Examiners' Report

Summer 2016

Pearson Edexcel GCE in Further Pure Mathematics 2 (6668/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2016
Publications Code 6668_01_1606_ER

Grade Boundaries

Grade boundaries for this, and all other papers, can be found on the website on this link:
http://www.edexcel.com/iwantto/Pages/grade-boundaries.aspx

GCE Mathematics Further Pure 2

Specification 6668/01

Introduction

Most of this paper was reasonably straightforward although there were some challenging parts, notably questions 4 and 8.

As always some students do not write down the formula they are about to use, opting for merely a substitution. With the general formula shown before substitution students can still gain the method mark if an error is made during the subsequent substitution; without the general formula the method mark is lost as examiners cannot assume the formula to be correct when it is not shown. If there are dependent method marks following, such an omission can be costly.

Report on individual questions

Question 1

A wide range of approaches were available and many of these were seen. Many students attempted to multiply by the square of the 2 denominators, often successfully. A number were not able to find the correct factors, getting $x^2 - 1$ instead of $x^2 - 2$, or sometimes even $x^2 + 2$. Finding the correct factors almost always led to the 4 correct critical values, and then most often to the correct intervals. A few felt that ***$-\sqrt{2}$*** was greater than –1, which gave two incorrect intervals.

The alternative methods in the mark scheme were less common. Putting the two fractions over a common denominator usually led to correct critical values, but again $x^2 - 1$ was seen. Such errors led to two incorrect intervals, and the loss of 3 marks. There were several attempts using graphical methods combined with algebra, and use of the possible intervals between the critical values to determine the correct answers. Some students recognised immediately that –1 and –2 were critical values due to the nature of the functions, and normally proceeded to complete the solution.

Question 2

Q02(a) proved an accessible start to the question with most students legitimately achieving the printed answer. The majority of students used the method in the scheme but there were some who used division successfully. Most students started with the LHS and were able to gain both marks. Those who started with the RHS were able to successfully divide out and then used partial fractions to get the LHS. There were a few errors but generally most students were able to use a common denominator and simplify and obtained both marks.

In Q02(b) the method of differences was well known for the fractional terms although some did not extract the two terms required. There were many students who did not know how to find $\sum_{r=1}^{n}(r-3)$ and this did lead to a few students giving up on this question. Some students sought to involve the linear term in the method of differences procedure and so failed to calculate the sum of those terms. A few did not show at least three lines and a cancelling pattern in their method of differences and lost marks if they failed to obtain a correct final answer. The summation of the two terms at the start when done correctly was most often done by using a formula for the summation of the first n positive integers and combining that with – –3n. There were a few cases where this term was seen as –3 in the summation. The collection of terms into a single fraction was generally well done.

Question 3

The overall technique needed for Q03(a) was well known but mistakes were made in calculating the modulus of z with 8 and $\sqrt{2}$ being seen as the fourth root of 16. Some students calculated the argument incorrectly as $\frac{\pi}{3}$, and this mistake meant a student could get a maximum of 2 marks out of 5. A good proportion of students got the full 5 marks, using De Moivre's theorem correctly and giving the answers in the required exponential form. The generation of the four roots was well done by most although there were students dividing the principal argument by 4 first and then adding the $2k\pi$. There was a fairly even split in the correct answers between those who took the argument to be between 0 and 2π and $-\pi$ and π.

Q03(b) proved more challenging. From the drawings produced, it seemed that students did not appreciate the geometrical relationship between the roots in the Argand diagram. It was rare to see the basic circle drawn which does reinforce this idea. The angular placement of the points showed that many students had little concept of the size of $\frac{\pi}{24}$ or an appreciation that the arguments differed by $\frac{\pi}{2}$ and how to show that on a diagram. Students need to appreciate that a sketch needs to be reasonably accurate in terms of distances and angles to get credit. However, some students presented correct responses very clearly. Those who gained only one mark identified vectors which were perpendicular but lost the final mark for not putting the vectors/points close enough to the axis or for not labelling their points.

Question 4

This was one of the more challenging questions on the paper, along with question 8; particularly Q04(ii) which tested students' integration techniques. The use of unknown constants in Q04(i) was also a difficulty for some students. However, nearly all were confident enough to have an attempt at this question, even if incomplete. The question as a whole seemed to work as a good discriminator between students' different levels of ability, with only a small minority achieving full marks.

In Q04(i) it is unfortunate that there was an omission of constraints given on the variables *p*, *q* and *r*, but this did not seem to cause problems for students, with very few (if any) cases of students not assuming they are all positive (as was the intention of the question).
Q04(a) was generally well done, although there were many students who did not use the initial conditions to find the constant of integration. This was perhaps due to the fact that there were already "unknown constants" in the equation, and so the presence of a constant of integration was just one more. The subtle difference of the role of the constants in the question and the role of the constant of integration (to be found in terms of the constants of in the question given the initial condition) was lost on these students. Many of the students who stopped at the general solution Q04(i) but who had a good attempt at Q04(ii) did attempt the constant in the second part.

The majority of students used an integrating factor method to answer this part of the question. A few did identify that the variables could be separated, and usually went on to successfully complete the integration, although for some the separation attempt was poor, yielding equations with a single term denominator. A very small number of students applied an auxiliary equation method (with successful outcome). For those using the integrating factor approach, most were successful in completing the first stage, although there were still a number of students who did not multiply the right-hand side by the integrating factor, or missed the variable *t* from the index of their term. There were also occasional errors in algebraic manipulation or mixing up the constants, with miscopying of the index between lines being a not infrequent error; students need to mind their *p*'s and *q*'s.

In Q04(b) the idea of a limiting value seemed to be well understood by the majority of students, along with the fact that expressions of the form e^{-kx} approach 0 as $x \to \infty$. Almost all students with a correct answer to Q04(i)(a) went on to gain this mark, even those who had not evaluated the constant of integration.

Q04(ii) was a discriminator for the paper.

The majority of the students used the intended integrating factor method, and almost all of these reached $y\mathrm{e}^{2\theta} = \int \mathrm{e}^{2\theta} \sin\theta \,\mathrm{d}\theta$. After this point the levels of progress made varied considerably, as this type of integral seemed to be beyond the scope of many students, although applications of integration by parts twice are within the C4 specification.

Question 4 continued

Of those who reached this stage, only a minority stopped and made no more progress, not even attempting the integral, while a similar number made incorrect approaches (eg substitutions which lead nowhere), or attempted a reduced simplicity integral (eg just integrating the sine term to yield $-e^{2\theta}\cos\theta$. The majority of students did, however, attempt integration by parts (not necessarily always correctly, with sign errors, or errors with the constant multiples occurring frequently) at least once.

For most, though, they stopped after the first application, not recognising where to go next, or they then regressed to an over simplification (similar to above). Of the minority who realised a second application of parts was necessary, a small number reversed the roles of the parts and so ended up back at the beginning, resulting in giving up or trying a different method, but most did apply the correct way a second time. For those doing so most did then recognise the original integral within their expression and proceed to replace by $ye^{2\theta}$, rearrange and hence find the expression. Of the students successfully completing the method, a few had made sign errors and ended up dividing through, for example, by 3 instead of 5, but most did achieve the correct expression and usually included a constant of integration at this stage. However, some divided through by the exponential before including the constant, and thus ended up with the incorrect answer. The inclusion, and attempt to find a constant of integration, was much more in evidence on this part than in Q04(i), even if incorrectly carried out.

Question 5

The great majority of students were able to access Q05(a) with success, those who chose to use the substitution $z - z^{-1} = 2\mathrm{i}\sin\theta$ were more successful, although a significant number omitted the i term. Those who used the $z = \cos\theta + \mathrm{i}\sin\theta$ substitution frequently failed to deal with the $\sin^3\theta$ term correctly. A very small number of students made use of either the $e^{i^{\theta}} - e^{-i^{\theta}}$ substitution or the compound angle approach.

Almost all students were able to deal with the integration in Q05(b) of the question successfully, dealing with the multiple angles and sign changes that were required. A surprising number of attempts failed to adequately show the correct use of the limits of $\frac{\pi}{3}$ and 0, given that the answer $\frac{53}{480}$ was given in the question, this omission cost them dearly.

Question 6

In Q06(a) was well answered by most students. The differentiation was done accurately in the main, up to the 3rd derivative, with a few variations in the route to this derivative, although there were some slips in the use of product and chain rule. Substitution was almost always correct, and substitution in the general series also well done. As usual, some students did not link the series to tan x, using an undefined f(x) = or y =.

Q06(b) was less well done. Although $\frac{5\pi}{12}$ was substituted in, it was often not shown or stated that it led to expressions in terms of $\frac{\pi}{6}$. Many others used $\frac{\pi}{6}$ without any justification. If students are asked to show a result, they need to support their working with sufficient evidence.

Question 7

Q07(a) was attempted by almost all students, with the majority showing confidence using the chain rule to express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}u}$ and $\frac{\mathrm{d}u}{\mathrm{d}x}$. The more successful students tended to use the method on the first page of the scheme (finding an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ then $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, then substituting these into the original equation). Problems arose in the application of the chain rule within the product rule in order to find the second derivative $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. It was not uncommon for students to use only the product rule, then on substituting into the equation realise a factor of e^{-2u} was missing and introduce it at this stage. Given that this was a proof, in order to obtain full marks students needed to fully demonstrate their method with no errors seen.

Students were very confident with Q07(b) and Q07(c) of this question, with almost all knowing that they needed to construct and solve the auxiliary quadratic, and choose the appropriate complementary function. Some students, having incorrectly solved the auxiliary equation and obtained a root of –2, then went on to use λe^{-2u} as a particular integral.
This is a method error and was not awarded the marks for this part of the solution. Those students who chose an appropriate for the PI tended to be successful in differentiating it twice and finding a correct value for λ. A mark was awarded to any student who put together their CF and PI in terms of u, provided this expression was identified as being y.
Students who had managed Q07(b) largely managed Q07(c) without any issues.

Question 8

The overwhelming majority of students achieved full marks for Q08(a) of this question, though there was a number who truncated the angle 0.7227 to 0.722 or rounded it to 2 significant figures and some who only gave the first quadrant answer for the angle.

Students however struggled with Q08(b); very few got the exact final answer and a relatively small number obtained the approximate decimal answer. Most students managed to integrate $49\cos^2\theta$ correctly. However, there were many students who expanded $\left(3+3\cos\theta\right)^2$ incorrectly, either forgetting to square the 3 or omitting the middle term. Those who took out a factor of 9 generally made fewer mistakes.

A number of students had a sign error in their trigonometric identity, losing all accuracy marks and many integration errors were caused by incorrect limits, or by substituting limits in incorrectly. A few students recognised that curve C_1 was a circle and were able to use the area formula; those who erroneously identified the curve C_2 between P and Q as an arc of a circle were completely unsuccessful.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London WC2R 0RL