Pearson Edexcel

Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Level In
Mechanics M3 (WME03/01)

Summer 2021
Question Paper Log Number P66392A
Publications Code WME03_01_2106_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A**marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.

- Use of $g = 9.81$ should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.
  N2L Newton's Second Law (Equation of Motion)
  NEL Newton's Experimental Law (Newton's Law of Impact)
  HL Hooke's Law
  SHM Simple harmonic motion
  PCLM Principle of conservation of linear momentum
  RHS, LHS Right hand side, left hand side.

Question Number	Scheme				Marks
1	Slant height = $5a$				B1
		Hemisphere	Cone	Total	
	Mass	$2\pi(4a)^2 k\lambda$	$\pi \times 4a \times 5a \times \lambda$	$32\pi a^2 k\lambda + 20\pi a^2 \lambda$	B1
	Mass ratio(m)	$8k$	5	$8k+5$	
	Distance from vertex (y)	$5a$	$2a$	$4a$	B1
	my	$40ka$	$10a$	$4a(8k+5)$	
	$40ka + 10a = 4a(8k+5)$				M1A1
	$k = 1.25$ oe				A1
					[6]
	Alternatives for the moments equation:				
ALT 1	Moments about the centre of mass (G) of the toy:				
	Distances:	$(-)a$	$2a$	0	B1
	(Accept a provided the minus appears in the equation)				
	Equation: $-8ka + 10a = 0$				M1A1
ALT 2	Moments about the lowest point of the hemisphere:				
	Distances:	$2a$	$5a$	$3a$	B1
	Equation: $16ka - 5a = a(8k+5)$				M1A1
ALT 3	Moments about the centre of the circular base of the conical shell/hemispherical shell				
	Distances:	$2a$	$(-)a$	a	B1
	Equation: $16ka + 25a = 3a(8k+5)$				M1A1

B1	Slant height = $5a$ seen anywhere (could be on diagram)	
B1	Correct masses/mass ratio for hemisphere, cone and combined shape.	
B1	Correct distances seen. If using Alts 1 or 3 minus signs not needed here.	
M1	Moments equation attempted with all 3 terms (2 terms if about *G*). Condone inconsistent mass dimensions, if they are clearly using what they consider to be a mass. The equation can be formed using actual masses or a ratio of masses.	
A1	Correct moments equation, with actual masses or a ratio of masses and all signs correct.	
A1	Correct value for *k*. 1.25, $\frac{5}{4}$, $1\frac{1}{4}$ or any other equivalent fraction.	

Question Number	Scheme	Marks
2a)	$\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$	
	$T\cos\theta = mg$	M1A1
	$\frac{4}{5}T = mg \to T = \frac{5mg}{4}$	A1
		(3)
2b)	$r = a + \frac{5a}{4} \times \frac{3}{5} = \frac{7a}{4}$	B1
	$T\sin\theta = m\omega^2\left(\frac{7a}{4}\right)$	M1A1A1
	$\frac{3}{5}\left(\frac{5mag}{4}\right) = \frac{7}{4}ma\omega^2$	DM1
	$\omega = \sqrt{\frac{3g}{7a}}$	A1 **(6)**
		[9]
a)		
M1	Resolving vertically. $T\cos\theta$ or $T\sin\theta$ accepted	
A1	Correct equation	
A1	Correct tension	
b)		
B1	Correct radius of motion seen explicitly or used in N2L	
M1	Attempt at horizontal equation of motion. Allow either form of acceleration. $T\cos\theta$ or $T\sin\theta$ accepted. Allow this mark if $r = \frac{3a}{4}$ used.	
A1	Correct LHS	
A1	Correct RHS. Acceleration must now be in $r\omega^2$ form.	
DM1	Substitute in trig and eliminate T to find value for ω or ω^2. Depends on first M mark in (b)	
A1	Correct value for ω. (Square root sign must cover all terms)	

Question Number	Scheme	Marks
3a)	$\int_0^9$	B1
	$V=(\pi)\int_0^9\left(3-\sqrt{x}\right)^2 \mathrm{d}x$	M1
	$V=(\pi)\int_0^9\left(9-6\sqrt{x}+x\right)\mathrm{d}x$	
	$V=(\pi)\left[9x-4x^{\frac{3}{2}}+\frac{x^2}{2}\right]_0^9$	A1
	$V=(\pi)\left[81-108+\frac{81}{2}\right] \quad \left(-\pi[0]\right)$	
	$V=\frac{27}{2}\pi$ *	A1*
		(4)
3b)	$(\pi)\int xy^2\mathrm{d}x=(\pi)\int\left(9x-6x^{\frac{3}{2}}+x^2\right)\mathrm{d}x$	M1
	$=(\pi)\left[\frac{9}{2}x^2-\frac{12}{5}x^{\frac{5}{2}}+\frac{x^3}{3}\right]_0^9$	A1
	$=(\pi)\left(\left[\frac{729}{2}-\frac{2916}{5}+\frac{729}{3}\right]-[0]\right)$	
	$=\frac{243}{10}(\pi)$	A1
	$\bar{x}=\frac{(\pi)\int xy^2\mathrm{d}x}{(\pi)\int y^2\mathrm{d}x}$	DM1
	$\bar{x}=\frac{\left(\frac{243}{10}\right)}{\left(\frac{27}{2}\right)}=1.8$	A1 **(5)**
		[9]

(a)		
B1	Identifying correct limits.	
M1	Attempt at $(\pi)\int_0^9\left(3-\sqrt{x}\right)^2\,\mathrm{d}x$. π not needed. Limits may be missing. Minimum accepted for the squaring is $9\pm k\sqrt{x}\pm x$. At least one term must be integrated (power increased) and none to be differentiated (power decreased)	
A1	Correct integration. π not needed but (correct) limits now needed.	
A1*	**Given** result reached from fully correct working. π must not just appear on final line without justification (as this is a "show that" question).	
(b)		
M1	$(\pi)\int xy^2\mathrm{d}x=(\pi)\int\left(9x-6x^{\frac{3}{2}}+x^2\right)\mathrm{d}x$. π not needed.	
A1	Correct integration. π not needed.	
A1	Correct result from substitution of correct upper limit. (Lower limit is 0 and substitution of 0 gives 0)	
DM1	Use of $\overline{x}=\dfrac{(\pi)\int xy^2\mathrm{d}x}{(\pi)\int y^2\mathrm{d}x}$. π must appear in both or neither. Depend on both previous M marks	
A1	1.8. Accept any exact equivalent eg $\dfrac{18}{10},\dfrac{9}{5},1\dfrac{4}{5}$ etc.	

Question Number	Scheme	Marks
4a)	Initially $T_{IN} = mg\cos 60$	M1
	$T_{IN} = \dfrac{mg}{2}$	A1
		(2)
4b)	Energy to the lowest point $\dfrac{1}{2}mv^2 - mg(l) = -mg(l\sin 30)$	M1A1A1
	$T_{FI} - mg = m\dfrac{v^2}{r}$	M1A1
	$T_{FI} = m\left(\dfrac{gl}{l}\right) + mg$	DM1
	$T_{FI} = 2mg = 4T_{IN}$ *	A1* (7)
		[9]
(a)		
M1	Equation of motion towards centre at A. Must have $v = 0$. Weight must be resolved and tension not resolved. Allow with cos or sin of 60° or 30°.	
A1	Correct tension	
(b)		
M1	Energy equation from A to the lowest point. One KE and a difference in GPE required.	
A2	Correct equation. -1 for each error.	
M1	Equation of motion at the lowest point, with acceleration in either form. Tension and weight needed.	
A1	Correct equation. Acceleration must now be in the correct form.	
DM1	Solve to find tension at the lowest point. Must reach $T =$ ... but need not be simplified. Depends on both M marks in (b)	
A1*	Achieve the **given** result, from fully correct working. c.s.o.	
NB	If the equations in (b) are found at a general point, mark the equations as above. The final M mark will require some evidence of maximising the tension.	

Question Number	Scheme	Marks
5a)	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\sin 2x$	M1
	$\int 0.5v \, \mathrm{d}v = \int -\sin 2x \, \mathrm{d}x$	DM1
	$0.25v^2 = \dfrac{1}{2}\cos 2x(+c)$	A1
	$v^2 = 2\cos 2x + c$	
	$x = 0, v = 2 \Rightarrow 4 = 2 + c$	DM1
	$v^2 = 2\cos 2x + 2\left(= 4\cos^2 x\right)$	A1
	$v = 2\cos x$ *	A1*
		(6)
ALT	*Using definite integration*	
	$0.5v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\sin 2x$	M1
	$\int_2^v 0.5v \, \mathrm{d}v = \int_0^x -\sin 2x \, \mathrm{d}x \quad \left(\text{or } \int_0^x \sin 2x \, \mathrm{d}x\right)$	DM1
	$\left[0.25v^2\right]_2^v = \left[\dfrac{1}{2}\cos 2x\right]_0^x \quad \left(\text{or } \left[-\dfrac{1}{2}\cos 2x\right]_x^0\right)$	A1
	$0.25\left(v^2 - 4\right) = \dfrac{1}{2}\cos 2x - \dfrac{1}{2}$	DM1A1
	$v = 2\cos x$ *	A*
		(6)

Question Number	Scheme	Marks
5b)	$\dfrac{dx}{dt} = 2\cos x$	M1
	$\int \sec x dx = \int 2dt$	
	$\ln\left\|\sec x + \tan x\right\| = 2t + k$	DM1
	$t = 0, x = 0 \;\; \ln 1 = 2(0) + k \Rightarrow k = 0$	A1
	$t = \dfrac{1}{2}\ln\left\|\sec x + \tan x\right\| = \dfrac{1}{2}\ln\left(\sec\dfrac{\pi}{4} + \tan\dfrac{\pi}{4}\right)$	DM1
	$t = \dfrac{1}{2}\ln\left(\sqrt{2} + 1\right)$ *	A1*
		(5)
ALT	*Using definite integration*	
	$\dfrac{dx}{dt} = 2\cos x$	M1
	$\int_0^{\frac{\pi}{4}} \sec x dx = \int_0^t 2dt$	
	$\left[\ln\left\|\sec x + \tan x\right\|\right]_0^{\frac{\pi}{4}} = \left[2t\right]_0^t$	DM1A1
	$2t = \ln\left(\sec\dfrac{\pi}{4} + \tan\dfrac{\pi}{4}\right)$	DM1
	. $t = \dfrac{1}{2}\ln\left(\sqrt{2} + 1\right)$ ..*	A1*
		(5)
		[11]

(a)	*Indefinite integration*	
M1	Equation of motion, with acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. Condone sign error.	
DM1	Separate variables to prepare for integration. Depends on the M mark above.	
A1	Correct integration. Constant not needed.	
DM1	Substitute $x = 0,\ v = 2$ to find the constant. Depends on both M marks above.	
A1	A correct result for v^2	
A1*	**Given** result reached through use of double angle formula. (Formula need not be shown.).	
ALT	*Definite integration*	
M1	Equation of motion, with acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. Condone sign error.	
DM1	Separate variables, to prepare for integration. Limits not needed for this mark. Depends on the M mark above.	
A1	Correct integration – limits not needed	
DM1	Correct substitution of correct limits in their integrated function. Limits must be "paired" correctly. Depends on both previous M marks in (a) (Formula need not be shown.).	
A1	Correct expression which can yield v^2	
A1*	**Given** result reached through use of double angle formula. (Formula need not be shown.).	
(b)		
M1	Use of $v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$	
DM1	Correct separation of variables and attempt integration (integral is in the formula book). Depends on first M of (b) Modulus signs may be missing.	
A1	Correct integration and use limits to find correct value for constant.	
DM1	Substitute $x = \dfrac{\pi}{4}$ and solve for t. Depends on both previous M marks in (b)	
A1*	Given result reached from fully correct working. (Modulus signs may be missing throughout.).	

ALT	*Definite integration*	
M1	Use of $v=\dfrac{\mathrm{d}x}{\mathrm{d}t}$	
DM1	Correct separation of variables and attempt integration. Limits not needed. Depends on first M of (b). Modulus signs may be missing.	
A1	Correct integration including correct limits.	
DM1	Substitute their limits and solve for t. Depends on both previous M marks in (b)	
A1*	Given result reached from fully correct working. (Modulus signs may be missing throughout.).	

Question Number	Scheme	Marks
6a)	$F_r = \frac{1}{7}\times 0.4\times 9.8\ \left(=0.56\right)\left(\text{N}\right)$	B1
	$\frac{1}{2}\times 0.4v^2 = \frac{1}{2}\times 0.4(1.8)^2 - 0.8\times "0.56"$	M1A1A1 (ft their F_r)
	$v^2 = 1.00 \Rightarrow v = 1.0$ or $1.00\,(\text{m s}^{-1})$	A1
		(5)
6b)	$\frac{1}{2}\times 0.4(1.0)^2 = 0.56x + \frac{0.6x^2}{2(0.8)}$	M1A1B1
	$0.375x^2 + 0.56x - 0.2 = 0$	DM1
	$x = 0.2977...$	A1
	Total distance = 1.1 (m) (or 1.10)	A1
		(6)
ALT 1	*Work from A to C with total distance as the unknown*	
	$\frac{1}{2}\times 0.4(1.8)^2 = 0.56y + \frac{0.6\left(y-0.8\right)^2}{2(0.8)}$	M1A1B1
	$0.375y^2 - 0.046y - 0.408 = 0$	DM1
	$y = 1.0977...$	A1
	y = 1.1 or 1.10	A1
ALT 2	*Work from A to C with distance BC as the unknown*	
	$\frac{1}{2}\times 0.4(1.8)^2 = 0.56\left(y+0.8\right) + \frac{0.6y^2}{2(0.8)}$	M1A1B1
	Rest as main scheme	DM1A1A1
6c)	$T = \frac{0.6\times "0.2977"}{0.8}\ \left(=0.223\right)$	M1A1ft
	$0.223 < 0.56$ Tension less than F_{max}. Therefore particle will not move. cso.*	A1cso*
		(3)
		[14]

(a)		
B1	Correct friction seen. (Might be contained in WD) g or 9.8 acceptable.	
M1	Work-Energy equation with 2 KE terms and their WD by friction. All terms must be dimensionally correct.	
A1A1	One each for the KE terms.	
A1	$v = 1.0$ or 1.00 (2 or 3 sf as g has been used to obtain the speed at B)	
(b)		
M1	Work-Energy equation with KE, WD and EPE. EPE term to be of the form $\dfrac{\lambda x^2}{k \times \text{natural length}}$ with $k = 1$ or 2	
B1	Correct EPE term. M mark for the equation not needed for this mark.	
A1ft	Fully correct equation. Follow through their EPE	
DM1	Reducing to a 3 term quadratic in x, terms in any order. Depends on the first M of (b)	
A1	$x = 0.2977...$	
A1	1.1 (m) or 1.10 (m) (2 or 3 sf as g has been used to obtain the speed at B)	
ALT 1		
	First 4 marks as main scheme notes.	
A1A1	Award A1A1 if final answer is correct, from a correct equation. Award A1A0 if answer correct but not rounded.	
ALT 2	Notes as for main scheme.	
(c)		
M1	Use of Hooke's Law for their extension at C.	
A1ft	Correct tension. Follow through their extension. No need to simplify.	
A1cso*	Correct conclusion, from fully correct working including evaluation of tension.	

Question Number	Scheme	Marks
7a)	$\dfrac{4(x-2)}{2}=\dfrac{2(7-x-3)}{3}$	M1A1A1
	$6(x-2)=2(4-x)$	
	$x=2.5\,(\text{m})$ *	A1
		(4)
ALT	$\left(T_A=\right)\dfrac{4x}{2}=\dfrac{2y}{3}\left(=T_B\right)\quad x+y=2$	M1A1
	$x=\dfrac{1}{2}$ or $y=1.5$	A1
	Distance AO = 2.5 (m)	A1
7bi)	$2\ddot{y}=\dfrac{2(1.5-y)}{3}-\dfrac{4(y+0.5)}{2}$ OR $2\ddot{y}=\dfrac{4(-y+0.5)}{2}-\dfrac{2(1.5+y)}{3}$ (y measured towards B) (y measured towards A)	M1A1
	$\ddot{y}=-\dfrac{4}{3}y=-\omega^2 y\quad\therefore$ SHM	M1A1
		(4)
7bii)	$\omega^2=\dfrac{4}{3}$	B1
	$\left(2v_{\max}=6\Rightarrow\right)v_{\max}=3\left(\text{ms}^{-1}\right)$	B1
	$v_{\max}=a\omega=\dfrac{2a}{\sqrt{3}}$ Accept $1.2a$ or better	M1
	$3=a\dfrac{2}{\sqrt{3}}\qquad a=\dfrac{3\sqrt{3}}{2}$ (m) Accept 2.6 or better	A1
		(4)
7c)	$v^2=\dfrac{4}{3}\left(\dfrac{27}{4}-\left(\dfrac{3}{2}\right)^2\right)$	M1
	$v=\sqrt{6}\left(\text{ms}^{-1}\right)$ Accept 2.4 or better	A1 (2)

Question Number	Scheme	Marks
7d)	$\frac{3}{2}=\frac{3\sqrt{3}}{2}\sin\left(\omega t\right)$	M1A1
	$t=0.53\left(\text{s}\right)$ or better	A1
		(3)
ALT	$x=a\cos\omega t\Rightarrow 1.5=\frac{3\sqrt{3}}{2}\cos\left(\frac{2}{\sqrt{3}}\right)t$	
	time $=\frac{\pi}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	M1A1
	t = 0.53 or better	A1
		[17]

(a)		
M1	Equate tension in the two strings. Must be using $k\lambda\frac{x}{l}$ and the sum of the extensions must be 2	
A1	Correct LHS	
A1	Correct RHS	
A1*	Given result found from fully correct working.	
ALT		
M1	Obtain 2 equations using the tensions in the 2 strings and sum of extensions = 2 Must be using $k\lambda\frac{x}{l}$	
A1	2 correct equations	
A1	Correct extension for either string	
A1*	Obtain given result from fully correct working.	

b i)		
M1	Equation of motion with two variable tensions. Allow a or $\ddot{y}$	
A1	Correct equation. Allow a or $\ddot{y}$	
M1	Rearrange to required form. Must now be $\ddot{y}$	
A1	Correct result, from fully correct working **and** concluding statement.	
(ii)		
B1	Correct ω or ω^2	
B1	$v_{\max} = 3$ or $2v_{\max} = 6$ seen explicitly or used	
M1	Use $v_{\max} = a\omega$ with their ω or use $v^2 = \omega^2\left(a^2 - x^2\right)$ with their ω and $x = 0$	
A1	$a = \dfrac{3\sqrt{3}}{2}$ (allow 2.6 or better)	
(c)		
M1	Use of $v^2 = \omega^2\left(a^2 - x^2\right)$ with $x = 1.5$ and their ω and a OR attempt an energy equation with the correct number of terms	
A1	$v = \sqrt{6}$ (2.4 or better)	
7(d)		
M1	Use of $x = a\sin\left(\omega t\right)$, with $x = 1.5$ and their ω and a	
A1ft	Correct equation. Ft their ω and a	
A1	$t = 0.53$ or better $\left(t = 0.533021.....\right)$	
ALT		
M1	Complete method using cosine.	
A1ft	Correct equation (or equations) follow through their ω and a	
A1	$t = 0.53$ or better $\left(t = 0.533021.....\right)$	

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom