Mark Scheme (Results)

Summer 2016

Pearson Edexcel International A Level in Mechanics 3 (WME03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

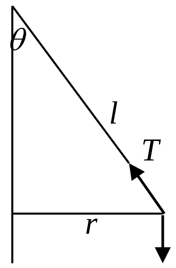
  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

June 2016
WME03/01 M3 (IAL)
Mark Scheme

Question Number	Scheme	Marks
1.		
	$T\cos\theta = mg$	M1A1
	$T\sin\theta = mr\omega^2$	M1A1
	$\tan\theta = \dfrac{r\omega^2}{g}$	DM1
	$\tan\theta = \dfrac{r}{\sqrt{l^2 - r^2}}$	B1
	$\dfrac{r\omega^2}{g} = \dfrac{r}{\sqrt{l^2 - r^2}}$	DM1
	$\omega^2 = \dfrac{g}{\sqrt{l^2 - r^2}}$ *	A1cso
		[8]

M1 Attempting to resolve vertically
A1 Completely correct equation
M1 Attempting NL2 along the radius, acceleration in either form
A1 Completely correct equation inc acceleration as $r\omega^2$
DM1 Eliminate T Dependent on 1st and 2nd M marks
B1 $\tan\theta$ or $\cos\theta$ (ie vert side of triangle used)
DM1 Eliminate the angle Dependent on 1st and 2nd M marks
A1cso Obtain the GIVEN result.

$mg\sin\theta = mr\omega^2\cos\theta$ as initial equation scores M2A2
Triangle of forces methods must show the triangle (not just the equations) for the working to be complete.

Question Number	Scheme	Marks
2(a)	$T = 2mg = \frac{10mgx}{5a}$	M1
	$x = a$	A1
	$AO = 5a + a = 6a$	A1ft (3)
(b)	$2mg - T = 2m\ddot{x}$ or $2ma$	M1
	$2mg - \frac{10mg(a+x)}{5a} = 2m\ddot{x}$	M1A1
	$\ddot{x} = -\frac{gx}{a}$ $\therefore$ SHM	A1cso (4)
(c)	Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{a}{g}}$ oe	M1A1 (2)
		[9]

(a)M1 Resolve vertically using HL for the tension
A1 Obtaining $x = a$
A1ft Add their extension to 5*a*

(b)M1 Forming an equation of motion, acceleration $\ddot{x}$ or a
M1 Use HL to express the tension in terms of *x*, (centre not nec at the correct point), acceleration $\ddot{x}$ or a.
A1 Fully correct equation acceleration to be $\ddot{x}$ now
A1cso Simplify to $\ddot{x} = -\frac{gx}{a}$ **and** state SHM
All marks available for an algebraic solution (ie no sub made for any/all of mass, λ, *l*.)

(c)M1 Using period $= \frac{2\pi}{\omega}$ with their ω from an "SHM" equation. (ie $\ddot{x}$ or $a = \pm\omega^2 x$)
A1cso Correct period, any equivalent form but not fractions within fractions. ω must have come from a correct equation and substitutions for mass, λ, *l* made.

Question Number	Scheme	Marks
3(a)	EPE at $A = \dfrac{4mg\left(\frac{1}{4}l\right)^2}{2l} \quad \left(= \dfrac{mgl}{8}\right)$	B1
	Work done against friction from A to natural length $= \dfrac{2}{5}mg\left(\dfrac{1}{4}l\right)$	B1
	$\dfrac{mgl}{8} > \dfrac{mgl}{10}$	M1
	$\therefore$ P is still moving when string goes slack, ie $OB < l$	A1 cso (4)
(b)	$\dfrac{2}{5}mgx = \dfrac{mgl}{8}$	M1
	$x = \dfrac{5}{16}l$	A1
	$OB = \dfrac{5}{4}l - \dfrac{5}{16}l = \dfrac{15}{16}l$	A1ft (3)
		[7]

(a)B1 Correct EPE at start

B1 Correct work done against friction from release at A to string becoming slack.

M1 Either showing the inequality above or using an equation to show P has KE at natural length. Comparing the 2 energy terms even if incorrect scores M1 EPE to be dimensionally correct

A1cso Drawing the required conclusion, with evidence, from correct working.

NB The two B marks can be awarded in (b) - Award B1 for work done against friction from release to coming to rest again.

(b)M1 Work-energy from start to B

A1 Correct distance moved

A1ft Subtract their distance moved from $\dfrac{5}{4}l$

ALT for (b): Work from natural length to *B*:

M1 Find KE at natural length (may have been done in (a)) and then find further distance moved by any valid method.

A1 Correct distance moved from natural length $= \dfrac{1}{16}l$

A1ft Subtract their distance moved from l.

Question Number	Scheme	Marks
4 (a)	$F = (-)\dfrac{k}{x^2}$	
	$x = R \quad F = (-)mg$	
	$mg = (-)\dfrac{k}{R^2}$	M1
	Mag of $F = \dfrac{mgR^2}{x^2}$ *	A1cso (2)
(b)	$(m)v\dfrac{dv}{dx} = -(m)\dfrac{gR^2}{x^2}$	M1
	$\int v dv = -\int gR^2 x^{-2} dx$	
	$\frac{1}{2}v^2 = gR^2x^{-1} + (c)$	DM1A1ft
	$x = R \;\; v = U \qquad \frac{1}{2}U^2 = gR + c$	M1
	$\frac{1}{2}v^2 = \dfrac{gR^2}{x} + \frac{1}{2}U^2 - gR$ oe	A1
	$x = \frac{21}{20}R \;\; v = 0 \qquad 0 = \dfrac{gR^2}{\frac{21}{20}R} + \frac{1}{2}U^2 - gR$	M1
	$\frac{1}{2}U^2 = gR - \frac{20}{21}gR$	
	$U = \sqrt{\dfrac{2gR}{21}}$ oe	A1 (7)
		[9]

(a)M1	Using NL2 at the surface of the Earth with the proportionality condition, force to be *mg*, distance to be *R*. Can use $F = \dfrac{GMm}{x^2}$
A1cso	Find the constant of proportionality and hence deduce the stated magnitude.
(b)M1	Using NL2 with acceleration in form $v\dfrac{dv}{dx}$. Minus sign may be missing. *m* may be cancelled.
DM1	Attempt integration of their expression. v^2 or x^{-1} must be seen.
A1ft	Correct integration follow through a missing minus sign. Constant of integration may be missing.
M1	Using $x = R \;\; v = U$ OR substitute $x = \frac{21}{20}R \;\; v = 0$ in an equation containing *c*
A1	Substituting a **correct** constant to obtain a correct expression for $\frac{1}{2}v^2$
M1	Substitute $x = \frac{21}{20}R \;\; v = 0$ OR substitute $x = R \;\; v = U$
A1	Complete to a correct expression for *U*.

Question Number	Scheme	Marks

Definite integration in (b):

$(m)v\frac{dv}{dx} = -(m)\frac{gR^2}{x^2}$ M1

$\int_U^0 v\,dv = -\int_R^{\frac{21R}{20}} gR^2x^{-2}dx$

$\left[\frac{1}{2}v^2\right] = \left[gR^2x^{-1}\right]$ DM1A1ft (ft missing sign, as main scheme) Limits not needed

$\left[\frac{1}{2}v^2\right]_U^0 = \left[gR^2x^{-1}\right]_R^{\frac{21R}{20}}$ M1 limits seen, correct "values" but may be paired incorrectly

A1 correct limits, 0 and $\frac{21R}{20}$ at top or bottom on both integrals, U and R in the other positions

M1 Substitute their limits A1 correct answer.

NB It is possible to appear to omit the minus sign and reverse the limits on one integral. However, without some explanation this will be insufficient working and scores as a missing minus case.

Question Number	Scheme	Marks
5(a)	Period $=12.25$ hours	B1
	$\dfrac{2\pi}{\omega}=12.25$	
	$\omega=\dfrac{2\pi}{12.25}$ oe eg $\omega=\dfrac{8\pi}{49}$	M1A1ft
	Amplitude $=5\text{ m}$	B1
	$v^2=\omega^2\left(a^2-x^2\right)$	
	$x=4 \Rightarrow v^2=\left(\dfrac{2\pi}{12.25}\right)^2\left(5^2-4^2\right)$	M1A1
	$v=\left(\dfrac{2\pi}{12.25}\right)\times 3=1.53873...=1.5\text{ m h}^{-1}$ (1.5 or better or $\dfrac{24\pi}{49}$)	A1cao (7)
(b)	$x=4 \quad 4=5\cos\omega t$	
	$t=\dfrac{12.25}{2\pi}\cos^{-1}0.8$ (= 1.2546...)	M1A1
	Time ladder in the water $=12.25-2\times\dfrac{12.25}{2\pi}\cos^{-1}0.8$	DM1
	$=9.7407...=9.7$ hours (9.7 or better) (9hr 44 min or 35 066 875 seconds)	A1 cao (4) [11]

(a)B1 Correct period for the SHM (Allow 44100 s)

M1 Using period $=\dfrac{2\pi}{\omega}$ to obtain an expression for ω or any other complete method

A1ft A correct expression for ω follow through their period, inc with units changed.

B1 Correct amplitude seen explicitly or used

M1 Using $v^2=\omega^2\left(a^2-x^2\right)$ with their ω, amplitude and $x=\left(\text{their amp }-1\right)$

A1 Correct expression for v^2

A1cao Correct value of v, 2 sf or better Units must be m h^{-1}.

(b)M1 Using $x=a\cos\omega t$ or $a\sin\omega t$ with $x=\pm\left(\text{their amp }-1\right)$, their ω and amplitude

A1 Finding a correct time, either from the end point to reaching the bottom step or from the bottom step to the centre of the oscillation. (in hours or seconds)

DM1 Using their time to obtain an expression for the required time

A1cao Time = 9.7 hrs or better

Sine used: Time = $2\times\dfrac{\text{period}}{4}+2\times\text{time found}$

Question Number	Scheme	Marks
6(a)	Energy A to B $\frac{1}{2}\times 0.2v^2 - \frac{1}{2}\times 0.2u^2 = 0.2g\times 0.1$	M1A1
	$v^2 - u^2 = 0.2g$	
	$v^2 = u^2 + 1.96$ *	A1 cso (3)
(b)	$mg\cos\theta \ \ (-R) = m\frac{v^2}{r}$	M1
	$R = 0 \Rightarrow g\times\frac{4}{5} = \frac{(u^2+1.96)}{0.5}$	A1
	$0.4g = u^2 + 1.96$	
	$u^2 = 1.96,\ \ u = 1.4$ or 1.40	DM1,A1 (4)
(c)	Vert speed $= v\sin\theta = \frac{3}{5}\sqrt{0.4g}\ \ (=1.1879...)$ or $\frac{21\sqrt{2}}{25}$	M1
	$0.4 = \frac{3}{5}\sqrt{0.4g}\,t + \frac{1}{2}gt^2$	M1A1ft
	$4.9t^2 + 1.1879...t - 0.4 = 0$	
	$t = \frac{-1.1879 \pm \sqrt{1.1879^2 + 4\times 4.9\times 0.4}}{9.8}$	
	$t = 0.1891...\ \ (t > 0)$	A1
	Horiz speed $= v\cos\theta = \frac{4}{5}\sqrt{0.4g}$	M1
	Horiz distance $= \frac{4}{5}\sqrt{0.4g}\times 0.1891... = 0.2995$	A1
	$OC = 0.3 + 0.2995... = 0.5995...\ \ \ OC = 0.60$ or 0.600 m	A1ft (7) [14]

Question Number	Scheme	Marks

(a)M1 Energy equation from A to B: A difference of KE terms = loss of GPE mass m, 0.2 or already cancelled. **Must** be clearly an energy equation and not uniform acceleration.

A1 Completely correct equation mass m, 0.2 or already cancelled.

A1cso Rearrange to the GIVEN form mass m, 0.2 or previously cancelled.

(b)M1 Attempt NL2 at B. Normal contact force is zero here so need not be seen in the equation

A1 Fully correct equation with $R = 0$ and substitution for v^2

DM1 Solve for a numerical value for u^2 or u

A1 Correct value for u, must be 1.4

(c)M1 Attempt the vertical speed, $v\sin\theta$ or $v\cos\theta$, with their value for v (not their value of u) and an attempt at a numerical value for the trig function.

M1 Use $s = ut + \frac{1}{2}at^2$ with their vertical speed. Must have attempted to resolve the speed.

A1ft Correct equation, follow through their vertical speed

A1 Correct value of t (no ft here)

M1 Attempt the horizontal speed, (trig function used to be different to that used for the vertical) with their value for v and an attempt at a numerical value for the trig function. Allow if value of u is used, provided there is an attempt to resolve.

A1 Correct horizontal distance

A1ft Add their horizontal distance to 0.3. Answer must be 2 or 3 sf.

Question Number	Scheme	Marks
7(a)	$y=\frac{rx}{h}$	
	$(\pi)\int_0^h xy^2\mathrm{d}x=(\pi)\int_0^h \frac{r^2x^3}{h^2}\mathrm{d}x$	M1A1
	$(\pi)\left[\frac{r^2x^4}{4h^2}\right]_0^h=(\pi)\frac{r^2h^2}{4}$	A1
	$\bar{x}=\frac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}=\frac{r^2h^2}{4}\div\frac{r^2h}{3}=\frac{3}{4}h$ *	M1A1cso (5)
(b)	Mass $\quad \frac{1}{3}\pi\times5r^3 \quad \frac{2}{3}\pi r^3 \quad \frac{7}{3}\pi r^3$ $\quad 5 \quad 2 \quad 7$	B1
	Dist from O $\quad \frac{1}{4}\times5r \quad -\frac{3}{8}r \quad \bar{x}$	B1
	$\frac{25r}{4}-\frac{3r}{4}=7\bar{x}$	M1A1ft
	$\bar{x}=\frac{11r}{14}$	A1 cao (5)
(c)	$\tan\theta=\frac{\frac{11r}{14}}{r}=\frac{11}{14}$	M1A1ft
	$\theta=38.15...^\circ$ (38 or better) (0.6659... rad; 0.67 or better)	A1 cao (3)
(d)	Mass of cone $=\frac{5}{2}M$ or Mass of $S=\frac{7}{2}M$ or mass of the particle $=\frac{2}{3}\pi r^3\rho$ and work with masses in volume form	B1
	$kMr(g)=\frac{7}{2}M\times\frac{11r}{14}(g)$	M1A1ft
	$k=\frac{11}{4}$ or 2.75	A1 cao (4)
		[17]

Question Number	Scheme	Marks
(a)M1	Attempting $(\pi)\int_0^h xy^2\,\mathrm{d}x$ π not needed. No integration needed for this mark but must move to a function of x only. limits not needed.	
A1	Correct integral $\int_0^h \frac{r^2x^3}{h^2}\mathrm{d}x$ with or without π limits needed	
A1	Integrate correctly and substitute correct limits	
M1	Using $\bar{x} = \frac{\int xy^2\mathrm{d}x}{\int y^2\mathrm{d}x}$ π in both or neither integral. Denominator need not be in integral form.	
	If mass/unit volume included, must appear in both or neither.	
A1cso	Correct GIVEN answer. cso so $y = \frac{rx}{h}$ must have been used.	
	If $h = r$ is assumed: M marks only available. If $h = 5r$ is assumed max available is 4/5 (Deduct final A mark)	
(b)B1	Correct mass ratio	
B1	Correct distances from O or any other point. Both known distances may be positive here, but one must have a minus both here and in the equation or just in the equation.	
M1	Attempt a moments equation using their mass ratio and distances (signs may be incorrect)	
A1ft	Correct equation, follow through their mass ratio and distances but signs to be correct	
A1cao	Correct distance for the c of m	
(c)M1	Using tan to find the required angle. Ratio can be either way up.	
A1ft	Tan ratio to be correct way up and have correct numbers, follow through their ans to (b)	
A1cao	Correct size of angle in degrees or radians, 2 sf or better	
(d)B1	Correct mass for cone or for S	
M1	Moments equation about O or another point. This may include an $\bar{x}$ for the c of m of S + particle	
A1ft	Correct nos in the equation, follow through their ans to (b) and mass of $S \neq M$ $\bar{x}$ to be 0 now	
A1cao	Correct value, 11/4 or 2.75	
ALTs	**For (d)**	
1	Use $M = \frac{2}{3}\pi r^3$ so mass of particle is $\frac{2}{3}\pi r^3 k$ and use volumes in the work. Mass of the particle scores B1 provided all the work is using volumes.	
2	Can work with the cone, hemisphere and particle so the equation will have 3 terms (or 4 if $\bar{x}$ for the c of m of S + particle is not assumed to be zero.	

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