Examiners' Report

Summer 2016

Pearson Edexcel GCE in Core Mathematics 4 (6666/01)

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Mathematics Unit Core Mathematics 4

Specification 6666/01

General Introduction

This paper proved accessible to all and discriminated well.

There were certain questions on this paper where some students lost marks. In Q04, some students obtained $\int x\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$ by splitting up the variables incorrectly and so lost seven marks for this question.

In Q06(i), some students lost 6 marks for not realising that they needed to split up $\frac{3y-4}{y(3y+2)}$ as partial fractions. In Q08(e) and Q08(f) some students did not draw a diagram to understand the problem and so potentially lost a number of marks.

The standard of algebra seen was generally good, although a number of students made basic sign, bracketing or manipulation errors in Q01, Q02(c), Q03, Q06, Q08(d) and Q08(f).

Report on Individual Questions

Question 1

Most students started by manipulating $\frac{1}{(2+5x)^3}$ to give $\frac{1}{8}\left(1+\frac{5x}{2}\right)^{-3}$, with the $\frac{1}{8}$ outside the brackets sometimes written incorrectly as either 2, 8 or $\frac{1}{2}$ and a few incorrectly used a power of 3. The majority used a correct method for expanding a binomial expression of the form $(1+ax)^n$. A variety of incorrect values of a, such as $\frac{2}{5}$, $\frac{5}{3}$, 5 or 1, were seen at this stage. Some students, who expanded $\frac{1}{(2+5x)^3}$ to give $1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+\ldots$, then forgot to multiply their expansion by $\frac{1}{8}$. Sign errors, bracketing errors and simplification errors were also seen in this question.

Question 2

The majority of students found Q02(a) and Q02(b) to be accessible and gained full marks. In Q02(a), it was extremely rare to see anything other than 0.6595 for the missing value, with occasionally a few students writing 0.6594. The application of the trapezium rule was usually correct and the final answer usually rounded to 3 decimal places. Errors seen included the use of an incorrect multiplier of $\frac{1}{5}$ or $\frac{1}{12}$ or a missing/extra term in the bracket. Some miscalculated their answer from a correctly written expression for the approximate area. It was rare for students to find the approximate area as a sum of separate trapezia.

In Q02(c) most students recognised that integration by parts was required and this was often completed correctly. Some students labelled u and $\frac{\mathrm{d}v}{\mathrm{d}x}$ the wrong way round and a common error was to integrate x^2 to give $2x$. Following the first stage of integration by parts, some students attempted to integrate $\frac{x^3}{3}.\frac{1}{x}$ using 'by parts' a second time rather than simplifying this expression first to give $\frac{x^2}{3}$. Other students incorrectly obtained $\frac{x^3}{6}$ after integrating $\frac{x^2}{3}$. Whilst the majority applied the limits of 2 and 1 correctly and subtracted the correct way round to give a correct exact answer of $\frac{8}{3}\ln 2 - \frac{7}{9}$, some used a lower limit of 0 or made bracketing or sign errors whilst others gave a decimal answer to varying degrees of accuracy including some who possibly believed that an answer of 1.070614704 was exact.

Question 3

In Q03(a), the trigonometric term in the implicit equation caused problems for a number of students. These students differentiated –cos (πy) with respect to x to give incorrect expressions such as $-\pi\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{1}{\pi}\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x}$, $\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pi\sin(\pi y)$. A minority did not apply the product rule correctly to differentiate $2x^2y$ and a small number left the constant term of 17 on the right hand side of their differentiated equation. Only about half of the students differentiated the equation correctly, factorised out $\frac{\mathrm{d}y}{\mathrm{d}x}$, and rearranged their resulting equation to obtain a correct expression for the gradient function.

In Q03(b), the majority of students demonstrated the complete method of finding the equation of the normal at $(3, \frac{1}{2})$ followed by substituting $y = 0$ and solving for x. A significant number of students made sign or manipulation errors when working with a rational fraction containing π, whilst others applied an incorrect method of finding the value of their normal gradient from their tangent gradient. Only a minority of students used correct algebra to achieve $x = \frac{3\pi + 62}{\pi + 22}$.

Question 4

This question on the topic of differential equations discriminated well across students of all abilities.

In Q04(a), the majority of students separated the variables correctly to give $\int \frac{1}{x}\mathrm{d}x = \int -\frac{5}{2}\mathrm{d}t$ and integrated both sides to give a correct equation containing a constant of integration. A minority who separated their variables to obtain either $\int x\,\mathrm{d}x = \int -\frac{5}{2}\mathrm{d}t$ or $\int \frac{2x}{5}\mathrm{d}x = \int -1\mathrm{d}t$ made little progress with the rest of the question. Although the majority of students used $t = 0$, $x = 60$ to find the value of their constant of integration, some wrote their final answer as $\ln x = -\frac{5}{2}t + \ln 60$ without progressing to make x the subject whilst others used incorrect algebraic manipulation to obtain a final answer of $x = \mathrm{e}^{-\frac{5}{2}t} + 60$.

Those students who obtained either $x = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln x = -\frac{5}{2}t + \ln 60$ in Q04(a) were often able to complete Q04(b) successfully, with most converting their time in days to a time in minutes, although some made rounding errors. Although most students substituted $x = 20$ into their Q04(a) equation there were a minority who substituted $x = 40$ after misunderstanding the question posed in Q04(b).

Question 5

In Q05(a), the majority of students used the correct method of parametric differentiation followed by the substitution of $t = \frac{\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Common errors in this part included simplifying a correct $\frac{\mathrm{d}x}{\mathrm{d}t} = 4\sec^2 t$ to give $\frac{1}{4\cos^2 t}$, substituting $t = \frac{\pi}{6}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$, or substituting both $t = \frac{\pi}{3}$ and $t = \frac{\pi}{6}$ to give two values for $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P.

In Q05(b), most students scored either two marks or no marks. A considerable number of students set $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero, produced correct algebra leading to $t = \frac{\pi}{4}$ and then substituted this back into $x = 4\tan t$ and $y = 5\sqrt{3}\sin 2t$ to find the correct coordinates of Q. Only a few students used the sketch given in the question to deduce that Q was the maximum turning point and so used $y = 5\sqrt{3}\sin 2t$ to write down $y_Q = 5\sqrt{3}$ followed by $t_Q = \frac{\pi}{4}$ and then $x_Q = 4$.

Question 6

In Q06(i), the majority of students identified the strategy of splitting up $\frac{3y-4}{y(3y+2)}$ as partial fractions and proceeded to complete this part correctly. Common errors for these students included either integrating $\frac{9}{3y+2}$ to give $9\ln(3y+2)$ or poor bracketing; eg writing $3\ln 3y+2$ instead of $3\ln(3y+2)$. Few students attempted to rewrite $\frac{3y-4}{y(3y+2)}$ as $\frac{3y}{y(3y+2)}-\frac{4}{y(3y+2)}$ and then simplified this further to give $\frac{3}{(3y+2)}-\frac{4}{y(3y+2)}$. A significant number of these students did not realise they needed to split $\frac{4}{y(3y+2)}$ into partial fractions. Some students assumed the integral was already in the form $\frac{\mathrm{f}'(y)}{\mathrm{f}(y)}$ and integrated $\frac{3y-4}{y(3y+2)}$ to give an answer of the form $\lambda\ln(3y^2+2y)$. Other erroneous methods included attempts at integration by parts and attempts at integrating expressions involving y as a constant outside the integral sign.

In Q06(ii)(a), many students started by either differentiating $x=4\sin^2\theta$ to give $\frac{\mathrm{d}x}{\mathrm{d}\theta}=8\sin\theta\cos\theta$ or by writing $4\sin^2\theta$ in terms of $\cos 2\theta$ and differentiating to give $\frac{\mathrm{d}x}{\mathrm{d}\theta}=4\sin 2\theta$. Many students substituted $x=4\sin^2\theta$ into $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\sqrt{\left(\frac{4\sin^2\theta}{4-4\sin^2\theta}\right)}$ and some used the trigonometric identity $1-\sin^2\theta=\cos^2\theta$ correctly in order to simplify this further to give either $\frac{\sin\theta}{\cos\theta}$ or $\tan\theta$. Some students failed to substitute for the $\mathrm{d}x$ part of the integral with some replacing '$\mathrm{d}x$' with '$\mathrm{d}\theta$'. Common errors in this part included simplifying $\sqrt{\left(\frac{4\sin^2\theta}{4-4\sin^2\theta}\right)}$ to give either $\frac{2\sin\theta}{2-2\sin\theta}$, $\frac{2\sin\theta}{\cos\theta}$ or $2\tan\theta$. Those students who simplified $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\frac{\sin\theta}{\cos\theta}$ or $\tan\theta$ and found $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ correctly usually proceeded to obtain the correct result. Some students lost the final mark in this part by not providing sufficient evidence to show the change in limits.

Question 6 continued

In Q06(ii)(b), the majority of students who attempted to integrate $\lambda \sin^2\theta$ realised the need for using $\cos 2\theta \equiv 1 - 2\sin^2\theta$. Whilst the double angle formula was generally quoted correctly, this did not always lead to a correct expression for integration as a result of sign or bracketing errors. A number of students, because of either incorrect integration or failure to use the correct value of λ, did not find the correct answer of $\frac{4\pi}{3} - \sqrt{3}$. Those students who gained 0 marks in Q06(ii)(b) usually integrated $\sin^2\theta$ to give expressions such as $\frac{\sin^3\theta}{3}$ or $\frac{\sin^3\theta}{3\cos\theta}$.

Question 7

In Q07(a), successful students either applied $\int (ax+b)^n \, \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)}$, or applied a substitution of $u = (2x-1)$. Many achieved an answer in the form $\lambda(2x-1)^{\frac{5}{2}}$, with some incorrectly giving λ as $\frac{2}{5}$, $\frac{4}{5}$ or 1 and others not simplifying $\frac{2}{10}$ to give $\frac{1}{5}$. There were a few students who differentiated $y = (2x-1)^{\frac{5}{2}}$ to give $\frac{\mathrm{d}y}{\mathrm{d}x} = 5(2x-1)^{\frac{3}{2}}$ who then proceeded to deduce the correct answer.

Many fully correct solutions to Q07(b) were seen, with the majority of students attempting to solve $(2k-1)^{\frac{3}{4}} = 8$. Two errors were commonly seen: the first was rearranging $(2k-1)^{\frac{3}{4}} = 8$ to give $k = \frac{1}{2}(8^{\frac{3}{4}} + 1)$ and the second was an error in the solution of $2k - 1 = 16$ to give $k = \frac{15}{2}$.

In Q07(c), the majority of students were able to apply the volume formula $\pi\int y^2 \, \mathrm{d}x$ correctly to give $\pi\int (2x-1)^{\frac{3}{2}} \mathrm{d}x$, although a few used incorrect formulae such as $2\pi\int y^2 \mathrm{d}x$, $\int y^2 \mathrm{d}x$ or even $\int y \, \mathrm{d}x$. Some students applied incorrect limits such as 0 and 8.5, 0 and 8 or 0.5 and 8 to their integrated expression. Many students applied correct limits of 0.5 and 8 to achieve $\frac{1024\pi}{5}$, but a significant number of them did not make any further progress. Of those who did, only a minority realised that they needed to find the volume of a cylinder with radius 8 and height 8.5 to give 544π, whilst some used a height of 8 and others tried to find the area of a rectangle of length 8.5 and width 8. Few students used the formula for the volume of a cone. Those students who found both 544π and $\frac{1024\pi}{5}$ usually proceeded to find the correct exact answer.

Question 8

This question on the topic of vectors discriminated well across students of all abilities. Students generally scored well on Q08(a) to Q08(d) with parts Q08(e) and Q08(f) proving to be effective discriminators.

In Q08(a), most students substituted $\mu = 1$ into l_1 to find the correct coordinates of A.

In Q08 (b), most students used the information given in the question to write down a correct equation for l_2, but some students were penalised for writing an expression for l_2 rather than an equation for l_2.

In Q08(c) most students used correct algebra to find an expression for $\overrightarrow{AP}$ and used Pythagoras to calculate the length of this vector. A small number, however, stopped after finding (the vector) $\overrightarrow{AP}$.

In Q08(d), the majority of students applied the dot product formula between $\overrightarrow{AP}$ and the direction vector of l_2 and found $\cos\theta = \frac{4}{5}$. Some students, however, who applied the dot product formula between $\overrightarrow{PA}$ and the direction vector of l_2 found $\cos\theta = -\frac{4}{5}$, but only a few then argued that $\cos\theta = \frac{4}{5}$ because the angle θ is acute. Other students lost the final mark in Q08(d) by finding θ as 36.87° without making reference to $\cos\theta = \frac{4}{5}$. A minority of students applied the dot product formula between a pair of non-relevant vectors which sometimes included $\overrightarrow{OA}$, $\overrightarrow{OB}$ or $8\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

Many students struggled to make progress in Q08(e) and Q08(f). Successful solutions almost always followed a good diagram of the situation and students need to be encouraged to set out the given information in such a form before attempting to write up their solution.

In Q08(e), the area of a triangle APE was usually found using $x = \frac{1}{2}ab\sin C$ with students using their answers to Q08(c) and Q08(d). Some students incorrectly assumed triangle APE was right-angled and applied the formula $\frac{1}{2}$(base)(height).

Q08(f) was often not attempted. Students who formed an appropriate equation in λ also usually knew how to use their values of λ to find the coordinates of the two possible positions of E. Some errors were made with the algebraic solution of their equation in λ and some arithmetical errors were made in calculating the possible coordinates of E.

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