

# Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level  
in Further Pure Mathematics  
(WFM01/01)

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Summer 2015

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# PEARSON EDEXCEL IAL MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\surd$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d... or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- $\square$  or d... The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

### **Method mark for solving 3 term quadratic:**

#### **1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$ , where  $|pq| = |c|$ , leading to  $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$

#### **2. Formula**

Attempt to use the correct formula (with values for a, b and c).

#### **3. Completing the square**

Solving  $x^2 + bx + c = 0$ :  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$

### **Method marks for differentiation and integration:**

#### **1. Differentiation**

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### **2. Integration**

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

### **Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

### **Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.



June 2015  
Further Pure Mathematics F1 WFM01  
Mark Scheme

Question Number	Scheme	Marks
<b>1.(a)</b>	$2z^3 - 5z^2 + 7z - 6 = (2z - 3)(z^2 + az + b)$	
	$a = -1$ and $b = 2$	B1: One of $a = -1$ or $b = 2$
		B1: Both $a = -1$ and $b = 2$
	Values may be implied by a correct quadratic e.g. sight of $z^2 - z + 2$	
		(2)
<b>(b)</b>	$z = 1\frac{1}{2}$	$z = 1.5$ or equivalent
		M1: Solves their 3 term quadratic (usual rules) as far as $z = \dots$
	$z = \frac{1}{2} \pm \left(\frac{1}{2}\sqrt{7}\right)i$	A1: Allow $z = \frac{1 \pm i\sqrt{7}}{2}$ or equivalent
		e.g. $z = \frac{1}{2} \pm \left(\sqrt{\frac{7}{4}}\right)i$
	<b>Answers must be exact and accept correct answers only for both marks. Answers that are not exact with no working score M0A0</b>	
		(3)
		<b>[5 marks]</b>

Question Number	Scheme		Marks
2.	$(3r - 2)^2 = 9r^2 - 12r + 4$	Correct expansion	B1
	$\sum_{r=1}^n (3r - 2)^2 = \sum_{r=1}^n 9r^2 - \sum_{r=1}^n 12r + \sum_{r=1}^n 4$ $= 9 \frac{n}{6} (n+1)(2n+1) - 12 \frac{n}{2} (n+1) + 4n$	B1ft: "4" = "4"n	B1ft M1
		M1: Uses valid formulae for sum of squares <b>and</b> sum of integers (their 9 or 12 may be followed through from their coefficients)	
	$= \frac{n}{2} (3(n+1)(2n+1) - 12(n+1) + 8)$ <p>or</p> $\frac{n}{6} (9(n+1)(2n+1) - 36(n+1) + 24)$	Takes out factor $\frac{n}{2}$ or $\frac{n}{6}$ . Dependent on the B1ft having been scored.	dM1
	$= \frac{n}{2} (6n^2 - 3n - 1)$	Correct result or states $a = 6$ , $b = -3$ , $c = -1$	A1
		(5)	
	<b>You should always award marks as in the scheme but generally there are no marks for proof by induction</b>		
			<b>[5 marks]</b>

Question Number	Scheme		Marks
<b>3.(a)</b>	$\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = 2$	Allow $\frac{4}{2}$ for 2	B1
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \left(\frac{7}{2}\right)^2 - 2(2) = \frac{33}{4}$	M1: Uses $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ A1: $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25	M1 A1
			(3)
<b>(b)</b>	Sum of roots is $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{33/4}{2} = \frac{33}{8}$ and product of roots is $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$	M1: Attempts sum <b>or</b> product of new roots correctly (may be implied) A1: Sum = $\frac{33}{8}$ and product = 1	M1 A1
	$x^2 - \frac{33}{8}x + 1 = 0 \therefore 8x^2 - 33x + 8 = 0$	$8x^2 - 33x + 8 = 0$ or any integer multiple including the “= 0”	A1
			(3)
			<b>[6 marks]</b>
<b>Alternative – finds roots explicitly:</b>			
<b>(a)</b>	$\alpha, \beta = \frac{1}{4}(7 \pm \sqrt{17})$	Correct exact roots including $\sqrt{17}$	B1
	$\alpha^2 + \beta^2 = 2\left(\frac{49}{16}\right) + 2\frac{17}{16} = 2 \times \frac{66}{16} = \frac{33}{4}$	M1: Squares and adds their roots A1: cao $\frac{33}{4}$ or $8\frac{1}{4}$ or 8.25	M1 A1
			(3)
<b>(b)</b>	$\left(x - \frac{7 + \sqrt{17}}{7 - \sqrt{17}}\right)\left(x - \frac{7 - \sqrt{17}}{7 + \sqrt{17}}\right) = \dots$	Uses $\left(x - \frac{\alpha}{\beta}\right)\left(x - \frac{\beta}{\alpha}\right)$ with numerical $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and attempts to expand. There are no marks until numerical values are used.	M1
	$= x^2 - \frac{33}{8}x + 1$		A1
	$8x^2 - 33x + 8 = 0$	This answer with no errors or any integer multiple including the “= 0”	A1cso
			(3)
	<b>Note:</b> <b>Roots of the form <math>\frac{1}{k}(7 \pm \sqrt{17})</math>, <math>k \neq 4</math> will give a correct answer – in this case lose the final mark as not cso.</b>		

Question Number	Scheme		Marks
<b>4.(a)</b>	$(PQ =) 13$	Sight of 13 (Must be seen in (a))	B1 (1)
<b>(b)</b>	$9 + a = 13 \Rightarrow a = \dots$ or $(9 \pm a)^2 + 36a = 169 \Rightarrow a = \dots$	M1: Uses $9 \pm a = 13$ or $(9 \pm a)^2 + 36a = 169$ to obtain a value for $a$ A1: $a = 4$ only	M1 A1 (2)
<b>(c)</b>	$y = 12$ Uses Area of triangle = $\frac{1}{2} \times 13 \times "y"$ or $\frac{1}{2} \times \begin{vmatrix} -4 & 9 & 4 & -4 \\ y & y & 0 & y \end{vmatrix}$ = 78	Correct $y$ coordinate of $P$ . A correct triangle area method cao	M1 M1 A1 (3)
	<p style="text-align: center;"><b>Alternative method for area of triangle using midpoint of QS (M)</b></p> <p style="text-align: center;"><math>\text{Area} = \frac{1}{2} \times QS \times MP = \frac{1}{2} \times \sqrt{208} \times \sqrt{117}</math></p> <p style="text-align: center;"><b>The method for QS and MP must be correct for their values</b></p> <p style="text-align: center;"><b>There are other methods for the area and the method should be correct for their values to score the M1 e.g. Box – Triangles = <math>156 - 48 - 30 = 78</math></b></p>		
			<b>[6 marks]</b>

Question Number	Scheme		Marks																		
<b>5.(a)</b>	$f(2) = \dots$ and $f(3) = \dots$	Attempts to evaluate both $f(2)$ and $f(3)$ (ignore use of degrees for this mark) <b>NB degrees usually scores M1A0M0A0</b> <b>NB <math>f(2) \approx 2</math> and <math>f(3) \approx -3</math> for degrees</b>	M1																		
	$f(2) = 2.3\dots$ , $f(3) = -1.4\dots$	Needs accuracy to 1 figure truncated or rounded	A1																		
	$f(2.5) = 0.5\dots$ and $f(2.75) = -0.4\dots$	Evaluates both $f(2.5)$ and $f(2.75)$ (and not $f(2.25)$ )	M1																		
	$(2.5, 2.75)$	$2.5 \leq x \leq 2.75$ or $2.5 < x < 2.75$ or $2.5 \leq \alpha \leq 2.75$ or $2.5 < \alpha < 2.75$ or $[2.5, 2.75]$ or $(2.5, 2.75)$ or equivalent in words. Allow a mixture of 'ends' but not incorrect statements such as $2.75 < x < 2.5$ . Needs accuracy to 1 figure truncated or rounded for $f(2.5)$ and $f(2.75)$ and conclusion	A1																		
			(4)																		
<p><b>Note that some candidates only indicate the sign of f not its value. In this case the M's can still score as defined but not the A's. However if f(2) and f(3) are correctly evaluated in (b) then the first A1 can be given retrospectively.</b></p>																					
<p align="center"><b>Common Approach in the form of a table:</b></p> <table border="1"> <thead> <tr> <th><math>a</math></th> <th><math>f(a)</math></th> <th><math>b</math></th> <th><math>f(b)</math></th> <th><math>\frac{a+b}{2}</math></th> <th><math>f\left(\frac{a+b}{2}\right)</math></th> </tr> </thead> <tbody> <tr> <td>2</td> <td>2.31576...</td> <td>3</td> <td>-1.428...</td> <td>2.5</td> <td>0.5151...</td> </tr> <tr> <td>2.5</td> <td>0.5151...</td> <td>3</td> <td>-1.428...</td> <td>2.75</td> <td>-0.4472...</td> </tr> </tbody> </table> <p align="center"><math>2.5 &lt; \alpha &lt; 2.75</math></p> <p align="center"><b>Would score full marks in (a)</b></p>			$a$	$f(a)$	$b$	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$	2	2.31576...	3	-1.428...	2.5	0.5151...	2.5	0.5151...	3	-1.428...	2.75	-0.4472...	
$a$	$f(a)$	$b$	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$																
2	2.31576...	3	-1.428...	2.5	0.5151...																
2.5	0.5151...	3	-1.428...	2.75	-0.4472...																
<b>(b)</b>	$\frac{\alpha - 2}{2.3158} = \frac{3 - \alpha}{1.4280}$ or $\frac{\alpha - 2}{2.3158} = \frac{3 - 2}{3.7438}$	Correct equation involving $\alpha$ or $x$ and <b>their</b> values even in degrees. Use of negative lengths scores M0	M1																		
	$\alpha(1.4280 + 2.3158) = 3 \times 2.3158 + 2 \times 1.4280$ so $\alpha = \dots$	Makes $\alpha$ or $x$ the subject. Dependent on the previous M but condone poor algebra.	dM1																		
	$(\alpha =) 2.62$	cao and cso (Allow $x =$ )	A1																		
	<b>A correct statement followed by 2.62 scores 3/3</b>																				
				(3)																	
	<b>Using <math>y = mx + c</math>:</b>																				
$m = f(2) - f(3) = -3.74\dots$ $c = f(2) - 2m = 9.80\dots$	Correct method to find equation of straight line	M1																			
$y = 0 \Rightarrow x = \dots$	Substitutes $y = 0$ and makes $x$ or $\alpha$ the subject. Dependent on the previous M	dM1																			
$(\alpha =) 2.62$	cao and cso (Allow $x =$ )	A1																			
<p><b>Also allow candidates to find the value of e.g <math>3 - \alpha</math> or <math>\alpha - 2</math> and then add to 2 or subtract from 3: M1 for a correct method for <math>3 - \alpha</math> or <math>\alpha - 2</math>, dM1 for adding to 2 or subtracting from 3 and A1 for 2.62 cao and cso.</b></p>																					
			[7 marks]																		

Question Number	Scheme		Marks
<b>6(a)</b>	Gradient of $PQ$ is $\frac{\frac{6}{p} - \frac{6}{q}}{6p - 6q} \left( = -\frac{1}{pq} \right)$	Correct gradient in any form	B1
	Equation of $PQ$ is $y - \frac{6}{q} = \frac{\frac{6}{p} - \frac{6}{q}}{6p - 6q} (x - 6q)$	M1: Uses straight line equation in any form correctly for their gradient or uses $y = mx + c$ and attempts to find $c$ in terms of $p$ and $q$	M1 A1
		A1: Correct line in any form	
	$y - \frac{6}{q} = \frac{-1}{pq} (x - 6q)$		
	$pq(y - \frac{6}{q}) = -(x - 6q) \Rightarrow pqy + x = 6(p + q)^*$	Cso. Reaches the given answer with at least one intermediate step.	A1*
			(4)
<b>Alternative simultaneous equations</b>			
	$\frac{6}{p} = m(6p) + c, \quad \frac{6}{q} = m(6q) + c$	Correct equations	B1
	$m = \frac{q - p}{pq(p - q)}, \quad c = \frac{6}{p} + \frac{6}{q}$	M1: Solves simultaneously to obtain <b>either</b> “ $m$ ” or “ $c$ ” in terms of $p$ and $q$	M1A1
		A1: $m = \frac{q - p}{pq(p - q)}$ <b>and</b> $c = \frac{6}{p} + \frac{6}{q}$	
	$pqy + x = 6(p + q)^*$	Cso. Reaches the given answer with at least one intermediate step.	A1*
<b>(b)</b>	$y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -36x^{-2}$ and uses $x = 6r$	$\frac{dy}{dx} = kx^{-2}$ and uses $x = 6r$	M1
	$x \frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ and use $x = 6r, y = 6/r$	$\frac{dy}{dx} = k \frac{y}{x}$ and uses $x = 6r, y = 6/r$	
	$\frac{dx}{dr} = 6 \Rightarrow \frac{dy}{dr} = -\frac{6}{r^2}$ and uses $\frac{dy}{dx} = \frac{dy}{dr} \div \frac{dx}{dr}$	$\frac{dx}{dr} = k \Rightarrow \frac{dy}{dr} = \frac{k}{r^2}$ and uses	
		$\frac{dy}{dx} = \frac{dy}{dr} \div \frac{dx}{dr}$	
	So at $R$ gradient of curve = $-\frac{1}{r^2}$	Allow any un-simplified correct form e.g. $-36(6r)^{-2}, -\frac{6}{6r}, -\frac{6}{r^2} \div 6$	A1
	So gradient of normal = $r^2$	Correct use of perpendicular gradient rule	M1
	$-\frac{1}{pr} \times -\frac{1}{qr} = -1$	M1: Uses gradient $PR$ perpendicular to gradient $QR$	M1 A1
		A1: Correct equation connecting $p, q$ and $r$	
	So $r^2 = \frac{-1}{pq}$ which is the gradient of $PQ$ so the normal at $R$ is <b>parallel</b> to $PQ$	Conclusion with all previous marks scored. <b>Must see the word ‘parallel’ used.</b>	A1cso
			(6)
			[10 marks]

Question Number	Scheme		Marks
<b>7.(a)</b>	$ z  = k\sqrt{13}$	Accept $\sqrt{13k^2}$ but not $\sqrt{9k^2 + 4k^2}$	B1
	$\arg z = \pi + \arctan\left(\frac{2}{3}\right) = \pi + 0.588$ $= 3.73 \text{ or } -2.55$	M1: Uses $\arctan\left(\pm\frac{2}{3}\right)$ ( $\pm 0.588^\circ \dots / \pm 33.6^\circ \dots$ ) or $\arctan\left(\pm\frac{3}{2}\right)$ ( $\pm 0.98^\circ \dots / \pm 56.3^\circ \dots$ )	M1 A1
		A1: 3.73 or -2.55 only	
			(3)
<b>(b)(i)</b>	$\frac{4}{z+3k} = \frac{4}{-2ki} = \frac{2}{k}i$	M1: Substitutes $z$ and multiplies numerator and denominator by conjugate of denominator or equivalent	M1 A1
		A1: $\frac{2}{k}i$ oe (Allow un-simplified e.g. $\frac{8k}{4k^2}i$ ). Allow $0 + \frac{2}{k}i$	
<b>(ii)</b>	$z^2 = (-3k - 2ki)(-3k - 2ki) = 9k^2 + 12ik^2 + 4i^2k^2$	Multiplies out obtaining 3 term quadratic in $i$	M1
	$= 5k^2 + 12k^2i$	M1: Uses $i^2 = -1$ (may be implied) A1: cao	M1A1
			(5)
<b>(c)</b>		Plots $z$ in 3 <sup>rd</sup> quadrant and $z^*$ as mirror image in 2 <sup>nd</sup> quadrant and both correctly labelled	B1
		Plots a complex number on positive imaginary axis and correctly labelled	B1
		Plots and labels D in the first quadrant, positioned correctly relative to the other points and further from the origin than all the other points.	B1
		<p><b>Notes:</b></p> 1. Penalise the omission of labels once and penalise it the first time it occurs. 2. For labels allow letters, in terms of $z$ , coordinates or labels on axes. 3. If there are separate Argand Diagrams, imagine them superimposed. 4. Accept points, lines or arrows.	(3)
			(3)
			<b>[11 marks]</b>

Question Number	Scheme		Marks
8.(a)	$\mathbf{P}^{-1} = \frac{1}{25a^2} \begin{pmatrix} 3a & 4a \\ -4a & 3a \end{pmatrix} \text{ or } \frac{1}{25a} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$	M1: Switches signs on minor diagonal	M1 B1 A1
		B1: Correct determinant. Allow simplified or un-simplified e.g. $3a(3a)-(-4a)(4a)$ , score when first seen.	
		A1: Completely correct inverse with determinant simplified.	
			(3)
(b)	$\frac{1}{25a} \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} -3a & 6a & -20a \\ -4a & 8a & 15a \end{pmatrix}$	Sets up correct multiplication including $\frac{1}{25a}$ or equivalent	M1
	$= \begin{pmatrix} -1 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$	Correct matrix	A1
	(-1,0), (2,0) and (0,5)	Follow through their matrix but must be written as coordinates	A1ft
(c)	Area of triangle $T_1 = \frac{1}{2} \times 3 \times 5$ o.e. $\left(\frac{15}{2}\right)$	Correct area for triangle $T_1$ .	M1
	Area scale factor is $25a^2$ so Area of triangle $T_2 = \frac{15}{2} \times 25a^2 = 187.5a^2$ oe	M1: Multiplies their area of $T_1$ by their det $\mathbf{P}$ to find required area A1: cao	M1A1
			(3)
<b>Alternative 1: Shoelace method</b>			
	area $T_2 = \frac{1}{2} \times \begin{vmatrix} -3a & 6a & -20a & -3a \\ -4a & 8a & 15a & -4a \end{vmatrix}$	Correct statement.	M1
	$\frac{1}{2} \times \begin{vmatrix} -3a \times 8a + (6a \times 15a) + (-20a \times -4a) \\ -\{(-4a \times 6a) + (-8a \times 20a) + (15a \times -3a)\} \end{vmatrix}$	Correct calculation	M1
	$= 187.5a^2$ oe	cao	A1
<b>Alternative 2: Encloses <math>T_2</math> by a rectangle and subtracts triangles:</b>			
	Rectangle area = $494a^2$ and <b>one</b> triangle area of $161.5a^2, 91a^2$ or $54a^2$	Correct values	M1
	$494a^2 - 161.5a^2 - 91a^2 - 54a^2$	Complete method for area	M1
	$= 187.5a^2$ oe	cao	A1
(d)	$\mathbf{Q} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$	M1: $\begin{pmatrix} \frac{3}{5} & \alpha \\ \beta & \frac{3}{5} \end{pmatrix}$ $\frac{3}{5}$ in both entries of main diagonal and $\alpha \neq 0$ and $\beta \neq 0$	M1A1
		A1: Correct matrix	
			(2)
(e)	$\mathbf{R} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 3a & -4a \\ 4a & 3a \end{pmatrix} = \begin{pmatrix} 5a & 0 \\ 0 & 5a \end{pmatrix} \text{ oe}$	M1: Sets up correct multiplication in correct order. "Their $\mathbf{Q}$ " $\times$ $\mathbf{P}$	M1 A1
		A1: cao	
			(2)
			<b>[13 marks]</b>



Question Number	Scheme	Marks	
<b>9.(i)</b>	If $n = 1$ , $\sum_{r=1}^n r^2(2r-1) = 1$ and $\frac{1}{6}n(n+1)(3n^2+n-1) = 1$ , <b>LHS=RHS</b> so true for $n = 1$ .	B1	
	$\sum_{r=1}^{k+1} r^2(2r-1) = \frac{1}{6}k(k+1)(3k^2+k-1) + (k+1)^2(2(k+1)-1)$ (Adds the $(k+1)^{\text{th}}$ term to the sum of the first $k$ terms)	M1	
	$= \frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$	dM1: Attempt factor of $\frac{1}{6}(k+1)$ A1: $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$	dM1A1
	$= \frac{1}{6}(k+1)(k+2)(3k^2+7k+3) = \frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$		A1
	Achieves this result with no errors and $3k^2+7k+3$ seen <b>Allow work that shows equivalence between</b> e.g. $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ and $\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$		
	<b>True for <math>n = k + 1</math> if true for <math>n = k</math>, and as true for <math>n = 1</math> true by induction for all <math>n</math>.</b>	A1cso	
	Full conclusion and all previous marks scored	(6)	
<b>(ii)</b>	$n = 1: \begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^1 = \begin{pmatrix} 6+1 & -12 \\ 3 & 1-6 \end{pmatrix}$ so true for $n = 1$	Shows true for $n = 1$	B1
	$\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^{k+1} = \begin{pmatrix} 6k+1 & -12k \\ 3k & 1-6k \end{pmatrix} \begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix} \text{ or } \begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} 6k+1 & -12k \\ 3k & 1-6k \end{pmatrix}$		M1
	Either statement scores M1		
	$\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ or e.g. $\begin{pmatrix} 7(6k+1)+3(-12k) & -12(6k+1)+(-12k)(-5) \\ 3k(7)+3(1-6k) & -12(3k)+(1-6k)(-5) \end{pmatrix}$	M1: Correct attempt at multiplication (if unclear, at least 2 terms must be correct)	M1A1
		A1: Correct matrix possibly un-simplified	
	If the previous A1 was awarded for $\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ then allow the next A mark for the matrix as shown. If the previous A1 was awarded for e.g. $\begin{pmatrix} 7(6k+1)+3(-12k) & -12(6k+1)+(-12k)(-5) \\ 3k(7)+3(1-6k) & -12(3k)+(1-6k)(-5) \end{pmatrix}$ then this must be simplified to $\begin{pmatrix} 6k+7 & -12k-12 \\ 3k+3 & -6k-5 \end{pmatrix}$ before the next A mark can be awarded.		
	$\begin{pmatrix} 6(k+1)+1 & -12(k+1) \\ 3(k+1) & 1-6(k+1) \end{pmatrix}$	States or shows by <b>equivalence</b> that the result is true for $n = k + 1$	A1
	<b>True for <math>n = k + 1</math> if true for <math>n = k</math>, and as true for <math>n = 1</math> true by induction for all <math>n</math>.</b>	A1	
	Full conclusion and all previous marks scored	(6)	
		<b>[12 marks]</b>	

