Pearson Edexcel

Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Subsidiary/Advanced Level
In Pure Mathematics P3 (WMA13/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\sqsubset$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

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General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right)2x\cos\left(\dfrac{1}{2}x\right)-\dfrac{1}{2}x^2\sin\left(\dfrac{1}{2}x\right)$	M1 A1
	Sets $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\tan\left(\dfrac{1}{2}x\right)=\dfrac{4}{x}\Rightarrow x=2\arctan\left(\dfrac{4}{x}\right)$ *	**d**M1 A1*
		(4)
(b)	$x_2=2\arctan\left(\dfrac{4}{2}\right)$	M1
	$x_2=\text{awrt }2.214$ $\quad$ $x_6=2.155$ cao	A1, A1
		(3)
		(7 marks)

(a)

M1: Differentiates using the product rule to obtain an expression of the form $Ax\cos\left(\dfrac{1}{2}x\right)+Bx^2\sin\left(\dfrac{1}{2}x\right)$

If the product rule is quoted it must be correct.

A1: For a correct simplified or unsimplified derivative.

dM1: Sets their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ and proceeds to an equation involving $\tan\left(\dfrac{1}{2}x\right)$. This may be implied by their working.

The M1 must have been scored.

A1*: Correctly achieves the given answer of $x=2\arctan\left(\dfrac{4}{x}\right)$

An intermediate line of $\tan\left(\dfrac{1}{2}x\right)=\dfrac{4}{x}$ or $\dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}=\dfrac{2x}{\frac{1}{2}x^2}$ must be seen

Note that some may work backwards having differentiated e.g.

$$x=2\arctan\left(\frac{4}{x}\right)\Rightarrow\frac{x}{2}=\arctan\left(\frac{4}{x}\right)\Rightarrow\tan\left(\frac{x}{2}\right)=\frac{4}{x}\Rightarrow\frac{x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}=4\Rightarrow4\cos\left(\frac{x}{2}\right)=x\sin\left(\frac{x}{2}\right)$$

$$\Rightarrow2x\cos\left(\frac{x}{2}\right)=\frac{1}{2}x^2\sin\left(\frac{x}{2}\right)\Rightarrow2x\cos\left(\frac{x}{2}\right)-\frac{1}{2}x^2\sin\left(\frac{x}{2}\right)=0$$

Score M1 for a complete method to reach $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ and A1 if correct with (minimal) conclusion.

(b)

M1: Attempts to use the given iteration formula. Award for $x_2=2\arctan\left(\dfrac{4}{2}\right)$ or awrt 2.21

Note that this mark may also be implied by awrt 127 if they are using degrees and no working is shown.

A1: x_2 = awrt 2.214

A1: x_6 = 2.155 (Note that this is not awrt)

Question Number	Scheme	Marks
2(a)	States or uses $\sin 2x = 2\sin x\cos x$	B1
	$\dfrac{1-\cos 2x}{2\sin 2x} = \dfrac{1-\left(1-2\sin^2 x\right)}{4\sin x\cos x} = \dfrac{2\sin^2 x}{4\sin x\cos x} = \dfrac{1}{2}\tan x$	M1 A1
		(3)
(b)	$\dfrac{9\left(1-\cos 2\theta\right)}{2\sin 2\theta} = 2\sec^2\theta \Rightarrow 9\times "\dfrac{1}{2}"\tan\theta = 2\sec^2\theta$	M1
	Attempts $1+\tan^2\theta = \sec^2\theta \Rightarrow 4\tan^2\theta - 9\tan\theta + 4 = 0$	**d**M1 A1
	$\Rightarrow \tan\theta = \dfrac{9\pm\sqrt{17}}{8}\left(1.64038...,0.60961...\right)$	M1
	$\Rightarrow \theta = \text{awrt } 31.4°, 58.6°$	A1 A1
		(6)
(b) Way 2	$\dfrac{9\left(1-\cos 2\theta\right)}{2\sin 2\theta} = 2\sec^2\theta \Rightarrow 9\times "\dfrac{1}{2}"\tan\theta = 2\sec^2\theta$	M1
	$\dfrac{9}{2}\tan\theta = 2\sec^2\theta \Rightarrow \dfrac{9\sin\theta}{\cos\theta} = \dfrac{4}{\cos^2\theta} \Rightarrow 9\sin\theta\cos\theta = 4 \Rightarrow \sin 2\theta = \dfrac{8}{9}$	**d**M1A1
	$\sin 2\theta = \dfrac{8}{9} \Rightarrow 2\theta = 62.7333...,117.266... \Rightarrow \theta = ...$	M1
	$\Rightarrow \theta = \text{awrt } 31.4°, 58.6°$	A1 A1
		(9 marks)

(a)

B1: States or uses $\sin 2x = 2\sin x\cos x$

M1: Attempts to use $1 - \cos 2x = \pm 2\sin^2 x$ and $2\sin 2x = A\sin x\cos x$ and to proceeds to $k\tan x$

A1: Fully correct solution with correct bracketing if seen

(b)

M1: Uses part (a) to form an equation of the form $A\tan\theta = B\sec^2\theta$

dM1: Replaces $\sec^2\theta$ by $\pm 1 \pm \tan^2\theta$ and proceeds to form a 3TQ equation in just $\tan\theta$

A1: Correct 3TQ. E.g. $9\tan\theta = 4 + 4\tan^2\theta$

M1: Correct attempt to find at least one value for $\tan\theta$ for their 3TQ in $\tan\theta$

Allow solutions from calculators and condone decimals as long as accuracy is to at least 2sf.

A1: One of either awrt 31.4° **or** awrt 58.6°

A1: Both awrt 31.4° **and** awrt 58.6°. Withhold this mark if there are extra solutions in range.

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(b) Way 2

M1: Uses part (a) to form an equation of the form equation $A\tan\theta = B\sec^2\theta$

dM1: Uses $\sec^2\theta = \dfrac{1}{\cos^2\theta}$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = ...\sin\theta\cos\theta$ to produce an equation of the form $\sin 2\theta = C$

A1: $\sin 2\theta = \dfrac{8}{9}$

M1: Full method to find at least one value of θ from $\sin 2\theta = C$ where $-1 < C < 1$.

E.g. $2\theta = \sin^{-1}C$, $\theta = \frac{1}{2}\sin^{-1}C$. (NB radians gives 0.547…, 1.02… and implies the M mark)

A1, A1: As above

Question Number	Scheme	Marks
3(i)	$\displaystyle\int \frac{12}{(2x-1)^2}\,\mathrm{d}x = \frac{-6}{(2x-1)}(+c)$	M1 A1
		(2)
(ii)(a)	(a) $\dfrac{4x+3}{x+2} = 4 - \dfrac{5}{x+2}$	M1, A1
		(2)
(ii)(b)	(b) $\displaystyle\int \frac{4x+3}{x+2}\,\mathrm{d}x = \int 4 - \frac{5}{x+2}\,\mathrm{d}x = 4x - 5\ln\lvert x+2\rvert\ (+c)$	M1 A1ft
	Hence $\displaystyle\int_{-8}^{-5} \frac{4x+3}{x+2}\,\mathrm{d}x = (-20-5\ln 3)-(-32-5\ln 6)$	dM1
	$=12+5\ln 2$	A1
		(4)
		(8 marks)

(i)

M1: Achieves $\dfrac{A}{(2x-1)}$ or equivalent e.g. $A(2x-1)^{-1}$

A1: Achieves $\dfrac{-6}{(2x-1)}$ or $-6(2x-1)^{-1}$. There is no requirement for the $+c$

(ii)(a)

M1: For using division or any other suitable method to find either A or B

Using division look for $x+2\overline{\smash{)}4x+3}$ with quotient 4

Using an identity look for $4x + 3 = A(x + 2) + B$ followed by a value for either A or B

It may be implied by either $A = 4$ or $B = -5$

A1: Correct value for A and B or a correct expression $4-\dfrac{5}{x+2}$ or $4+\dfrac{-5}{x+2}$

(ii)(b)

M1: Integrates $\dfrac{B}{x+2} \to \ldots\ln\lvert x+2\rvert$. Condone $\dfrac{B}{x+2} \to \ldots\ln(x+2)$ and condone $\dfrac{B}{x+2} \to \ldots\ln x+2$

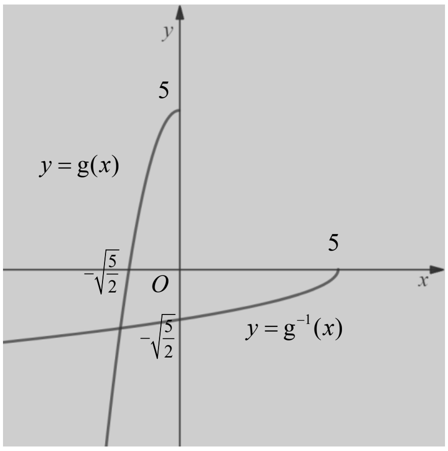
A1ft: Integrates $A+\dfrac{B}{x+2} \to Ax+B\ln\lvert x+2\rvert$. Condone $A+\dfrac{B}{x+2} \to Ax+B\ln(x+2)$

But do **not** condone missing brackets unless they are implied by subsequent work.

dM1: Substitutes -5 and -8 into their integrated function containing $\ln\lvert x+2\rvert$ or $\ln(x + 2)$

A1: $12+5\ln 2$ or equivalent exact answer such as $12+\ln 32$

SC: For candidates who proceed to $12+5\ln 2$ or equivalent exact answer via $(-20-5\ln(-3))-(-32-5\ln(-6))$ without sight of modulus signs withhold just the final A1

Question Number	Scheme		Marks
4(a)	$$\text{fg}(x)=\frac{4\left(5-2x^2\right)+6}{5-2x^2-5}$$		M1
	$$\text{fg}(x)=3\Rightarrow\frac{26-8x^2}{-2x^2}=3\Rightarrow x=-\sqrt{13}$$		**d**M1 A1 A1
			(4)
(b)	$$y=\frac{4x+6}{x-5}\Rightarrow yx-5y=4x+6\Rightarrow yx-4x=5y+6$$		M1
	$$\Rightarrow x=\frac{5y+6}{y-4}$$		A1
	$$\Rightarrow \text{f}^{-1}(x)=\frac{5x+6}{x-4}\qquad x\in\mathbb{R},\ x\neq 4$$		A1
			(3)
(c)		Shape and position for $y = \text{g}(x)$. A parabola starting on the y-axis that extends down in quadrant 2 to at least the x-axis. Condone slips of the pen e.g. if the curve heads back towards the y-axis in quadrant 3 slightly.	B1
		Shape and position for $y = \text{g}(x)$ as above that crosses the x-axis **and** $y = \text{g}^{-1}(x)$ a parabola starting on the x-axis with graphs crossing as shown in quadrant 3. Condone slips of the pen e.g. if $\text{g}^{-1}(x)$ heads back towards the x-axis in quadrant 3 slightly.	B1
		Correct intercepts. This mark is for the 4 correct intercepts as shown. Allow as values or coordinates and allow coordinates the wrong way round (e.g. (0, 5) for (5, 0)) as long as they are in the correct places. The negative intercepts must be exact as shown or equivalent. The graphs must cross the axes at these points and you can ignore any other intercepts.	B1
	Note that labels for g(x) and g^{-1}(x) are not required		
			(3)
			(10 marks)

(a)

M1: For attempting $\text{fg}(x)$ in the correct order.

dM1: For proceeding to $x^2 = \ldots$ **Depends on the first mark**.

A1: For $x^2 = 13$ which may be implied by $x=\sqrt{13}$

A1: For $x=-\sqrt{13}$ only

Note that in (a) some candidates may take an alternative approach for the first mark:

$$\mathrm{fg}(x)=3 \Rightarrow \mathrm{g}(x)=\mathrm{f}^{-1}(3)=-21 \Rightarrow 5-2x^2=-21$$

Score M1 for finding $\mathrm{f}^{-1}(3)$ and setting $\mathrm{g}(x)=\mathrm{f}^{-1}(3)$ then as main scheme

or

$$\mathrm{fg}(x)=3 \Rightarrow \mathrm{g}(x)=\mathrm{f}^{-1}(3)=-21 \Rightarrow x=\mathrm{g}^{-1}(-21)$$

Score M1 for finding $\mathrm{f}^{-1}(3)$ and setting $\mathrm{g}^{-1}\mathrm{f}^{-1}(3)=x$ then as main scheme

(b)

M1: For attempting to change the subject. In the attempt shown in the mark scheme score when both terms in x have been isolated and set on one side of the equation

A1: For $x=\dfrac{5y+6}{y-4}$ or equivalent such as $x=5+\dfrac{26}{y-4}$

If the x and y have been swapped award for $y=\dfrac{5x+6}{x-4}$ or equivalent.

A1: Requires both the expression in x **and** the domain.

$\mathrm{f}^{-1}(x)=\dfrac{5x+6}{x-4}$ $\quad x\neq 4$ or $\quad \mathrm{f}^{-1}(x)=\dfrac{5x+6}{x-4}$ $\quad x\in\mathbb{R}, x\neq 4$ $\quad$ (Condone $\mathrm{f}^{-1}=\ldots$)

Do **not** allow $y=\ldots$

(c) See scheme

Question Number	Scheme	Marks
5(a)	States or implies that $\log_{10} p = 0.32$ **or** $\log_{10} q = \frac{0.56-0.32}{8}$	M1
	$p =$ awrt 2.089 **or** $q =$ awrt 1.072	A1
	States or implies that $\log_{10} p = 0.32$ **and** $\log_{10} q = \frac{0.56-0.32}{8}$	M1
	$p =$ awrt 2.089 **and** $q =$ awrt 1.072	A1
		(4)
(b)	States or implies that $\frac{dA}{dt} = p\ln q \times q^t$ with their values for p and q	M1 A1
	Rate of increase in pond weed after 6 days is 0.22 (m^2/day)	A1
		(3)
		(7 marks)

(a)

M1: States or implies that $\log_{10} p = 0.32$ **or** $\log_{10} q = \frac{0.56-0.32}{8}$ or equivalent equations

A1: $p =$ awrt 2.089 **or** $q =$ awrt 1.072

M1: States or implies that $\log_{10} p = 0.32$ **and** $\log_{10} q = \frac{0.56-0.32}{8}$ or equivalent equations

A1: $p =$ awrt 2.089 **and** $q =$ awrt 1.072

(b)

M1: Uses $\frac{d}{dt}q^t \to kq^t \quad k \neq 1$

A1: States or implies that $\frac{dA}{dt} = p\ln q \times q^t$ with their values for p and q

A1: awrt 0.22 Units are not required

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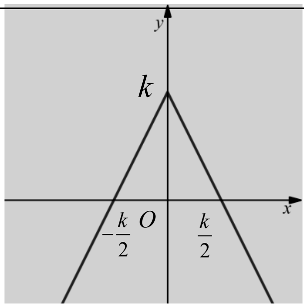
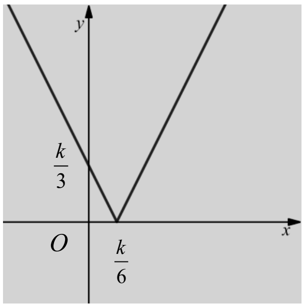
Alt (b) using $\log_{10} A = 0.03t + 0.32$ as a starting point

M1: Attempts to differentiate and reaches $\frac{1}{A}\frac{dA}{dt} = k$ or equivalent

A1: $\frac{1}{A\ln 10}\frac{dA}{dt} = 0.03$

A1: awrt 0.22 Units are not required

Question Number	Scheme		Marks
6(a)(i)		Shape	M1
		Correct intercepts.	A1
(ii)		Shape	M1
		Correct intercepts.	A1
			(4)
(b)	Attempts to solve either $k+2x=-2x+\frac{k}{3}$ **or** $k-2x=2x-\frac{k}{3}$		M1
	Achieves either $x=-\frac{1}{6}k$ **or** $x=\frac{1}{3}k$ oe		A1
	Attempts to solve both $k+2x=-2x+\frac{k}{3}$ **and** $k-2x=2x-\frac{k}{3}$		M1
	Achieves either $x=-\frac{1}{6}k$ **and** $x=\frac{1}{3}k$		A1
			(4)
			(8 marks)

(a)

M1: For the shape of $y=k-2|x|$. Score for an upside down V with maximum on the y-axis with the branches at least reaching the x-axis.

A1: Intercepts at k on the y-axis and $\pm\frac{k}{2}$ on the x-axis. Allow as values or coordinates and allow coordinates the wrong way round (e.g. $(k, 0)$ for $(0, k)$) as long as they are in the correct places. The graph must cross the axes at these points.

M1: For the shape of $y=\left|2x-\frac{k}{3}\right|$. Score for a V shape in quadrants 1 and 2 with a minimum on the x-axis and at least reaches the y-axis.

A1: Intercepts at $\frac{k}{3}$ on the y-axis and $\frac{k}{6}$ on the x-axis. Allow as values or coordinates and allow coordinates the wrong way round (e.g. $(k/3, 0)$ for $(0, k/3)$) as long as they are in the correct places. The graph must cross the y-axis and touch the x-axis.

(b)

M1: Attempts to solve one correct equation not involving moduli (or equivalent equations)

A1: One correct solution. Allow unsimplified forms such as $-\frac{2k}{12}$

M1: Attempts to solve two correct equations not involving moduli (or equivalent equations)
A1: Two correct solutions (simplified) and no other solutions given.

Some may square in (b) e.g.

$$\left(2x-\frac{k}{3}\right)^2=\left(k-2x\right)^2 \Rightarrow 4x^2-\frac{4kx}{3}+\frac{k^2}{9}=k^2-4kx+4k^2 \Rightarrow \frac{8kx}{3}=\frac{8k^2}{9} \Rightarrow x=\frac{k}{3}$$

Score M1 for squaring to obtain at least 3 terms on both sides and solving for x and A1 for $x=\frac{k}{3}$

Candidates are unlikely to find the other root using this approach.

Question Number	Scheme	Marks
7	$x=6\sin^2 2y \Rightarrow \dfrac{dx}{dy}=24\sin 2y\cos 2y$	M1 A1
	Attempts to use $\dfrac{dy}{dx}=1\div\dfrac{dx}{dy}$	M1
	Attempts to use both $\sin 2y=\sqrt{\dfrac{x}{6}}$ and $\cos 2y=\sqrt{1-\sin^2 2y}=\sqrt{1-\dfrac{x}{6}}$	M1
	$\dfrac{dy}{dx}=\dfrac{1}{24\sin 2y\cos 2y}=\dfrac{1}{24\times\sqrt{\dfrac{x}{6}}\times\sqrt{1-\dfrac{x}{6}}}=\dfrac{1}{4\sqrt{6x-x^2}}$	A1
		(5)
Way 2	$x=6\sin^2 2y=3-3\cos 4y \Rightarrow \dfrac{dx}{dy}=12\sin 4y$	M1A1
	Attempts to use $\dfrac{dy}{dx}=1\div\dfrac{dx}{dy}$	M1
	Attempts to use $\sin 4y=\dfrac{\sqrt{6x-x^2}}{3}$	M1
	$\dfrac{dy}{dx}=\dfrac{1}{12\sin 4y}=\dfrac{1}{12\times\dfrac{\sqrt{6x-x^2}}{3}}=\dfrac{1}{4\sqrt{6x-x^2}}$	A1
Way 3	$x=6\sin^2 2y=24\sin^2 y\cos^2 y \Rightarrow \dfrac{dx}{dy}=48\sin y\cos^3 y-48\sin^3 y\cos y$	M1A1
	Attempts to use $\dfrac{dy}{dx}=1\div\dfrac{dx}{dy}$	M1
	$\dfrac{dy}{dx}=\dfrac{1}{48\sin y\cos^3 y-48\sin^3 y\cos y}=\dfrac{1}{48\sin y\cos y\left(\cos^2 y-\sin^2 y\right)}$ $\dfrac{dy}{dx}=\dfrac{1}{24\sin 2y\cos 2y}=\dfrac{1}{24\times\sqrt{\dfrac{x}{6}}\times\sqrt{1-\dfrac{x}{6}}}$	M1
	$=\dfrac{1}{4\sqrt{6x-x^2}}$	A1
		(5 marks)

In general, apply the following marking guidance for this question:

M1: Attempts to differentiate to obtain $\frac{dx}{dy}$ in a correct form

A1: Correct derivative

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

M1: Change fully to a function of x.
A1: All correct

E.g.

M1: Attempts to use the chain rule on the rhs to achieve $k\sin 2y \cos 2y$

A1: Fully correct derivative $\frac{dx}{dy} = 24\sin 2y \cos 2y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 24\sin 2y \cos 2y \Rightarrow \frac{dy}{dx} = \frac{24}{\sin 2y \cos 2y}$

Do not condone slips/errors on the variable. E.g. $\frac{dx}{dy} = 24\sin 2y \cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{24\sin 2x \cos 2x}$

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x.

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

Way 2
M1: Differentiates to obtain $k\sin 4y$

A1: Fully correct derivative $\frac{dx}{dy} = 12\sin 4y$

M1: Attempts to use $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$.

Condone a slip on the coefficient. E.g. $\frac{dx}{dy} = 12\sin 4y \Rightarrow \frac{dy}{dx} = \frac{12}{\sin 4y}$

M1: Attempts to change both $\sin 4y$ to a function in x.

Expect to see $\sin 4y = p\sqrt{6x-x^2}$ or equivalent e.g. $\sin 4y = p\sqrt{9-\left(9-6x+x^2\right)}$

A1: CSO $\frac{dy}{dx} = \frac{1}{4\sqrt{6x-x^2}}$

Way 3

M1: Differentiates to obtain $A\sin y\cos^3 y + B\sin^3 y\cos y$

A1: Fully correct derivative $\frac{\mathrm{d}x}{\mathrm{d}y} = 48\sin y\cos^3 y - 48\sin^3 y\cos y$

M1: Attempts to use $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \frac{\mathrm{d}x}{\mathrm{d}y}$.

M1: Attempts to change both $\sin 2y$ and $\cos 2y$ to functions in x.

Expect to see $\sin 2y = p\sqrt{x}$ and $\cos 2y = \sqrt{1-qx}$

A1: CSO $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sqrt{6x-x^2}}$

A possible alternative:

$$x = 6\sin^2 2y \Rightarrow \sin 2y = \sqrt{\frac{x}{6}} \Rightarrow 2y = \sin^{-1}\sqrt{\frac{x}{6}} \Rightarrow y = \frac{1}{2}\sin^{-1}\sqrt{\frac{x}{6}}$$

M1: For a correct attempt to make y or $2y$ the subject

A1: Correct rearrangement

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times \frac{1}{\sqrt{1-\frac{x}{6}}} \times \frac{1}{12}\left(\frac{x}{6}\right)^{-\frac{1}{2}}$$

M1: For $\frac{A}{\sqrt{1-\frac{x}{6}}} \times \ldots$ M1: For $\ldots \times B\left(\frac{x}{6}\right)^{-\frac{1}{2}}$

A1: CSO $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sqrt{6x-x^2}}$

Question Number	Scheme	Marks
8(a)	200	B1
		(1)
(b)	600	B1
		(1)
(c)	$500=\dfrac{600e^{0.3t}}{2+e^{0.3t}}\Rightarrow 100e^{0.3t}=1000,\Rightarrow e^{0.3t}=10$	M1, A1
	$\Rightarrow t=\dfrac{\ln 10}{0.3}=7$ years 8 months	dM1 A1
		(4)
(d)	$N=\dfrac{600e^{0.3t}}{2+e^{0.3t}}\Rightarrow\dfrac{dN}{dt}=\dfrac{\left(2+e^{0.3t}\right)\times 180e^{0.3t}-180e^{0.3t}\times e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$	M1 A1
	$\Rightarrow\dfrac{dN}{dt}=\dfrac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$	A1 **(3)**
(e)	$8=\dfrac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}\Rightarrow 8\left(e^{0.3t}\right)^2-328\left(e^{0.3t}\right)+32=0$	M1
	$e^{0.3t}=\dfrac{41+\sqrt{1665}}{2}$	dM1
	$(t=)\dfrac{\ln\left(\dfrac{41+\sqrt{1665}}{2}\right)}{0.3}$	ddM1
	$(T=)$ awrt 12.4	A1
		(4)
		(13 marks)

(a)

B1: 200

(b)

B1: 600

(c)

M1: Sets $N=500$ and proceeds to $Ce^{0.3t}=D$ or equivalent

A1: $e^{0.3t}=10$. If ln's are taken earlier it would be for $\ln 100+0.3t=\ln 1000$

dM1: Full method to find a value for t using correct log work.

A1: 7 years 8 months. Accept 7 years 9 months following a correct value of t.

(d)

M1: Uses the quotient rule to obtain an expression of the form $\frac{\left(2+e^{0.3t}\right)\times ae^{0.3t}-be^{0.3t}\times e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

In general condone missing brackets for the M mark.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\frac{vu'-uv'}{v^2}$ **etc.**

E.g. if they quote $u = 600e^{0.3t}$ and $v = 2 + e^{0.3t}$ and do not make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct derivative in any form e.g. $\frac{dN}{dt}=\frac{\left(2+e^{0.3t}\right)\times 180e^{0.3t}-180e^{0.3t}\times e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

A1: Correctly obtains $\frac{dN}{dt}=\frac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

Withhold this mark if you see $e^{0.3t}\times e^{0.3t}$ written as $e^{0.3t^2}$ or $e^{0.09t}$

....

Alt (d)

M1 A1: $N=\frac{600e^{0.3t}}{2+e^{0.3t}}\Rightarrow N=600-\frac{1200}{2+e^{0.3t}}\Rightarrow\frac{dN}{dt}=\frac{1200\times 0.3e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

Score M1 for splitting $\frac{600e^{0.3t}}{2+e^{0.3t}}$ into $A\pm\frac{B}{2+e^{0.3t}}$ leading to $\frac{dN}{dt}=\frac{ke^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

and A1 for correct derivative in any form.

A1: $\frac{dN}{dt}=\frac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$ (following fully correct work)

....

May also see product rule in (d):

$$N=\frac{600e^{0.3t}}{2+e^{0.3t}}=600e^{0.3t}\left(2+e^{0.3t}\right)^{-1}\Rightarrow\frac{dN}{dt}=180e^{0.3t}\left(2+e^{0.3t}\right)^{-1}-180e^{0.3t}\times e^{0.3t}\left(2+e^{0.3t}\right)^{-2}$$

$$=\frac{180e^{0.3t}}{2+e^{0.3t}}-\frac{180e^{0.6t}}{\left(2+e^{0.3t}\right)^2}=\frac{360e^{0.3t}+180e^{0.6t}-180e^{0.6t}}{\left(2+e^{0.3t}\right)^2}=\frac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$$

Score M1 for an expression of the form $ae^{0.3t}\left(2+e^{0.3t}\right)^{-1}-be^{0.3t}\times e^{0.3t}\left(2+e^{0.3t}\right)^{-2}$ and A1 A1 as above.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $vu'+uv'$ **etc.**

(e)

M1: Sets $\frac{dN}{dt}=8$ and proceeds to a quadratic in $e^{0.3t}$

dM1: Correct attempt to solve the quadratic in $e^{0.3t}$. Condone both roots to be found

ddM1: Correct attempt to find the value of *t* for $e^{0.3t}=k$ where $k>0$ using correct log work

A1: Achieves $(T=)$ awrt 12.4

Question Number	Scheme	Marks
9(a)	$R=13$	B1
	$\tan\alpha=\frac{5}{12}\Rightarrow\alpha=\text{awrt }0.395$	M1A1
		(3)
(b)	$g(\theta)=10+13\sin\left(2\theta-\frac{\pi}{6}-0.395\right)$	
(i)	(i) Minimum value is -3	B1 ft
(ii)	$2\theta-\frac{\pi}{6}-0.395=\frac{3\pi}{2}\Rightarrow\theta=\text{awrt }2.82$	M1 A1
		(3)
(c)	$h(\beta)=10-169\sin^2(\beta-0.395)$	
	$-159\leqslant h\leqslant 10$	M1 A1
		(2)
		(8 marks)

(a)

B1: $R=13$ ($R=\pm13$ is B0)

M1: $\tan\alpha=\pm\frac{5}{12}$, $\tan\alpha=\pm\frac{12}{5}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{5}{R}$ or $\cos\alpha=\pm\frac{12}{R}\Rightarrow\alpha=...$

A1: $\alpha=\text{awrt }0.395$ Note that the degree equivalent $\alpha=\text{awrt }22.6°$ is A0

(b)(i)

B1ft: States the value of $10-R$ following through their R.

(b)(ii)

M1: Attempts to solve $2\theta-\frac{\pi}{6}\pm\text{"}0.395\text{"}=\frac{3\pi}{2}\Rightarrow\theta=...$

A1: $\theta=\text{awrt }2.82$. No other values should be given

(c)

M1: Achieves one of the end values, either -159 (or 10 – (their R)^2 evaluated) or 10

A1: Fully correct range $-159\leqslant h\leqslant 10$, $-159\leqslant h(\beta)\leqslant 10$, $-159\leqslant \text{range}\leqslant 10$, $-159\leqslant h(x)\leqslant 10$, $[-159,10]$ or equivalent correct ranges.

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