Examiners' Report

Principal Examiner Feedback

January 2017

Pearson Edexcel International A Level
In Further Pure Mathematics F1 (WFM01)

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General Introduction

This paper was accessible and there was plenty of opportunity for the majority of students to gain some marks in each of the 9 questions. There was some testing material involving coordinate systems and mathematical induction that allowed the paper to discriminate well across the higher ability levels.

In summary, Q1(a), Q2, Q3, Q4(a), Q5, Q6(a), Q7(ii)(c) and Q8(a) were a good source of marks for the average student, mainly testing standard ideas and techniques and Q1(b), Q4(b), Q6(b), Q7 and Q9 were discriminating at the higher grades. Q8(c) and Q8(d) proved to be the most challenging questions on the paper.

Question 1

This question was accessible with the majority of students scoring full marks.

In Q1(a), most students evaluated $\mathrm{f}(2)$ and $\mathrm{f}(3)$ correctly with most indicating the idea of a sign change followed by an acceptable conclusion. Some students, however, incorrectly applied $\mathrm{f}(x) = 2^x - 10\sin x - 2$ as $\mathrm{f}(x) = 2^x - 10\sin(x-2)$ when they evaluated $\mathrm{f}(2)$ and $\mathrm{f}(3)$.

In Q1(b), the majority of students used similar triangles in order to form a correct equation in α and proceeded to solve it correctly. Some students, however, formed an equation in α with one of their fractions the wrong way round, while others still used negative lengths in their working. Occasionally, students went back to first principles and found the equation of the line joining the points $(2, -7.09...)$ and $(3, 4.58...)$, before proceeding to find where the line crossed the *x*-axis. Few students attempted to use interval bisection instead of linear interpolation.

Question 2

A significant number of students were generally well prepared for this question with just over half of them scoring full marks. There were the occasional algebraic, manipulation and bracketing errors seen in some students' solutions. A small minority of students, who did not adhere to the instruction given in the question, found and used $\alpha, \beta = \frac{1 \pm \sqrt{23}\,\mathrm{i}}{4}$.

In Q2(a), the most common mistake was the inability to recall that the sum and product of roots in a quadratic equation $ax^2 + bx + c = 0$ were $-\frac{b}{a}$ and $\frac{c}{a}$ respectively.

In Q2(b), most students used the correct method of substituting their $\alpha+\beta$ and their $\alpha\beta$ into $\dfrac{\alpha+\beta}{\alpha\beta}$ in order to find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$.

In Q2(c), some students applied $\left(x-\left(2\alpha-\dfrac{1}{\beta}\right)\right)\left(x-\left(2\beta-\dfrac{1}{\alpha}\right)\right)=0$, while most students found the values for the sum and product of $\left(2\alpha-\dfrac{1}{\beta}\right)$ and $\left(2\beta-\dfrac{1}{\alpha}\right)$. The main source of error was in forming the new quadratic equation. The three main errors in establishing the required quadratic were applying the incorrect method of $x^2+(\text{sum})x+(\text{product})$, the omission of "= 0" and not giving integer coefficients.

Question 3

This question was accessible with the vast majority of students scoring full marks.

Almost all students wrote down the complex conjugate root $-1-3\text{i}$, although a few did not state this root explicitly. Most students used the conjugate pair to write down and multiply out $(x-(-1+3\text{i}))(x-(-1-3\text{i}))$ in order to identify the quadratic factor $x^2+2x+10$. Some students achieved this quadratic factor using the sum and product of roots method. Most students either used the methods of long division or comparing coefficients to find the remaining quadratic factor, although some manipulation errors were seen at this stage. A majority of students achieved the second quadratic factor x^2+16, although a surprising number of them failed to solve $x^2+16=0$ correctly, with $\pm 16\text{i}$ or ± 4 commonly seen.

Question 4

This question was accessible with the majority of students scoring full marks.

In Q4(a), almost all students expanded the cubic expression $r(2r+1)(3r+1)$ and substituted the standard formulae for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ into $\sum_{r=1}^{n}(6r^3+5r^2+r)$. Students who directly factorised out $\dfrac{1}{6}n(n+1)$ were usually more successful in obtaining the correct answer. A number of students who expanded to give $\dfrac{3}{2}n^4+\dfrac{14}{3}n^3+\dfrac{9}{2}n^2+\dfrac{4}{3}n$ were less likely to obtain the correct answer. Common errors included expanding $9n(n+1)$ as part of their algebra to give $9n^2+9$ or writing $\sum_{r=1}^{n} r$ as n.

In Q4(b), the majority of students applied $\mathrm{f}(20)-\mathrm{f}(9)$, where $\mathrm{f}(n) = \frac{1}{6}n(n+1)(9n^2+19n+8)$, and obtained the correct answer. A small number of unsuccessful students either applied $\mathrm{f}(20)-\mathrm{f}(10)$ or $\mathrm{f}(20)-\mathrm{f}(11)$.

Question 5

This question was accessible with the majority of students scoring full marks.

Q5(a) was accessible to almost all students. The majority were able to apply Pythagoras' Theorem to find a correct value for $|z|$. A small minority of students subtracted rather than added the two squared terms.

Q5(b) was answered less well than the other parts. A significant number of students did not realise that the required angle was in the second quadrant and many of these found an angle in the first quadrant. Most students worked in radians but there was a significant minority who gave their answers in degrees.

There were many fully correct solutions in Q5(c). Most students multiplied the numerator and denominator of $\frac{z}{1+\mathrm{i}}$ by $(1-\mathrm{i})$ and then rearranged the resulting equation. The alternative approach of multiplying both sides by $(1+\mathrm{i})$ was less successful, as *w* was often not multiplied by $(1+\mathrm{i})$.

Q5(d) was accessible to most students, and almost all students were able to plot *z* and their *w* on an Argand diagram, and the majority labelled the points usually as a coordinate or a complex number. Marks were usually lost in Q5(d) by students not labelling their points.

Question 6

This question was accessible with the majority of students scoring 6 of the 7 marks available. Q6(b) was often not attempted.

In Q6(a), most students differentiated $\mathrm{f}(x)$ and applied the Newton-Raphson procedure to give a correct second approximation for α as 0.642. In some cases, however, a lack of working did mean that it was sometimes difficult for examiners to tell if the Newton-Raphson procedure was applied correctly. Few students struggled to differentiate $-\frac{1}{2x}$ correctly.

In Q6(b), only a minority of students were able to confirm α as 0.642 to 3 decimal places. The majority of these students usually evaluated both $\mathrm{f}(0.6415)$ and $\mathrm{f}(0.6425)$, demonstrated a change of sign and gave a full conclusion. Incorrect methods included evaluating both $\mathrm{f}(0.641)$ and $\mathrm{f}(0.643)$ or just evaluating $\mathrm{f}(0.642)$.

Question 7

This question discriminated well across students of all abilities.

In Q7(i)(a) there were many fully correct answers with most students realising that the matrix represented a reflection in the y-axis. A significant number of students added the comment "through" or "about" $(0, 0)$. Some students thought the reflection was in the x-axis, or the line $y = x$ or the line $y = -x$. A few students incorrectly thought that the transformation was a rotation. A small number of students, probably prompted by the next part of the question, described the matrix as a stretch parallel to the x-axis. Those who used this approach invariably gave the correct scale factor of -1.

Q7(i)(b) proved to be a little more challenging. Although there were many correct answers, common incorrect answers included $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ or $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$.

In Q7(ii)(a), many students applied a correct method to find the value of k, either by applying Pythagoras' Theorem or by using trigonometry. Even though the question stated $k > 0$, some students included a negative value for k in their final answer.

A significant number of students struggled with Q7(ii)(b). Many students attempted to use trigonometry to find an angle, but many found an angle in either the first or second quadrant and so gained no marks in this part. Most students were working in radians but there were a number who gave their answer in degrees.

There were many fully correct answers to part Q7(ii)(c) with most students applying a correct method for finding the inverse. A small minority of students applied $16 - 9$ (rather than $16 + 9$) to find an incorrect determinant of 7.

Question 8

Part (a) and part (b) were answered well by almost all students, but part (c) proved to be demanding, with success in part (d) then tending to be linked to success in part (c). Many students made no creditable attempt in parts (c) and (d).

In Q8(a), students used a variety of methods to find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Most students wrote y in terms of x and were able to differentiate and find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t correctly. Some students used implicit differentiation or the chain rule with the parametric equations. Most students were then successful in finding the equation of the normal and obtained the given result. A small number of students who did not use a calculus method but just stated that the gradient of the normal at P is $-t$ lost all the marks this part.

In Q8(b), most students used the fact that the focus is at $(a, 0)$ to identify the coordinates of B as $(5a, 0)$.

Only the minority of students were able to score any marks in part (c) or part (d). A clear correct diagram of the situation seemed to be helpful in assisting some students to formulate a correct strategy for finding the points *Q* and *R*.

In Q8(c), the most popular successful strategy was substituting their $x = 5a$ and $y = 0$ into the given normal equation. Those students who used this strategy usually proceeded to find $t = \pm\sqrt{3}$ and obtained the correct coordinates for *Q* and *R*. A related (occasionally seen) method (with equivalent algebra) was to realise that the gradient of the normal at *P* is equal to the gradient of radius *PB*. In another strategy, some students substituted $y^2 = 4ax$ into the circle equation $(x - 5a)^2 + y^2 = r^2$ and used algebra to achieve $x^2 - 6ax + 25a^2 - r^2 = 0$. The majority of these students failed to set the discriminant of this quadratic equation equal to zero and so made no creditable progress. The strategy of minimising the value of PB^2 was rarely seen.

In Q8(d), those students who had found the correct values of *B*, *Q* and *R* usually made a correct attempt at finding the area of triangle *BQR*. Some erroneously used a base length of 5*a* rather than 2*a* (i.e. $5a - 3a$).

Question 9

This question discriminated well across the higher ability students with part (i) more successfully attempted than part (ii). A considerable number of students proved the general result in part (i) by using standard formula for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ rather than proving the general result by induction.

In part Q9(i), most students showed that for $n = 1,$ both the LHS and RHS of the general result were both equal to 2. A large number of students assumed the general result was true for $n = k$ and attempted to add the $(k+1)^{\text{th}}$ term to the sum of *k* terms. Many students who got this far went on to show that the general result was true for $n = k + 1$, with some students expanding $k^3(k+1) + 4(k+1)^3 - 3(k+1)^2 + (k+1)$ instead of initially factorising but producing correct algebra to achieve $(k+1)^3(k+1+1).$ Some students lost marks by jumping from $k^3(k+1) + 4(k+1)^3 - 3(k+1)^2 + (k+1)$ to $(k+1)^3(k+1+1)$ with no supporting evidence.

In Q9(ii), many students successfully proved $\mathrm{f}(n) = 5^{2n} + 3n - 1$ was divisible by 9 for $n = 1$. There were then varying approaches to the proof with $\mathrm{f}(k+1) - \mathrm{f}(k)$ being the most popular. There were also other valid methods that met with varying degrees of success, such as attempts to find $\mathrm{f}(k+1) - m\mathrm{f}(k)$ with a suitable value for *m* or attempts to find $\mathrm{f}(k+1)$ directly.

Although the majority of students were able to write down a correct expression for either $\mathrm{f}(k+1)-\mathrm{f}(k)$ or $\mathrm{f}(k+1)$, a significant number could not manipulate their expression to a correct result for $\mathrm{f}(k+1)$ of either $25(5^{2k}+3k-1)-9(8k-3)$ or $27(5^{2k})+9k-2\mathrm{f}(k)$ or equivalent.

In Q9(i) and Q9(ii), some students did not bring all strands of their proof together to give a fully correct proof. A minimal acceptable proof, following on from completely correct work, would incorporate the following parts: assuming the general result is true for $n=k$; then showing the general result is true for $n=k+1$; showing the general result is true for $n=1$; and finally concluding that the general result is true for all positive integers.

Grade Boundaries

Grade boundaries for this, and all other papers, can be found on the website on this link:

http://qualifications.pearson.com/en/support/support-topics/results-certification/grade-boundaries.html