

Mark Scheme (Results)

January 2014

Pearson Edexcel International
Advanced Level

Further Pure Mathematics 1
(6667A/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Notes	Marks
<p>1.</p> <p>(a)</p> <p>(b)</p>	<p>$f(x) = 2x - 5\cos x$, x measured in radians</p> <p>$f(1) = -0.7015115293\dots$</p> <p>$f(1.4) = 1.950164285\dots$</p> <p>Sign change (and $f(x)$ is continuous) therefore a root α exists between $x = 1$ and $x = 1.4$</p> <p>$f(1.2) = 0.5882112276\dots \Rightarrow 1 \leq \alpha \leq 1.2$</p> <p>$f(1.1) = -0.06798060713\dots$ $\Rightarrow 1.1 \leq \alpha \leq 1.2$</p>	<p>Either any one of $f(1) = \text{awrt } -0.7$ or $f(1.4) = 1.9$ or awrt 2.0</p> <p>both values correct, sign change and conclusion</p> <p>$f(1.2) = \text{awrt } 0.6$</p> <p>Attempt to find $f(1.1)$</p> <p>$f(1.1) = -0.06$ or awrt -0.07 with $1.1 \leq \alpha \leq 1.2$ or $1.1 < \alpha < 1.2$ or $[1.1, 1.2]$ or $(1.1, 1.2)$.</p>	<p>M1</p> <p>A1</p> <p>[2]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>5</p>

Question Number	Scheme	Notes	Marks
<p>2.</p> <p>(i)</p> <p>(ii)</p>	<p>$\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}$, where k is a constant</p> <p>$\det \mathbf{A} = (-4)(k) - (-3)(10)$ $\Rightarrow -4k + 30 = 2$ or $-4k + 30 = -2$ $\Rightarrow k = 7$ or $k = 8$</p> <p>$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$</p> <p>$\mathbf{BC} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}$</p>	<p>Applies "$ad \pm bc$" to \mathbf{A}</p> <p>Equates their $\det \mathbf{A}$ to either 2 or -2</p> <p>Either $k = 8$ or $k = 7$</p> <p>Both $k = 8$ and $k = 7$</p> <p>Writes down a complete 2×2 matrix.</p> <p>Any 3 out of 4 elements correct</p> <p>Correct answer.</p>	<p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p>7</p>

Question Number	Scheme	Notes	Marks
3.	$x = 2t, y = \frac{2}{t}, t \neq 0$ $t = \frac{1}{2} \Rightarrow P(1, 4), t = 4 \Rightarrow Q\left(8, \frac{1}{2}\right)$ $m(PQ) = \frac{\frac{1}{2} - 4}{8 - 1} \left\{ = -\frac{1}{2} \right\}$ $m(L) = 2$ <p>So, $L: y = 2x$</p>	<p>Coordinates for either P or Q are correctly stated. (Can be implied).</p> <p>An attempt to find the gradient of the chord PQ.</p> <p>Applying $m(L) = \frac{-1}{\text{their } m(PQ)}$ $y = 2x$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1 oe</p> <p style="text-align: right;">[4] 4</p>

Question Number	Scheme	Notes	Marks
4.	$f(x) = 2\sqrt{x} - \frac{6}{x^2} - 3, \quad x > 0$ $f'(x) = x^{-\frac{1}{2}} + 12x^{-3} \quad \{+ 0\}$ $f(3.5) = 0.2518614684\dots$ $\{f'(3.5) = 0.8144058657\dots\}$ $\beta = 3.5 - \left(\frac{"0.2518614684\dots"}{"0.8144058657\dots"} \right)$ $= 3.190742075\dots$ $= 3.191 \text{ (3dp)}$	$\pm \lambda x^{-\frac{1}{2}} \text{ or } \pm \mu x^{-3}$ <p>Correct differentiation $f(3.5) = \text{awrt } 0.25$</p> <p>Correct application of Newton-Raphson using their values.</p> <p style="text-align: right;">3.191</p>	<p>M1 A1 B1</p> <p>M1</p> <p>A1 cao</p> <p style="text-align: right;">[5] 5</p>

Question Number	Scheme	Notes	Marks
<p>5.</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$z = 5 + i\sqrt{3}, \quad w = \sqrt{3} - i$ $ w = \left\{ \sqrt{(\sqrt{3})^2 + (-1)^2} \right\} = 2$ $zw = (5 + i\sqrt{3})(\sqrt{3} - i)$ $= 5\sqrt{3} - 5i + 3i + \sqrt{3}$ $= 6\sqrt{3} - 2i$ $\frac{z}{w} = \frac{(5 + i\sqrt{3})}{(\sqrt{3} - i)} \times \frac{(\sqrt{3} + i)}{(\sqrt{3} + i)}$ $= \frac{5\sqrt{3} + 5i + 3i - \sqrt{3}}{3 + 1}$ $\left\{ = \frac{4\sqrt{3} + 8i}{4} \right\} = \sqrt{3} + 2i$ $z + \lambda = 5 + i\sqrt{3} + \lambda = (5 + \lambda) + i\sqrt{3}$ $\left\{ \arg(z + \lambda) = \frac{\pi}{3} \Rightarrow \right\} \frac{\sqrt{3}}{5 + \lambda} = \tan\left(\frac{\pi}{3}\right)$ $\left\{ \frac{\sqrt{3}}{5 + \lambda} = \frac{\sqrt{3}}{1} \Rightarrow 5 + \lambda = 1 \Rightarrow \right\} \lambda = -4$	<p>2</p> <p>Either the real or imaginary part is correct. $6\sqrt{3} - 2i$</p> <p>Multiplies by $\frac{(\sqrt{3} + i)}{(\sqrt{3} + i)}$</p> <p>Simplifies realising that a real number is needed on the denominator and applies $i^2 = -1$ on their numerator expression and denominator expression.</p> <p>$\sqrt{3} + 2i$</p> <p>$\frac{\sqrt{3}}{\text{their combined real part}} = \tan\left(\frac{\pi}{3}\right)$</p> <p>-4</p>	<p>B1</p> <p>[1]</p> <p>M1 A1</p> <p>[2]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1 oe</p> <p>A1</p> <p>[2]</p> <p>8</p>

Question Number	Scheme	Notes	Marks
6. (a)	$\sum_{r=1}^n r(r+1)(r-1) = \sum_{r=1}^n (r^3 - r)$ $= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$ $= \frac{1}{4}n(n+1)(n(n+1) - 2)$ $= \frac{1}{4}n(n+1)(n^2 + n - 2)$ $= \frac{1}{4}n(n+1)(n-1)(n+2)$	<p>An attempt to use at least one of the standard formulae correctly. Correct expression.</p> <p>An attempt to factorise out at least $n(n+1)$.</p> <p>Achieves the correct answer. (Note: $a = 2$).</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>
(b)	$\sum_{r=1}^n r(r+1)(r-1) = 10 \sum_{r=1}^n r^2$ $\frac{1}{4}n(n+1)(n-1)(n+2) = \frac{10}{6}n(n+1)(2n+1)$ $\frac{1}{4}(n-1)(n+2) = \frac{5}{3}(2n+1)$ $3(n^2 + n - 2) = 20(2n + 1)$ $3n^2 - 37n - 26 = 0$ $(3n + 2)(n - 13) = 0$ $n = 13$	<p>Sets their part (a) = $\frac{10}{6}n(n+1)(2n+1)$</p> <p>Manipulates to a "3TQ = 0". $3n^2 - 37n - 26 = 0$</p> <p>A valid method for factorising a 3TQ. Only one solution of $n = 13$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p> <p>9</p>

Question Number	Scheme	Notes	Marks
<p>7.</p> <p>(a)</p> <p>(b)</p>	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$ $\mathbf{P}^{-1} = \frac{1}{4ab} \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix}$ <p>M = PQ</p> $\Rightarrow \mathbf{P}^{-1}\mathbf{M} = \mathbf{P}^{-1}\mathbf{P}\mathbf{Q} \Rightarrow \mathbf{Q} = \mathbf{P}^{-1}\mathbf{M}$ $\mathbf{Q} = \frac{1}{4ab} \begin{pmatrix} 2b & 2a \\ b & 3a \end{pmatrix} \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$ $= \frac{1}{4ab} \begin{pmatrix} -8ab & 12ab \\ 0 & 4ab \end{pmatrix}$ $= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$	<p style="text-align: right;">$\frac{1}{4ab}$</p> <p>Two out of four elements correct. Correct matrix.</p> <p>Multiples their \mathbf{P}^{-1} by \mathbf{M}</p> <p>Two out of four elements correct. Correct matrix.</p>	<p>B1; M1 A1</p> <p>[3]</p> <p>M1</p> <p>A1 A1</p> <p>[3] 6</p>

Question Number	Scheme	Notes	Marks
<p>8.</p> <p>(a)</p>	<p>$y^2 = 4ax$, at $P(ap^2, 2ap)$.</p> <p>$y = 2\sqrt{a}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}x^{-\frac{1}{2}}$</p> <p>or (implicitly) $2y\frac{dy}{dx} = 4a$</p> <p>or (chain rule) $\frac{dy}{dx} = 2a \times \frac{1}{2ap}$</p> <p>When $x = ap^2$, $\frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{ap^2}} = \frac{\sqrt{a}}{\sqrt{a}p} = \frac{1}{p}$</p> <p>or $\frac{dy}{dx} = \frac{4a}{2(2ap)} = \frac{1}{p}$</p> <p>So $m_N = -p$</p> <p>N: $y - 2ap = -p(x - ap^2)$</p> <p>N: $y - 2ap = -px + ap^3$</p> <p>N: $y + px = ap^3 + 2ap$</p> <p>(b)</p> <p>$(6a, 0) \Rightarrow 0 + p(6a) = ap^3 + 2ap$</p> <p>$\Rightarrow 4ap = ap^3 \Rightarrow p = 2$</p> <p>$x = -a, p = 2 \Rightarrow y + 2(-a) = a(2)^3 + 2a(2)$</p> <p>$\Rightarrow y = 8a + 4a + 2a = 14a \Rightarrow D(-a, 14a)$</p> <p>(c)</p> <p>When $p = 2$, $x = a(2)^2 = 4a$</p> <p>Area(XPD) = $\frac{1}{2}(14a)(5a) = 35a^2$</p>	<p>$\frac{dy}{dx} = \pm kx^{-\frac{1}{2}}$</p> <p>or $k y \frac{dy}{dx} = c$</p> <p>or $\frac{\text{their } \frac{dy}{dx}}{\text{their } \frac{dx}{dx}}$</p> <p>$\frac{dy}{dx} = \frac{1}{p}$</p> <p>Applies $m_N = \frac{-1}{\text{their } m_T}$</p> <p>Applies $y - 2ap = (\text{their } m_N)(x - ap^2)$</p> <p>Correct solution.</p> <p>Substitutes $x = 6a, y = 0$ into N</p> <p>$p = 2$</p> <p>Substitutes $x = -a$ and their p into N</p> <p>$D(-a, 14a)$</p> <p>Substitutes their p into $x = ap^2$</p> <p>Applies $\frac{1}{2}(\text{their } 14a)(\text{their "4a" + a})$</p> <p>$35a^2$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 cs *</p> <p>[5]</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>12</p>

Question Number	Scheme	Notes	Marks
9.	$(3 - i)z^* + 2iz = 9 - i$ $(3 - i)(x - iy) + 2i(x + iy) = 9 - i$ $3x - 3iy - ix - y + 2ix - 2y \{= 9 - i\}$ Re part: $3x - y - 2y = 9$ Im part: $-3y - x + 2x = -1$ $3x - 3y = 9$ $x - 3y = -1$ $2x = 10 \Rightarrow x = 5$ $x - 3y = -1 \Rightarrow 5 - 3y = -1 \Rightarrow y = 2$ $\{z = 5 + 2i\}$	Substituting $z = x + iy$ and $z^* = x - iy$ into $(3 - i)z^* + 2iz = 9 - i$ Multiplies out $(3 - i)(x - iy)$ correctly. This mark can be implied by correct later working. Equating either real or imaginary parts. One set of correct equations. Correct equations. Attempt to solve simultaneous equations to find one of x or y . Either $x = 5$ or $y = 2$. Both $x = 5$ and $y = 2$.	M1 A1 M1 A1 A1 ddM1 A1 A1 [8] 8

Question Number	Scheme	Notes	Marks
10. (i)	<p>$u_{n+1} = 5u_n + 3$, $u_1 = 3$ and $u_n = \frac{3}{4}(5^n - 1)$</p> <p>$n = 1$; $u_1 = \frac{3}{4}(5^1 - 1) = \frac{3}{4}(4) = 3$</p> <p>So u_n is true when $n = 1$.</p> <p>Assume that for $n = k$ that, $u_k = \frac{3}{4}(5^k - 1)$ is true for $k \in \mathbb{Z}^+$.</p> <p>Then $u_{k+1} = 5u_k + 3$</p> $= 5\left(\frac{3}{4}(5^k - 1)\right) + 3$ $= \frac{3}{4}(5)^{k+1} - \frac{15}{4} + 3$ $= \frac{3}{4}(5)^{k+1} - \frac{3}{4}$ $= \frac{3}{4}(5^{k+1} - 1)$ <p>Therefore, the general statement, $u_n = \frac{3}{4}(5^n - 1)$ is true when $n = k + 1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction</p>	<p>Check that $u_n = \frac{3}{4}(5^n - 1)$ yields 3 when $n = 1$.</p> <p>Substituting $u_k = \frac{3}{4}(5^k - 1)$ into $u_{k+1} = 5u_k + 3$</p> <p>An attempt to multiply out in order to achieve $\pm \lambda(5^{k+1}) \pm \text{constant}$</p> <p>$\frac{3}{4}(5^{k+1} - 1)$</p> <p>True when $n = k + 1$, then by induction the result is true for all positive integers.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[5]</p>

Question Number	Scheme	Notes	Marks
10. (ii)	<p>$f(n) = 5(5^n) - 4n - 5$ is divisible by 16</p> <p>$f(1) = 5(5^1) - 4(1) - 5 = 16$, { which is divisible by 16 }. { $\therefore f(n)$ is divisible by 16 when $n = 1$. }</p> <p>Assume that for $n = k$,</p> <p>$f(k) = 5(5^k) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+$.</p> <p>$f(k+1) - f(k) = 5(5^{k+1}) - 4(k+1) - 5 - (5(5^k) - 4k - 5)$</p> $= 5(5^{k+1}) - 4k - 4 - 5 - 5(5^k) + 4k + 5$ $= 25(5^k) - 4k - 4 - 5 - 5(5^k) + 4k + 5$ $= 20(5^k) - 4$ $= 4(5(5^k) - 4k - 5) + 16k + 20 - 4$ $= 4(5(5^k) - 4k - 5) + 16k + 16$ $= 4f(k) + 16(k+1)$ <p>$\therefore f(k+1) = 5f(k) + 16(k+1)$</p> <p>{ $\therefore f(k+1) = 5f(k) + 16(k+1)$, which is divisible by 16 as both $5f(k)$ and $16(k+1)$ are both divisible by 16. }</p> <p>If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.</p>	<p>Shows that $f(1) = 16$</p> <p>Applies $f(k+1) - f(k)$. Correct expression for $f(k+1) - f(k)$.</p> <p>Achieves an expression in 5^k.</p> <p>$f(k+1) = 5f(k) + 16(k+1)$</p> <p>Correct conclusion</p>	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>A1 cso</p> <p>[6] 11</p>

Appendix

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
- ft denotes “follow through”
- cao denotes “correct answer only”
- oe denotes “or equivalent”

Other Possible Solutions

Question Number	Scheme	Notes	Marks
7.	$\mathbf{P} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix}$		
<i>Aliter</i>	$\mathbf{M} = \mathbf{PQ}$		
(b)	$\begin{pmatrix} -6a & 7a \\ 2b & -b \end{pmatrix} = \begin{pmatrix} 3a & -2a \\ -b & 2b \end{pmatrix} \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix}$		
Way 2	$\begin{aligned} -6 &= 3q_1 - 2q_3 & 7 &= 3q_2 - 2q_4 \\ 2 &= -q_1 + 2q_3 & -1 &= -q_2 + 2q_4 \end{aligned}$ $= \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix}$	<p>Writes down a relevant pair of simultaneous equations.</p> <p>Can be implied by later working.</p> <p>Two out of four elements correct.</p> <p>Correct matrix.</p>	<p>M1</p> <p>A1</p> <p>A1</p>
			[3]

Question Number	Scheme	Notes	Marks
<p><i>Aliter</i></p> <p>10. (ii)</p> <p>Way 2</p>	<p>$f(n) = 5(5^n) - 4n - 5$ is divisible by 16</p> <p>$f(1) = 5(5^1) - 4(1) - 5 = 16,$ { which is divisible by 16 }. { $\therefore f(n)$ is divisible by 16 when $n = 1.$ }</p> <p>Assume that for $n = k,$ $f(k) = 5(5^k) - 4k - 5$ is divisible by 16 for $k \in \mathbb{Z}^+.$</p> <p>$f(k+1) = 5(5^{k+1}) - 4(k+1) - 5$ $= 25(5^k) - 4k - 9$ $= 5(5(5^k) - 4k - 5) + 20k + 25 - 4k - 9$ $= 5(5(5^k) - 4k - 5) + 16(k+1)$ $\therefore f(k+1) = 5f(k) + 16(k+1)$</p> <p>{ $\therefore f(k+1) = 5f(k) + 16(k+1),$ which is divisible by 16 as both $5f(k)$ and $16(k+1)$ are both divisible by 16. }</p> <p>If the result is true for $n = k,$ then it is now true for $n = k+1.$ As the result has shown to be true for $n = 1,$ then the result is true for all $n.$</p>	<p>Shows that $f(1) = 16$</p> <p>Applies $f(k+1).$ Correct expression for $f(k+1).$ Achieves an expression in $5^k.$</p> <p>$f(k+1) = 5f(k) + 16(k+1)$</p> <p>Correct conclusion</p>	<p>B1</p> <p>M1 A1 M1</p> <p>A1</p> <p>A1 cso</p> <p style="text-align: right;">[6]</p>

