Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level
In Mathematics Mechanics 3 (WME03) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1.	$\omega = \dfrac{10\pi}{60}$ (rad s^{-1})	B1
	$F = mg\mu$ (N)	B1
	$F = m \times 0.2\left(\dfrac{\pi}{6}\right)^2 = \dfrac{m\pi^2}{180}$	M1A1ft
	$mg\mu \geq \dfrac{m\pi^2}{180}$	dM1
	$\mu_{\min} = \dfrac{\pi^2}{180g}, \quad (0.0056, \quad 0.00560)$	A1
		[6]
B1	Correct angular speed in radians per second, seen anywhere	
B1	Correct inequality or equation for Friction, seen or used anywhere	
M1	Attempt the equation of motion along the radius. Must only contain friction and resultant force (give BOD unless clearly not friction). Allow with their ω or just ω.	
A1ft	Correct equation. Follow through their ω	
dM1	Eliminate *F* and solve to find μ. Allow with an inequality or equation. Dependent on previous M1.	
A1	Correct answer, as shown or 2/3 sf decimal (0.00560). Must not be an inequality now.	

Special Case: If $F \geq mg\mu$ or $F < mg\mu$ used, leading to $\mu = \dfrac{\pi^2}{180g}$ award max B1B0 M1A1 M1A0

Question Number	Scheme	Marks
2(a)	$v=\dfrac{dx}{dt}=\dfrac{1}{(4x+3)}$	
	$\dfrac{dt}{dx}=4x+3$	
	$t=\int (4x+3)dx, =\dfrac{1}{2}\times 4x^2+3x+c$	
	OR $\int_0^2 dt=\int_0^X (4x+3)dx, =\left[\dfrac{1}{2}\times 4x^2+3x\right]_0^X$	M1,dM1A1
	$c=0$	
	$t=2=\dfrac{1}{2}\times 4x^2+3x \quad 2x^2+3x-2=0$	dM1
	$x=\dfrac{1}{2}\ (x=-2)$	A1cso (5)
(b)	$a=v\dfrac{dv}{dx} \qquad alt:\ a=\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	M1
	$=\dfrac{1}{4x+3}\times\dfrac{-4}{(4x+3)^2}$	dM1A1
	$\lvert F\rvert=\dfrac{1}{2}\times\dfrac{4}{(2+3)^3}=\dfrac{2}{125}=0.016\text{ N}$	M1 A1cso (5) **[10]**
(a)		
M1	Rewrite as $\dfrac{dx}{dt}$ **and** separate variables to reach a form ready for integration	
dM1	Attempt the integration (at least one power going up).	
A1	Correct integration. Constant/limits not needed.	
dM1	Use $t=2$ in their expression or substitute correct limits, and solve their 3 term quadratic to find x. If solving an incorrect quadratic, evidence of a correct method must be seen. Depends on the previous M mark.	
A1cso	Obtain $x=\dfrac{1}{2}$ (and reject -2 if seen) from completely correct work. Constant of integration must have been seen, although we do not need to see evidence of evaluation.	
(b)		
M1	Use $a=v\dfrac{dv}{dx}$	
dM1	Differentiate the given expression for v and obtain an expression for a. We need to see a power of 2 (or a power of 3 if using $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$). Depends on the previous M mark.	
A1	Correct expression, any form.	
M1	Use their acceleration in an equation of motion to obtain a value for F. Mass must be included and they must use their value of x. Independent, but must have found an expression for acceleration.	
A1cso	Correct magnitude of F. Correct solution only. Can be fraction or decimal. Must be positive.	
3 (a)	$\angle PBA=30°$	B1

Question Number	Scheme	Marks
	$R(\uparrow)\quad T\cos 30° + R\cos 60° = mg$	M1
	NL2 horizontally: $T\cos 60° + R\cos 30° = mr\omega^2,\ = ma\omega^2 \cos 30°$	M1A1,A1
	$T = \dfrac{m\sqrt{3}}{2}\left(2g - a\omega^2\right)$ o.e.	dM1A1 (7)
(b)	$R = 2mg - \dfrac{3m}{2}\left(2g - a\omega^2\right) = \dfrac{3ma\omega^2}{2} - mg$	M1A1
	Use $R \geq 0$	M1
	$\omega \geq \sqrt{\dfrac{2g}{3a}}$ *	A1cso (4)
		[11]

(a)	
B1	Correct angle, seen explicitly, implied by a correct trig ratio, or used.
M1	Attempt a vertical equation with 3 forces, *T* and *R* resolved. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces.
M1	Equation of motion horizontally, two forces resolved and acceleration in either form. Attempt at radius not needed. Angles can be algebraic. Condone sin/cos confusion and use of the same angle for both forces.
A1	Correct LHS
A1	Correct acceleration with correct radius (which might be seen later in part (a)).
dM1	Eliminate *R* and solve to find expression for *T.* Depends on both previous M marks. Allow this mark even if they have not found an angle.
A1	Correct expression for *T* (any correct equivalent).
(b)	
M1	Attempt to obtain an expression in *R*. Independent of the M marks in (a), but must have come from 2 equations in *T* and *R*.
A1	Correct unsimplified expression in R
M1	Use of the correct inequality for *R*
A1cso*	Obtain given result from fully correct working.

Question Number	Scheme	Marks
4(a)	$mg\sin\alpha\times\left(\frac{3l}{2}+e\right)=\mu mg\cos\alpha\times\left(\frac{3l}{2}+e\right)+\frac{1}{2}\times\frac{2mg}{l}e^2$	M1B1B1A1
	$\frac{3}{5}\left(\frac{3l}{2}+e\right)=\frac{4\mu}{5}\left(\frac{3l}{2}+e\right)+\frac{e^2}{l}$	
	$\mu=\frac{9l^2+6le-10e^2}{4l(3l+2e)}$ *	dM1A1cso (6)
(b)	$e=l\ \Rightarrow\ \mu=\frac{1}{4}$ or 0.25	B1
	$F=\frac{1}{5}mg$	B1ft
	Change in acceleration is due to change of direction of *F*	
	$F_1=2mg-mg\sin\alpha+F_r\left(=\frac{8}{5}mg\right)$ and $F_2=2mg-mg\sin\alpha-F_r\left(=\frac{6}{5}mg\right)$	M1
	Mag of change in accel $=\frac{F_1-F_2}{m}=\frac{2g}{5}=3.92$ or 3.9 (m s^{-2})	M1A1 (5)
		[11]

(a) M1	Attempt a work-energy equation with a GPE term, a single EPE term and the work done against friction. . (Allow $EPE=k\frac{\lambda x^2}{l}$)
B1	Correct EPE at C. (Ignore any extra EPE terms for this mark)
B1	Correct GPE
A1ft	Correct equation. Follow through their EPE and GPE terms providing they are of the correct form
dM1	At least one line of **correct** working to rearrange towards $\mu=$. They do not need to reach $\mu=$ for this mark.
A1cso*	Given result obtained with no errors seen and at least one line of correct rearrangement. Must be exactly as printed on paper.
(b)	
B1	Correct numerical value for μ seen anywhere in (b). This might be implied by later working.
B1ft	Correct value for *F*, seen anywhere in (b). Follow through their μ but must be dimensionally correct. μ
M1	Attempt 2 equations of motion to find resultant force. (Use of $Change=2F$) would imply this mark.
M1	Subtract and divide by *m* to obtain the mag of the change in the acceleration.
A1	Must be $\frac{2g}{5}$, or 3.9 or 3.92 (m s^{-2})

Question Number	Scheme	Marks
5(a)	$3amg = \frac{1}{2}m \times 7ag - \frac{1}{2}mv^2$	M1A2
	$v^2 = ag \quad v = \sqrt{ag}$	A1 (4)
(b)	$amg = \frac{1}{2}mw^2 - \frac{1}{2}m \times 7ag$	M1
	$w^2 = 9ag$	
	$T_1 - mg = \frac{mw^2}{4a}$	M1
	$T_1 = \frac{13mg}{4}$	A1
	Speed immediately after impact $= \frac{1}{2}\sqrt{ag}$	
	$4amg = \frac{1}{2}mV^2 - \frac{1}{2}m \times \frac{1}{4}ag$	M1
	$V^2 = \frac{33}{4}ag$	
	$T_2 - mg = \frac{mV^2}{4a}$	M1
	$T_2 = \frac{49}{16}mg$	A1
	$T_1 : T_2 = \frac{13}{4} : \frac{49}{16} = 52 : 49$	A1 (7)
		[11]
(a)		
M1	Energy equation from projection to reaching the ceiling. Must have at least one GPE term and 2 KE terms	
A2	Correct equation. -1 for each error.	
A1cso	Correct expression for v from fully correct work	
(b)		
M1	Energy equation from the point of projection to *B*. Must have all required terms	
M1	Form equation of motion at *B* and eliminate w^2 to obtain an expression for T_1 Must have attempted a velocity at *B*. Condone $r = a$.	
A1	Correct expression for T_1	
M1	Form energy equation from leaving the ceiling to reaching *B*. Must have attempted to use the coeff of restitution to find the initial speed for this equation. Condone $r = a$.	
M1	Attempt an equation of motion at *B* and eliminate V^2 to obtain an expression for T_2. Must have attempted a velocity at *B*.	
A1	Correct expression for T_2	
A1cao	Correct ratio. Question asks for simplest form, so must be 52:49 (Condone $\frac{52}{49}$)	

Question Number	Scheme	Marks
6(a)	$\frac{20(0.2-x)}{0.4}-\frac{20(0.2+x)}{0.4}=0.4\ddot{x}$	M1A1
	$-100x=0.4\ddot{x}$	
	$\ddot{x}=-250x \quad \therefore \text{SHM}$	dM1A1cso (4)
(b)	$\text{Period}=\frac{2\pi}{\sqrt{250}}$ oe	B1ft (1)
(c)	$v_{max}=\frac{2}{0.4}=5\text{ m s}^{-1}$	B1
	$a\omega=5 \quad a=\frac{5}{\sqrt{250}}=\frac{1}{\sqrt{10}}\ (=0.3162...)\text{m}$	M1A1ft (3)
(d)	$x=a\cos\omega t \quad 0.1=\frac{1}{\sqrt{10}}\cos\sqrt{250}t$ or $x=a\sin\omega t \quad 0.1=\frac{1}{\sqrt{10}}\sin\sqrt{250}t$	M1A1ft
	$t=\frac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $t=\frac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$	A1
	Time for which $AP>0.5$	
	$=\frac{2\pi}{\sqrt{250}}-2\frac{1}{\sqrt{250}}\cos^{-1}\left(0.1\times\sqrt{10}\right)$ or $=\frac{\pi}{\sqrt{250}}+2\frac{1}{\sqrt{250}}\sin^{-1}\left(0.1\times\sqrt{10}\right)$	dM1
	$=0.2393...\text{s}$	A1cso (5) [13]
(a)		
M1	Attempt an equation of motion using a difference of 2 tensions obtained from Hooke's law and having different variable extensions. $\ddot{x}$ or a allowed. Can be in algebraic form.	
A1	Correct equation. $\ddot{x}$ or a allowed but if a used the signs must indicate it is in the same direction as $\ddot{x}$ Can be in algebraic form.	
dM1	Rearrange their equation to the required form $\ddot{x}=-\omega^2x$. Must be $\ddot{x}$. They cannot just lose terms to get to the required form.	
A1ft	Correct equation, can be numerical as shown or algebraic $\left(e.g.\quad \ddot{x}=-\frac{4\lambda}{ml}x\right)$, **and** state conclusion. If algebraic this must include stating that their "ω^2" is positive.	
(b)		
B1ft	Correct period (numerical) as shown or equivalent. Follow through their ω from $\ddot{x}$ or $a=\pm\omega^2x$ (0.40 or better)	
(c)B1	Correct max speed, seen explicitly or used	
M1	Using $v_{max}=a\omega$ to obtain a value for a	
A1ft	Correct value, exact or decimal (0.32 or better)	
(d)		
M1	Use $x=a\cos\omega t$ or $x=a\sin\omega t$ with $x=\pm0.1$, their ω, a.	
A1ft	Correct equation, follow through their ω, a	
A1	Correct expression for time from their choice of equation (if only decimal seen, award for 2sf or better 0.078997.. for cos,0.020349… for sin) Must have come from fully correct work in (a)	
dM1	Complete correct method to obtain the required time. Dependent on previous M mark.	
A1cso	Correct final answer. 2s.f. or better. Must have come from fully correct work in (a) and (d), although they might not have used $\ddot{x}$	

Question Number	Scheme	Marks
7 (a)(i)	$V=\pi\int_1^2\left(x^2+4\right)^2\,\mathrm{d}x=\pi\int_1^2\left(x^4+8x^2+16\right)\mathrm{d}x$	
	$=\pi\left[\frac{1}{5}x^5+\frac{8}{3}x^3+16x\right]_1^2=\frac{613\pi}{15}$ (cm^3) *	M1A1 A1cso
(ii)	$(\pi)\int_1^2 x\left(x^2+4\right)^2\,\mathrm{d}x=(\pi)\int_1^2\left(x^5+8x^3+16x\right)\mathrm{d}x$	
	$=(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2$ $\quad$ *alt* $(\pi)\left[\frac{\left(x^2+4\right)^3}{6}\right]_1^2$	M1A1
	$\bar{x}=\frac{(\pi)\left[\frac{1}{6}x^6+2x^4+8x^2\right]_1^2}{\frac{613}{15}(\pi)}=\frac{\frac{129}{2}}{\frac{613}{15}}=1.578...=1.58$ (cm)	M1dM1A1 (8)
(b)	Mass $\quad \frac{613\pi}{15}M \quad 9\pi M \quad 45\pi M \quad \left(36\pi+\frac{613\pi}{15}\right)M=\frac{1153\pi}{15}M$	B1
	Dist from B $\quad$ 0.578 $\quad$ 0.5 $\quad$ 0.5 $\quad$ $\bar{y}$	B1ft
	$\frac{613\pi}{15}\times0.578-9\pi\times0.5+45\pi\times0.5=\left(36\pi+\frac{613\pi}{15}\right)\bar{y}$	M1A1ft
	$\bar{y}=\frac{1249}{2306}=0.5416...=0.54$ (cm)	A1 (5)
		[13]

(a)(i)M1	Attempt the squaring and integrating (at least one power going up). Allow w/o π
A1	Correct integration allow w/o π
A1*cso	Correct volume, with no errors seen. (Must include π and no $V = ...$ w/o π must have been seen.)
(ii)M1	Attempt $\int x\left(x^2+4\right)^2 dx$. Must either expand or obtain $k\left(x^2+4\right)^3$. π not needed. Limits not needed
A1	Correct algebraic integration, π not needed. Limits not needed
M1	Substitute the (correct) limits in their integrated function. Independent, but must have been attempting $\int xy^2 dx$
M1	Divide the two integrals (correct way up). Depends on the 1st and 2nd M marks. π and ρ in both or neither.
A1	Correct final result. Must be 3 sf.
	(SC Correct answer with no algebraic integration shown can score M0A0 M1 M0A0)
(b)	
B1	Correct masses seen explicitly or in an equation.
B1ft	Correct distances from B (or any vertical axis). Follow through distance from (a).
M1	Form a moments equation, with lighter cylinder subtracted and the heavier one added
A1ft	Correct equation, follow through their distance from (a).
A1	Correct distance from B, 2 sf or better

Alt (b) Find mass and CoM of S_1 first $\quad \text{Mass} = \dfrac{478\pi}{15}M \quad CoM = \dfrac{287}{478} \approx 0.6004$

Award B1B1 when all component masses and distances are seen. Complete method needed for M1.
Award first A1 for correct masses/distances initially used in forming both equations.

(Note: Use of 0.58 leads to $\overline{x} = 0.603$ (cm) for S_1. This gives a final answer 0.543. If they give 0.54, award full marks, as premature approximation does not affect final answer, but penalise 0.543)

SC – If the use M and $5M$ for the masses, award max B0B1 M1A0A0

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