Examiners' Report
Principal Examiner Feedback

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General

This was an accessible paper which clearly allowed candidates to show their abilities. Questions 2, 3, 5 and 7 were on familiar themes whereas stretch and challenge was provided by questions such as 9 and 10. Centres seem more confident as to how the content of these units is being assessed and there was a marked improvement in performance on question 8, on the topic of logarithmic graphs.

Reports to individual Questions

Question 1

Most candidates were able to make progress with this question achieving the first two marks via an answer of the form $a\ln x + bx^{-2}$. Only a minority however went on to score all three marks with most unable to find the simplest form as required by the question. Popular incorrect answers scoring two out of three were

- $\frac{1}{2}\ln 2x + \frac{5}{4}x^{-2} + c$ (Not the simplest form)
- $\frac{1}{2}\ln x + \frac{5}{4}x^{-2}$ (Omission of $+\,c$)
-
- $2\ln x + 5x^{-2} + c$ (Error introduced when multiplying, rather than dividing, each term by 2)

More serious errors were seen when candidates integrated x^{-1} to x^{0} or else used the quotient rule of differentiation in an attempt to integrate the given expression.

Question 2

This question involved the sketching of graphs and was correctly attempted by the majority of candidates.

In part (i), many candidates achieved full marks, correctly identifying the two way stretch, leaving the curve passing through the origin and adjusting both coordinates accordingly. Occasionally the horizontal stretch used an incorrect scale factor – most commonly multiplying the x-coordinates by 2, rather than dividing by 2.

Part (ii) proved to be more demanding with one or two marks more common than all three. Many candidates have clearly been trained to transform each point before sketching the graph, which is quite sensible, but in this case, many did not consider the origin as a relevant point. As a result, many graphs still seemed to pass through the origin and not $(0,-1)$ due to the vertical translation. Another issue was that in some diagrams some coordinates were clearly in the wrong place with $(-3,\,-1)$ appearing on the x-axis or in quadrant 2. A more sensible strategy would have been to reflect the graph in the y-axis, and then translate vertically. Only after having the correct shape and position consider the coordinates of the turning points.

Question 3

This question involved the manipulation of algebraic fractions followed by the ability to find an inverse function and its domain.

In part (a) the first requirement was to factorise the denominator $2x^2 - 3x - 5$ which nearly all were able to do. When a common denominator of just $(x+1)(2x-5)$ was used the majority were able to go on, using well-honed algebraic skills, to find the correct answer. Unfortunately, many poor (and incomplete) solutions resulted from an attempt using the common denominator of $(x+1)^2(2x-5)$.

Other common errors were introduced as a result of

- incorrectly dealing with the $-$ sign in front of $\frac{x-2}{x+1}$, especially common when terms two and three were combined before dealing with the "3"
- bracketing errors, again mainly due to the $-$ sign in term 2

In part (b) most candidates knew how to find an inverse function, usually by swapping x and y followed by a change of subject. Occasional sign errors were seen, but the majority who could do part (a) scored both marks in part (b). Worryingly a small number of candidates thought that $\mathrm{f}^{-1}(x)$ was the reciprocal of $\mathrm{f}(x)$ and some even differentiated finding $\mathrm{f}'(x)$.

Part (c) was much more demanding and very few fully correct responses were seen. The most straightforward approach was to use the fact that the domain of the inverse function is the range of the original function. Many candidates were able correctly identify f (4) = 17/ 3 as one end of the, domain but few were able to spot 2 as the lower boundary. A quick sketch of the graph of the function would probably have been beneficial in this part, and, indeed was seen, but only by a mere handful of candidates.

Question 4

Question 4 involved a pair of modulus graphs followed by the solution of a modulus equation. Two marks were easily achieved in part (a) by the prepared candidate. Knowing that the minimum value of $|3x+a|$ is zero was key to answering this part correctly. This enabled candidates to know that this occurs when $x = -\frac{a}{3}$ giving a minimum y value of a.

Part (b) was standard bookwork and answered well by the majority of candidates. Errors seen included

- sketching a V shaped graph with the vertex on the y-axis or the positive x-axis
- giving the coordinate of the intersection on the negative x-axis as $5a$

Solutions of the modulus equation in part (c) were mixed. Many candidates did not have a clear strategy and resorted to replacing $|x+5a|$ with $\pm(x+5a)$ and $|3x+a|+a$ with $\pm(3x+a)+a$ or sometimes $\pm(3x+a+a)$. This sometimes produced 4 different answers. Few candidates used the given graphs and solved **only** the two appropriate equations. A common mark trait in this part was M1 A1 A0 M1 A0 scored for finding the solution $\left(\frac{3}{2}a, \frac{13}{2}a\right)$

Question 5

This question, dealing with the temperature in a warming oven, tested a candidate's ability to handle exponential and logarithmic functions. Whilst parts (a), (b) and (c) were often answered well, for part (d), many candidates did not realise they needed to use calculus, or, realising the derivative was required, did not appear to know how to differentiate e^{-kt} with respect to *t*. Errors in parts (a), (b) and (c) were rare. Amongst the most common seen were

- in part (a) incorrect processing leading to wrong values for *A* including 162 and 196
- numerical slips in part (b) or else poor attempts to solve $180e^{-5k} = 108$ where lns were taken at incorrect/inappropriate times
- sign errors on the value of *k* leading to a use of $198 - 180e^{9\times k}$ as opposed to $198 - 180e^{-9\times k}$

As mentioned, it was part (d) that candidates found challenging, with many not even attempting a solution. It is important to note that the question asked for the *rate* at which the oven was warming **at the instant** 9 minutes after switching on. More importantly this is not the same as the average rate of warming over the 9 minutes found from the calculation (126 - 18)/9 = 12deg/min. The rate at an instant is calculated by finding the value of dθ/dt at the value of *t*. In this case the easiest way was to differentiate $\theta = 198 - 180e^{-kt}$ to $\frac{d\theta}{dt} = 180ke^{-kt}$ and substitute in the values of *k* and *t*. The differentiation function on calculators could not be used to find the answer without showing the appropriate method as referenced in the question.

Question 6

This question was based around finding the turning point of a trigonometric function.

Part (a) was bookwork and generally very well answered. Most of the candidates arrived at the correct expression for $f'(x)$.

Part (b) was a very effective discriminator. Whilst most candidates recognised the need to divide by cos(*x*/3) to arrive at an equation involving tan(*x*/3) there was much confusion between tan(*x*/3, cot(*x*/3) and arctan(*x*/3). Quite often we witnessed tan(*x*/3) being replaced by arctan(3/*x*). As with all show that questions all steps are required to be shown and, in this case, it was vitally important to see a penultimate line of tan(*x*/3) = 3/*x*

If part (b) was correct, then, often, so was part (c). Reasons for errors were either having an incorrect equation or using a calculator in degree mode.

Part (d) was another effective discriminator. Most candidates knew to use the upper bound and lower bound within a function, but in many they were unsure which function to use. Many tried to use their answer to part (b) and failed to make any further progress. For those that did choose a suitable function, which was typically their answer to (a), almost all knew to make a comment about a change in sign but it was very common for the accuracy mark to be lost due to a lack of reference to the function being continuous.

Question 7

This trigonometry question was tackled with confidence by most candidates, with many excellent responses seen.

Part (a) was a typical proof of a trigonometric identity: most knew the single angle formulae for $\sin 2x$ and $\cos 2x$, although sign errors for the $\cos 2x$ formulae or incorrectly expressing $\cos 2x$ as $1 - \sin^2 x$ were sometimes encountered. A full proof, with no errors, was required, and some candidates lost the final "A" mark because of omitting necessary steps or writing sin for $\sin x$ for example. Candidates who created a common denominator first and then observed that the numerator was $\cos(2x-x)$ almost always completed the proof successfully.

Part (b) involved using the identity from (a) and realising that the expression on the LHS reduced to $7 + \text{cosec } 2\theta$. A common error here was to see the LHS written as $7 + 2\text{cosec } \theta$. Use of the identity $\text{cosec}^2 \theta = 1 + \cot^2 \theta$ was also required. The mark scheme was drawn up so that, if the candidate could not re-write the original equation as a three-term quadratic equation in $\text{cosec } 2\theta$ [or its reciprocal, $\sin 2\theta$], no further progress was possible. Provided the quadratic equation was correct, candidates generally were able to solve it, almost invariably by factorisation, and thus obtain values for 2θ and hence θ itself. The main errors at this stage lay in omission of values for θ within the stated range with -1.25 very frequently missed.

Question 8

Candidates ability to solve questions on logarithmic graphs seems to be improving. The best way to start this question was by taking logs of both sides of $P = ab^t$ and proceeding to the equation $\log_{10} P = \log_{10} a + t \log_{10} b$ Then, using the data supplied for the gradient and intercept, form and solve equations in a and b. For those attempting the question, success at finding a was much more common than that of b, yet exactly the same technique was required.
Part (b) was not answered well, even for those who got parts (a) and (c) completely correct. A full interpretation of the constant 'a' was required. Students needed to phrase their answer using **all three** key components, that is "the percentage population", "with access to the internet" and "at the start of 2005". Common wrong answers included "a is the initial population at $t = 0$".

In part (c) almost all candidates attempted their $a \times b^{10}$ and most obtained answers which rounded to 38.

Question 9

This question required candidates to spot the result of chain rule differentiation and use the fact that integration is the opposite of differentiation.
This type of integration is sometime termed "function and derivative" type integration.
There were some excellent solutions but also very many candidates who failed to score any marks at all.

In part (i) candidates needed to realise that $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4x + 5\right) = 6x - 4 = 2(3x - 2)$. As a result

$$\int \frac{3x-2}{3x^2 - 4x + 5}\,\mathrm{d}x = \frac{1}{2}\int \frac{1}{3x^2 - 4x + 5} \times \left(6x - 4\right)\mathrm{d}x = \frac{1}{2}\ln\left(3x^2 - 4x + 5\right) + c$$

Common answers seen were

- $\frac{1}{6x-4}\ln\left(3x^2 - 4x + 5\right)$ scoring 0 marks
- $2\ln\left(3x^2 - 4x + 5\right) + c$ scoring one mark

Part (ii) was similar with candidates needed to realise this time that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{2x} - 1\right)^{-2} = -2\left(\mathrm{e}^{2x} - 1\right)^{-3} \times 2\mathrm{e}^{2}$

Hence $\int \frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x} - 1\right)^3}\,\mathrm{d}x = -\frac{1}{4}\left(\mathrm{e}^{2x} - 1\right)^{-2} + c$.

Common answers seen here were

- $-\frac{1}{4\mathrm{e}^{2x}}\left(\mathrm{e}^{2x} - 1\right)^{-2} + c$ scoring 0 marks
- $\ln\left(\mathrm{e}^{2x} - 1\right)^{2} + c$ scoring 0 marks
- $\frac{1}{4\mathrm{e}^{2x}}\left(\mathrm{e}^{2x} - 1\right)^{-2} + c$ scoring 1 mark

It was possible to do both parts via substitutions, in (i) via $u = 3x^2 - 4x + 5$ and (ii) via $u = \mathrm{e}^{2x} - 1$ and this was rewarded as can be seen in the mark scheme.

Question 10

As a whole this question proved to discriminate between the most capable candidates and those with less understanding.

Part (a) was generally well attempted. Errors in using the chain rule were still quite common with $6\sec^2 2y \tan 2y$ being a popular answer for one out of two marks. Another common answer omitted the squaring of sec(2*y*).

Part (b) proved more challenging. There were plenty of clear, concise solutions gaining full marks, but these were outnumbered by the more common approaches that struggled to deal with the tan(2*y*). In some cases, little to no working was shown, ignoring the requirement of the question to "show that", and instead relied heavily on the given form of the answer. Those without any justification for replacing the tan(2y) were unable to score any marks as a result. Many good candidates did however deal with the tan(2y) successfully, gaining the first two marks in many cases for a correct, acceptable form, following on from their answer to part (a). Many of these candidates then failed to deal with the $\sqrt{3}$ successfully, and as a result did not score the final mark.

Part (c) proved more accessible, with many who had struggled in part (b), going on to score three marks out of five and in some cases full marks. Nearly everyone gained the B1 for $x = 4$, and the majority who attempted the question scored the first M1 for their attempt at substituting $y = \pi/12$ or $x = 4$ into dy/dx or dx/dy to get a value of the gradient in some form.
It was from this point onwards that we saw much confusion regarding the gradient of the normal. The easiest and most direct way was to use $-$ dx/dy using their part (a) and $y = \pi/12$. Another option was to use the negative reciprocal of dy/dx using their (b) at $x = 4$. Unfortunately, many students struggled at this point and lacked the clarity of thought required to complete the question. Those that did showed excellent understanding and first-rate algebraic skills gaining all 5 marks.