Mark Scheme (Results)

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GCE Statistics S3 (6691) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

   - **B** marks are unconditional accuracy marks (independent of M marks)

   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt

   - ft – follow through

   - the symbol   will be used for correct ft

   - cao – correct answer only

   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark

   - isw – ignore subsequent working

   - awrt – answers which round to

   - SC: special case

   - oe – or equivalent (and appropriate)

   - dep – dependent

   - indep – independent

   - **dp decimal places**

   - **sf   significant figures**

   - ✱   The answer is printed on the paper

   - ⌐ The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
1.	$X_1, X_2, ... X_n$ is a random sample of size n , for large n, drawn from a population of any distribution with mean μ and variance σ^2 then $\bar{X}$ is (approximately) $N\left(\mu, \dfrac{\sigma^2}{n}\right)$	B1 B1 B1 (3) **3**
	1st B for large sample or equivalent 2nd B for 'population of any distribution' or 'any population' 3rd B require mean or symbol and normal (parameters not required)	

Question Number	Scheme	Marks
2. **(a)**		

Town	A	B	C	D	E	F	G
h rank	1	5	2	3	7	4	6
c rank	4	3	2	1	6	7	5
$\lvert d \rvert$	3	2	0	2	1	3	1
d^2	9	4	0	4	1	9	1

M1
M1

$$\sum d^2 = 28$$

M1A1

$$r_s = 1 - \frac{6 \times 28}{7 \times 48}$$

M1

$$= 0.5$$

A1

(6)

(b)

$\mathrm{H_0}: \rho = 0, \mathrm{H_1}: \rho \neq 0$ B1

Critical values are $r_s = \pm 0.7857$ B1ft

0.5<0.7857 insufficient evidence to reject $\mathrm{H_0}$ M1

Councillor's claim is supported.   A1ft

(4)

10

Question Number	Scheme	Marks

Notes
(a)
	1st M1 for an attempt to rank the hardship against calls
	
	2nd M1 for attempting d for their ranks. Must be using ranks.
	3rd M1 for attempting $\sum d^2$ (must be using ranks)
	1st A1 for sum of 28 (or 84)
	4th M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.
	2nd A1 for awrt 0.5 (or -0.5)
	
	1st B1 for both hypotheses in terms of ρ, H_1 must be two tail.
	2nd B1 for cv of ±0.7857 (or 0.7143 to ft from 1-tailed H_1)
	
	M1 for a correct statement relating their r_s with their cv but cv must be such that |cv|<1
	A1ft for a correct contextualised comment. Must mention "Councillor" and "claim" <u>or</u> "hardship" and "number of calls (to the emergency services)"
	
	       Follow through their r_s and their cv (provided it is |cv| <1
	       Condone use of "association" in conclusion for A1
	       Condone 'positive' in conclusion.	

Note: the **(b)** label appears aligned with the row for the hypotheses/cv/A1ft notes.

Question Number	Scheme	Marks
3.		

Defect Type Shift	D_1	D_2		
First Shift	47.25	15.75	63	
Second Shift	56.25	18.75	75	
Third Shift	46.5	15.5	62	
	150	50	200	M1A1

H_0 : Type of defect is independent of Shift (no association)

H_1 : Type of defect is not independent of Shift (association)   B1

O	E	$\dfrac{(O-E)^2}{E}$	$\dfrac{O_i^2}{E_i}$	
45	47.25	0.1071…	42.857...	
18	15.75	0.3214…	20.571..	
55	56.25	0.02777…	53.777...	
20	18.75	0.0833…	21.333...	
50	46.5	0.2634…	53.763...	
12	15.5	0.7903…	9.290...	M1A1

$\dfrac{(O-E)^2}{E} = 1.5934..$ or $\dfrac{O_i^2}{E_i}$ -200=201.5934-200=1.5934..   awrt1.59   A1

$\nu = (3-1)(2-1) = 2$ B1

$\chi_2^2(0.10) = 4.605$ B1ft

1.59<4.605 so insufficient evidence to reject H_0 M1

Insufficient evidence to support manager's belief /claim.   A1

10

Question Number	Scheme	Marks
Notes	1st M1 for some use of $\dfrac{\text{Row Total}\times\text{Col.Total}}{\text{Grand Total}}$ May be implied by correct E_i 1st A1 for all expected frequencies correct B1 for both hypotheses. Must mention "defect" and "shift" at least once Use of "relationship" or "correlation" or "connection" is B0 2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i 2nd A1 for all correct terms. May be implied by a correct answer.(2 dp or better-allow eg 0.10…) 3rd M1 for a correct statement linking their test statistic and their cv . Must be χ^2 not normal. 4th A1 for a correct comment in context - must mention "manager's belief" or "shift" and "defect type" - condone "relationship" or "connection" here but **not** "correlation". No follow through e.g. "There is evidence of a relationship between shift and type of defect"	

Question Number	Scheme	Marks
4. **(a)**	$\bar{x} = \dfrac{5320}{80} = 66.5$	M1,A1
	$s^2 = \dfrac{392000 - 80 \times (66.5)^2}{79}$	M1A1ft
	$= 483.797\ldots$ awrt 484	A1
		(5)
(b)	$H_0: \mu_m = \mu_{nm}, \quad H_1: \mu_m > \mu_{nm}$ (accept μ_1, μ_2 with definition)	B1B1
	$z = \dfrac{69.0 - 66.5}{\sqrt{\dfrac{483.797}{80} + \dfrac{446.44}{60}}}$	M1dM1
	$= 0.6807$ awrt 0.681	A1
	One tailed cv 1.6449 (Probability is awrt 0.752)	B1
	$0.6807 < 1.6449$ (or $0.248 > 0.05$) insufficient evidence to reject H_0	dM1
	Mean money spent is not greater with music playing.	A1ft
		(8)
		13

Question Number	Scheme	Marks
(b)	**Notes** No definition award B1B0. 1st M1 for attempt at s.e. - condone one number wrong or switched 60 & 80 . 2nd dM1 for using their s.e. in correct formula for test statistic. 3rd dM1 **dep. on 2nd M1** for a correct statement based on their normal cv and their test statistic 2nd A1 for correct comment in context. Must mention "money spent" and "music playing". Allow ft. Critical Region for (b) Standard error x z value for 2nd M1 Standard error x 1.6449= awrt 6.04 for 1st A1 2.5<6.04	

Question Number	Scheme	Marks
5. **(a)**	Hurricanes: occur singly / are independent or occur at random /are a rare event / at a constant rate	B1B1 (2)
(b)	From data $\dfrac{1\times2+2\times5+3\times17+..+7\times12}{80}=4.4875$	M1A1 (2)

(c)

No of hurricanes, h	0	1	2	3	4	5	6	7+
$80P(X=h)$	0.9	4..038	**$r=9.06...$**	13.55	**$s=15.205$**	13.647...	10.206...	13.388...
Combine to give expected frequencies >5	13.9991			13.55	15.205...	13.647...	10.206...	13.388...
Observed	7			17	20	12	12	12
$\dfrac{(O-E)^2}{E}$	3.499...			0.876...	1.511...	0.198...	0.315...	0.143...
$\dfrac{O_i^2}{E_i}$	3.500...			21.322...	26.306...	10.551...	14.108...	10.755..

M1A1A1 (3)

(d) M1

H_0: Poisson distribution is a good fit o.e.
H_1: Poisson distribution is not a good fit o.e.   B1

$\displaystyle\sum \frac{(O_i-E_i)^2}{E_i}=6.545..$ <u>or</u> $\dfrac{O_i^2}{E_i}=86.545-80=6.545..$ (awrt **6.55 or 6.54**)   A1

$v=6-2=4$ B1

cv is 9.488   (ft their v i.e. $\chi_v^2(0.05)$)   B1ft

6.545<9.488 so insufficient evidence to reject H_0

(Hurricanes) can be modelled by a Poisson distribution   A1 (6)

13

Question Number	Scheme	Marks
	Notes	
(b)	M for at least 2 terms on numerator. 359/80 only award M0A0	
(c)	M for 80xPoisson probability with 4.4875 and either 2 or 4.	
	1st A1 for awrt 9.06 and 2nd A1 for awrt 15.20 or 15.21	
(d)	1st M1 for some pooling and attempting $\dfrac{(O-E)^2}{E}$ or $\dfrac{O^2}{E}$, at least 3 correct expressions or values.	
	1st B1 no value for parameter permitted	
	2nd A1 for a correct comment suggesting that Poisson model is suitable. No ft	

Question Number	Scheme	Marks
6. **(a)**	$L = A_1 + A_2 + \ldots + A_6$ Mean is $\mathrm{E}(L) = 6 \times 20 = 120$ Standard deviation is $\sqrt{\mathrm{Var}(W)} = \sqrt{6 \times 5^2} = 5\sqrt{6} = 12.247\ldots$ awrt 12.2	B1 B1 (2)
(b)	$P(L > 110) \quad = P\left(Z > \left(\dfrac{110 - 120}{12.247\ldots}\right)\right)$ $ = P(Z < 0.8164\ldots)$ $ = 0.7939$ (or 0.7929 using interpolation or 0.79289 by calc)	M1 A1 (2)
(c)	Let $X = 4B - \sum_1^6 A_i$ $\mathrm{E}(X) = 140 - 120 = 20$ $\mathrm{Var}(X) = 16 \times 8^2 + 6 \times 5^2 = 1174$ $\mathrm{P}(X < 0) = \mathrm{P}\left(Z < \dfrac{-20}{\sqrt{1174}}\right) = \mathrm{P}(Z < -0.583\ldots)$ $\phantom{\mathrm{P}(X < 0)} = 0.2797 \ \left(\text{or } 0.2810 \text{ if no interpolation}\right) \text{ or } 0.27971 \text{ by calc.}$	B1 M1M1A1 M1 A1 (6) **10**

Question Number	Scheme	Marks
	Notes	
(b)	M1 for identifying a correct probability (they must have the 110) and attempting to standardise with their mean and sd. This can be implied by the correct answer.	
	A1 for awrt 0.794 or 0.793	
(c)	Accept ±20 for B mark. Only award for probability statement if 2 terms in var	
	1st M1 for 1024, 2nd M1 for 150	
	3rd M for standardising with their mean and 2 term sd and finding probability <0.5	
	2nd A1 for awrt 0.280 or 0.281	

Question Number	Scheme	Marks
7.		
(a)	$H_0:\mu=250$, $H_1:\mu<250$,	B1
	$z = \dfrac{248-250}{\dfrac{5.4}{\sqrt{90}}}$	M1
	$= -3.513...$ awrt - 3.51	A1
	Critical value -1.6449	B1
	-3.513..<-1.6449 so sufficient evidence to reject H_0	
	Manager's claim is justified.	A1
		(5)
(b)	98% CI for μ is	
	$248 \pm 2.3263 \times \dfrac{5.4}{\sqrt{90}}$	M1B1
	$=$ awrt (247,249) dependent upon z value awrt 2.33	A1A1
		(4)
(c)	Hypothesis test is significant or CI does not contain stated weight.	B1
	(Manager should ask the company to investigate if their) stated weight is too high o.e.	B1
		(2)
(d)	$P(|\bar{x}-\mu|<1)=0.98$	
	$\dfrac{1}{\dfrac{3}{\sqrt{n}}} = 2.3263$	M1 A1
	$n = (3 \times 2.3263)^2 = 48.7...$	dM1A1
	Sample size 49 required.	A1
		(5)
		16

Question Number	Scheme	Marks
	Notes	
(a)	1st B1 for H_0 and for H_1 (must be <250) They must use μ not x, p, λ or $\bar{x}$ etc	
	1st M1 for attempt at standardising using 248, 250 and sd. Can accept $\pm$.	
	Critical region: $250-0.936=249.064$ for M1A1 (and compare with 248.)	
	3rd B1 for ±1.6449 seen (or probability of 0.0002 or better)	
	2nd A1 for a correct contextualised comment. Must mention "Manager" and "claim" or "weight" and "stated weight". No follow through.	
(b)	2.3263 or better for B mark. Any z value replacing 2.3263 award M.	
(d)	1st M for LHS = z value >1	
	1st A for RHS awrt 2.33	
	2nd A1 for answers in the range 48.7-48.9	
	3rd A1 don't condone $\geq$	

Ofqual

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