Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 30 January 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 0 0 8 6 A 0 1 2 4

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PEARSON

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1. Given that $z_1 = 1 - \mathrm{i}$,

(a) find $\arg(z_1)$.

(2)

Given also that $z_2 = 3 + 4\mathrm{i}$, find, in the form $a + \mathrm{i}b$, $a, b \in \mathbb{R}$,

(b) $z_1 z_2$,

(2)

(c) $\dfrac{z_2}{z_1}$.

(3)

In part (b) and part (c) you must show all your working clearly.

Leave blank

Question 1 continued

Q1

(Total 7 marks)

P 4 0 0 8 6 A 0 3 2 4

Turn over

Leave blank

2. (a) Show that $f(x) = x^4 + x - 1$ has a real root α in the interval $[0.5, 1.0]$.

(2)

(b) Starting with the interval $[0.5, 1.0]$, use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(c) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

Leave blank

Question 2 continued

Q2

(Total 10 marks)

P 4 0 0 8 6 A 0 5 2 4

Turn over

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3. A parabola C has cartesian equation $y^2 = 16x$. The point $P\left(4t^2, 8t\right)$ is a general point on C.

(a) Write down the coordinates of the focus F and the equation of the directrix of C.

(3)

(b) Show that the equation of the normal to C at P is $y + tx = 8t + 4t^3$.

(5)

P 4 0 0 8 6 A 0 6 2 4

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(a) Find the coordinates of the vertices of T'.

(2)

(b) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(c) Find **QR**.

(2)

(d) Find the determinant of **QR**.

(2)

(e) Using your answer to part (d), find the area of T''.

(3)

P 4 0 0 8 6 A 0 8 2 4

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Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

P 4 0 0 8 6 A 0 1 1 2 4

Turn over

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5. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3.

(a) Given that $z_1 = 3 + \mathrm{i}$, find z_2 and z_3.

(4)

(b) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

Leave blank

Question 5 continued

Q5

(Total 6 marks)

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6. (a) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the result in part (a), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8)$$

(3)

(c) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

P 4 0 0 8 6 A 0 1 4 2 4

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

Leave blank

7. A sequence can be described by the recurrence formula

$$u_{n+1} = 2u_n + 1, \qquad n \geqslant 1, \quad u_1 = 1$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that $u_n = 2^n - 1$

(5)

P 4 0 0 8 6 A 0 1 8 2 4

Leave blank

Question 7 continued

Q7

(Total 7 marks)

P 4 0 0 8 6 A 0 1 9 2 4

Turn over

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8.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Show that **A** is non-singular.

(2)

(b) Find **B** such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

Leave blank

Question 8 continued

Q8

(Total 6 marks)

P 4 0 0 8 6 A 0 2 1 2 4

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9. The rectangular hyperbola H has cartesian equation $xy = 9$

The points $P\left(3p, \frac{3}{p}\right)$ and $Q\left(3q, \frac{3}{q}\right)$ lie on H, where $p \neq \pm q$.

(a) Show that the equation of the tangent at P is $x + p^2 y = 6p$.

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at the point P and the tangent at the point Q intersect at R.

(c) Find, as single fractions in their simplest form, the coordinates of R in terms of p and q.

(4)

Leave blank

Question 9 continued

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Question 9 continued

Q9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

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