Mark Scheme (Results)
January 2010

GCE

Further Pure Mathematics FP1 (6674)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

A PEARSON COMPANY

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

Ask The Expert can be accessed online at the following link:

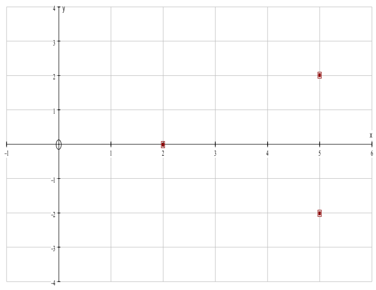
http://www.edexcel.com/Aboutus/contact-us/

January 2010

January 2010
6674 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Marks
Q1	(a) $\frac{a+\mathrm{i}b}{3-\mathrm{i}} \times \frac{3+\mathrm{i}}{3+\mathrm{i}} = \frac{(3a-b)+\mathrm{i}(a+3b)}{10}$ (A1 for numerator, A1 for 10)	M1 A1 A1 (3)
	(b) $\lvert z_1 \rvert = \sqrt{a^2+(-2a)^2} = \sqrt{5a^2} = a\sqrt{5}$ (*)	M1 A1 (2)
	(c) $\arg\frac{z_1}{z_2} = \arctan\frac{a+3b}{3a-b} = \arctan(\text{-}1), \quad = -\frac{\pi}{4} \quad \left(\text{or } \frac{7\pi}{4}, \text{ or } -45°, \text{ or } 315°\right)$	M1 A1ft, A1 (3)
		[8]
	(c) The <u>final</u> A1 requires a single answer, so for example: $\arctan(\text{-}1) = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$ is A0	

Question Number	Scheme	Marks
Q2	(a) $\mathrm{f}(2) = 2\cos 2 - 4 + 5 \qquad (= 0.1677...)$	
	$\mathrm{f}(2.1) = 2.1\cos 2.1 - 4.2 + 5 \qquad (= -0.2601...)$	M1
	Values correct (to 1 s.f.). Change of sign $\Rightarrow$ Root	A1 (2)
	(b) $\mathrm{f}'(x) = \cos x - x\sin x - 2$	M1 A1
	$x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} = 2 - \dfrac{0.1677}{-4.2347}, \qquad = 2.04$	M1 A1, A1 (5)
	(c) $\mathrm{f}(2.035) = ...$ and $\mathrm{f}(2.045) = ...$	M1
	$0.0189...$ and $-0.0238...$ Change of sign $\Rightarrow$ Correct to 2 d.p.	A1 (2) **[9]**
	(c) The M1 is also given for evaluating f at the ends of a 'tighter' interval.	

Question Number	Scheme	Marks
Q3	(a) $5-2\text{i}$ is a root	B1 (1)
	(b) $\left(x-(5+2\text{i})\right)\left(x-(5-2\text{i})\right)=x^2-10x+29$ $x^3-12x^2+cx+d=\left(x^2-10x+29\right)\left(x-2\right)$ $c=49,\qquad d=-58$	M1 M1 M1 A1, A1 (5)
	(c)  Conjugate pair in 1st and 4th quadrants, (symmetrical about real axis). Fully correct, labelled.	B1 B1 (2) **[8]**
	(b) 1st M: Form brackets using $(x-\alpha)(x-\beta)$ and expand. 2nd M: Achieve a 3-term quadratic with no i's. (b) Alternative: Substitute a root (usually $5+2\text{i}$) and expand brackets $(5+2\text{i})^3-12(5+2\text{i})^2+c(5+2\text{i})+d=0$ $(125+150\text{i}-60-8\text{i})-12(25+20\text{i}-4)+(5c+2c\text{i})+d=0$ (2nd M for achieving an expression with no powers of i) Equate real and imaginary parts $c=49,\qquad d=-58$	 M1 M1 M1 A1, A1

Question Number	Scheme	Marks
Q4	$m^2+6m+9=0 \qquad m=-3$	B1
	C.F. $(x=)\ (At+B)e^{-3t}$	M1 A1
	P.I. $x=p\cos t+q\sin t$	B1
	$\dfrac{dx}{dt}=-p\sin t+q\cos t \qquad \dfrac{d^2x}{dt^2}=-p\cos t-q\sin t$	M1
	$-p\cos t-q\sin t-6p\sin t+6q\cos t+9p\cos t+9q\sin t=5\cos t$	M1
	$-6p+8q=0$ and $8p+6q=5$	A1
	Solve simultaneously to find either p or q:	M1
	$p=\dfrac{2}{5}$ and $q=\dfrac{3}{10}$	A1
	General solution: $(x=)\quad (At+B)e^{-3t}+\dfrac{2}{5}\cos t+\dfrac{3}{10}\sin t$	A1ft
		[10]
	The final A1ft is dependent on the 3 preceding M marks (for the P.I.)	

Question Number	Scheme	Marks
Q5	(a) $1 = A(2r+5) + B(2r+1)$, and find values of A and B	M1
	$\dfrac{1}{4(2r+1)} - \dfrac{1}{4(2r+5)}$	A1 (2)
	(b) $r = 1: \quad \dfrac{1}{4}\left(\dfrac{1}{3} - \dfrac{1}{7}\right)$	B1ft
	$r = 2: \quad \dfrac{1}{4}\left(\dfrac{1}{5} - \dfrac{1}{9}\right)$	
	$r = n-1: \quad \dfrac{1}{4}\left(\dfrac{1}{2n-1} - \dfrac{1}{2n+3}\right)$	
	$r = n: \quad \dfrac{1}{4}\left(\dfrac{1}{2n+1} - \dfrac{1}{2n+5}\right)$	
	Sum: $\dfrac{1}{4}\left(\dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{2n+3} - \dfrac{1}{2n+5}\right)$	M1 A1
	$= \dfrac{1}{4}\left(\dfrac{8(2n+3)(2n+5) - 15(2n+5) - 15(2n+3)}{15(2n+3)(2n+5)}\right)$	M1
	$= \dfrac{1}{4}\left(\dfrac{32n^2 + 128n + 120 - 30n - 75 - 30n - 45)}{15(2n+3)(2n+5)}\right)$	A1
	$= \dfrac{1}{4}\left(\dfrac{32n^2 + 68n}{15(2n+3)(2n+5)}\right)$	
	$= \dfrac{n(8n+17)}{15(2n+3)(2n+5)} \qquad (c = 17)$	A1 (6)
		[8]
	(b) B1ft for one correct difference (ft A and B). M1 A1 M1 A1: The $\dfrac{1}{4}$ is not needed for these marks, only for the final A1.	

Question Number	Scheme	Marks
Q6	(a) $r = \frac{a}{2}$ $\quad \sin 2\theta = \frac{1}{2}$ $\quad 2\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}$	M1 A1, A1 (3)
	(b) $\sin^2 2\theta = \frac{1}{2}(1-\cos 4\theta)$	B1
	$\frac{a^2}{2}\int \sin^2 2\theta \, d\theta =$	M1
	$\pm\left[\theta - \frac{\sin 4\theta}{4}\right]$ (Correct integration of $\pm(1-\cos 4\theta)$)	A1
	$\left[\text{ ....}\right]_{\pi/12}^{5\pi/12} = \text{.......}, = \frac{a^2}{4}\left(\frac{5\pi}{12} - \left(-\frac{\sqrt{3}}{8}\right) - \frac{\pi}{12} + \frac{\sqrt{3}}{8}\right) = a^2\left(\frac{\pi}{12}, +\frac{\sqrt{3}}{16}\right)$	M1, A1, A1 (6)
		[9]
	(b) 1st M: Use of $\frac{1}{2}\int r^2 \, d\theta$ with some integration attempt. 2nd M: Correct use of their limits. N.B. Other methods are possible, e.g. $\left(\text{e.g. } 2\left[\text{ ....}\right]_{"\pi/12"}^{\pi/4}\right)$ Slips such as omitting the a or not squaring the a: just the final A mark is lost.	

Question Number	Scheme	Marks
Q7	(a) $10+3x-x^2=3x-1$ $x^2=\ldots$ or $x=\ldots$, $\sqrt{11}$ (β)	M1, A1
	$10+3x-x^2=1-3x$	M1
	$x^2-6x-9=0$ $x=\dfrac{6\pm\sqrt{72}}{2}$ or equiv.	A1
	$3-3\sqrt{2}$ or exact equiv. (α)	A1 (5)
	(b) $3-3\sqrt{2}<x<\sqrt{11}$	M1 A1ft (2)
	(c) Forming inequalities using all their four x values $\left(\pm\sqrt{11}\text{ and } 3\pm3\sqrt{2}\right)$	M1
	$-\sqrt{11}<x<3-3\sqrt{2}$, $\sqrt{11}<x<3+3\sqrt{2}$	B1, B1 (3)
		[10]
	Answers with decimals (3 s.f. accuracy) are acceptable in (b) and (c). (b) M: Answer including $x<\beta$ (positive β) or $x>\alpha$ (negative α). A1ft requires negative α and positive β.	

Question Number	Scheme	Marks
Q8	(a) $z=\frac{1}{y^2}$ $\qquad \frac{dz}{dx}=-\frac{2}{y^3}\frac{dy}{dx}$	B1
	$-\frac{y^3}{2}\frac{dz}{dx}+y=4xy^3$	M1
	$-\frac{y^2}{2}\frac{dz}{dx}+1=4xy^2 \qquad -\frac{1}{2z}\frac{dz}{dx}+1=\frac{4x}{z}, \qquad \frac{dz}{dx}-2z=-8x$ (*)	M1, A1 (4)
	(b) Integrating factor $e^{\int -2dx}=e^{-2x}$	B1
	$ze^{-2x}=-8\int xe^{-2x}\,dx$ or $\frac{d}{dx}\left(ze^{-2x}\right)=-8xe^{-2x}$	M1
	$\int xe^{-2x}\,dx=\left\{\frac{xe^{-2x}}{-2}+\frac{1}{2}\int e^{-2x}\,dx\right\}$	M1 A1
	$ze^{-2x}=4xe^{-2x}+2e^{-2x}+C, \qquad z=4x+2+Ce^{2x}$	M1, dM1
	(The second of these M marks is dependent on the first, and both are dependent on the use of an integrating factor).	
	$y=\frac{1}{\sqrt{4x+2+Ce^{2x}}}$ (or equiv.)	A1 (7)
	(c) $\frac{dy}{dx}=0: \quad y=4xy^3 \qquad y=\frac{1}{2\sqrt{x}}$ (*)	M1 A1 (2)
		[13]
	(b) Alternative for first 6 marks: C.F. $z=Ce^{2x}$ — B1 P.I. $z=px+q, \quad \frac{dz}{dx}=p$ — M1 $p-2px-2q=-8x$ — M1 $p=4 \quad q=2$ — M1 A1 $z=4x+2+Ce^{2x}$ — M1	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

January 2010

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH