Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

Wednesday 8 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6668/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. Use algebra to find the set of values of x for which

$$\frac{x}{x+1} < \frac{2}{x+2}$$

(6)

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Question 1 continued

Q1

(Total 6 marks)

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2. (a) Show that, for $r > 0$

$$r - 3 + \frac{1}{r+1} - \frac{1}{r+2} = \frac{r^3 - 7r - 5}{(r+1)(r+2)}$$

(2)

(b) Hence prove, using the method of differences, that

$$\sum_{r=1}^{n} \frac{r^3 - 7r - 5}{(r+1)(r+2)} = \frac{n(n^2 + an + b)}{2(n+2)}$$

where a and b are constants to be found.

(5)

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Question 2 continued

Q2

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3. (a) Find the four roots of the equation $z^4 = 8\left(\sqrt{3} + \mathrm{i}\right)$ in the form $z = r\mathrm{e}^{\mathrm{i}\theta}$

(5)

(b) Show these roots on an Argand diagram.

(2)

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Question 3 continued

Q3

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4. (i)

$$p\frac{\mathrm{d}x}{\mathrm{d}t} + qx = r \qquad \text{where } p, q \text{ and } r \text{ are constants}$$

Given that $x = 0$ when $t = 0$

(a) find x in terms of t

(4)

(b) find the limiting value of x as $t \to \infty$

(1)

(ii)

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} + 2y = \sin\theta$$

Given that $y = 0$ when $\theta = 0$, find y in terms of θ

(7)

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Question 4 continued

(Total 12 marks)

Q4

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5. (a) Use de Moivre's theorem to show that

$$\sin^5 \theta \equiv a \sin 5\theta + b \sin 3\theta + c \sin \theta$$

where a, b and c are constants to be found.

(5)

(b) Hence show that $\displaystyle\int_0^{\frac{\pi}{3}} \sin^5 \theta \, d\theta = \frac{53}{480}$

(5)

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Question 5 continued

(Total 10 marks)

Q5

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6. (a) Find the Taylor series expansion about $\frac{\pi}{4}$ of $\tan x$ in ascending powers of $\left(x - \frac{\pi}{4}\right)$ up to and including the term in $\left(x - \frac{\pi}{4}\right)^3$.

(7)

(b) Deduce that an approximation for $\tan \frac{5\pi}{12}$ is $1 + \frac{\pi}{3} + \frac{\pi^2}{18} + \frac{\pi^3}{81}$

(2)

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Question 6 continued

Q6

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7. (a) Show that the substitution $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = -x^{-2}, \quad x > 0 \qquad \text{(I)}$$

into the equation

$$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = -e^{-2u} \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II).

(7)

(c) Hence obtain the general solution of the differential equation (I) giving your answer in the form $y = f(x)$

(1)

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Question 7 continued

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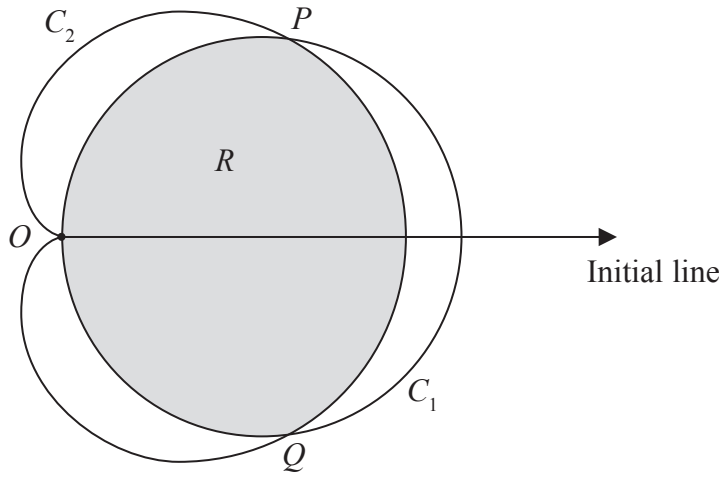

Figure 1

The curve C_1 with equation

$$r = 7\cos\theta, \quad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

and the curve C_2 with equation

$$r = 3(1 + \cos\theta), \quad -\pi < \theta \leqslant \pi$$

are shown on Figure 1.

The curves C_1 and C_2 both pass through the pole and intersect at the point P and the point Q.

(a) Find the polar coordinates of P and the polar coordinates of Q.

(3)

The regions enclosed by the curve C_1 and the curve C_2 overlap, and the common region R is shaded in Figure 1.

(b) Find the area of R.

(7)

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Q8

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TOTAL FOR PAPER: 75 MARKS

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