missing the "$x = ...$" A very small number did not replace e with a value or confused it with the base of natural logarithms.

Full marks were widely awarded for the proof in part (c). The method was well known to almost all and most used correct differentiation either parametrically or implicitly. Attempts at explicit differentiation were very rare and usually wrong. A range of precise straight line methods were seen but those who made the sensible choice of $y - y_1 = m(x - x_1)$ invariably proceeded easily to the given answer with sufficient working and no errors.

Marks were less common in (d) and – particularly – part (e). Many were able to produce the correct equation for l_2 with few errors obtaining the normal gradient. A very small number did not take account of the given fact that the line passed through the origin. The most common error was the absence of the "x" in the equation. Most who made an attempt at Q used the correct strategy although there were errors including some losing the x from l_2 when substituting into the tangent equation.

Attempts at (e) were not numerous and were exclusively via Way 2. Those who made an attempt often scored the first mark for substituting their Q into the left hand side of the given equation but only the very best candidates were able to simplify the resulting expression to a form where comparison with the right hand side could allow α and β to be clearly deduced.

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