Examiners' Report Summer 2009

GCE

GCE Mathematics (8371/8374,9371/9374)

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Contents

6663 Core Mathematics C1

Introduction

This proved to be a little harder than previous papers and this was reflected in the grade boundaries. Some candidates struggled with simple numerical tasks from addition and subtraction to the manipulation of surds, and centres should ensure that candidates are prepared to carry out these basic numerical calculations without the use of a calculator.

There were a larger number of cases of candidates using calculators in this examination this year and centres are reminded of the need for careful supervision as the use of a calculator on this paper has serious repercussions for the candidates.

Report on individual questions

Question 1

Some candidates could not square the surd terms correctly but nearly everyone attempted this question and most scored something.

In part (a) some failed to square the 3 and an answer of 21 was fairly common, others realised that the expression equalled 9×7 but then gave the answer as 56. A few misread the question and proceeded to expand $\left(3+\sqrt{7}\right)^2$. In part (b) most scored a mark for attempting to expand the brackets but some struggled here occasionally adding 8 + 2 instead of multiplying. Those with a correct expansion sometimes lost marks for careless errors, $-8\sqrt{5}+2\sqrt{5}=6\sqrt{5}$, and a small number showed how fragile their understanding of these mathematical quantities was by falsely simplifying a correct answer of $11-6\sqrt{5}$ to $5\sqrt{5}$.

Question 2

Many candidates could not deal with this test of indices. Two simple properties of indices were required: that a square root leads to a power of $\frac{1}{2}$ and the rule for adding the powers when multiplying. Those who identified these usually made good progress but the remainder struggled. Some re-wrote $32\sqrt{2}$ as $\sqrt{64}$ and then obtained a = 3 whilst others did obtain $\sqrt{2048}$ but usually failed to identify 2048 as 2^{11}. A number of candidates tried to use logs but this approach was rarely successful.

Question 3

This question was answered very well with most candidates knowing and applying the rules for differentiation and integration correctly although the use of the notation for this topic, especially the integration sign, is still poor.

Most differentiated $2x^3$ correctly in part (a) and although many wrote $\dfrac{3}{x^2}$ as $3x^{-2}$, some still thought the derivative was $-6x^{-1}$ or $+6x^{-3}$. Similar problems arose with the integration in part (b) and some lost marks through failing to simplify their expressions and of course others forgot the +c.

Question 4

Part (a) provided a simple start for the majority of the candidates and apart from a few arithmetic errors most scored full marks.

In part (b) the quadratic expression was factorised and the critical values were usually found correctly however many candidates were unable to identify the solution as a closed region. Many just left their answer as $x < -\frac{3}{2}$ and $x < 4$, others chose the outside regions and some just stopped after finding the critical values. Candidates who successfully answered part (b) often answered part (c) correctly as well although some repeated their previous working to achieve this result.

The use of a sketch in part (b) and a number line in part (c) was effective and is a highly recommended strategy for questions of this type.

Question 5

Many candidates did not seem to realise that this question was about Arithmetic Series initially and started with a simple arithmetic approach to obtain a difference of 60. When it came to part (b) they often did start to use arithmetic series formulae but confusion over the value of n and the sign of their 60 meant that an incorrect value for a was common. In part (c) most realised that the formula for S_{40} was required and their values were often substituted correctly.

Question 6

Some candidates opted out of this question but most realised that they needed to use the discriminant and made some progress. The most common error was to simplify $(3p)^2$ to $3p^2$ but there were many correct equations seen and usually these led to a correct answer. Many candidates chose to solve their equation by cancelling a p. In this case that was fine, since they were told that p was non-zero, but this is not good practice in general and factorising $9p^2 - 4p$ to $p(9p-4)$ is recommended.

Question 7

There were far fewer cases of candidates not understanding how an inductive formula like this works and many were able to answer parts (a) and (b) successfully. Part (b) required the candidates to "show" a given result and most gave the expression $2(2k-7)-7$ which was fine but a small minority thought the pattern must be $2 \times 2k - 3 \times 7$. Part (c) met with mixed success: many found a_4 but some then solved $a_4 = 43$ whilst others assumed that the series was arithmetic and attempted to use a formula such as $\frac{4}{2}(a_1 + a_4)$. Those who did attempt the correct sum occasionally floundered with the arithmetic but there were plenty of fully correct solutions seen.

Question 8

Most candidates had clearly learnt the coordinate geometry formulae and were able to give a correct expression for the gradient of AB although some had x and y the wrong way round. The perpendicular gradient rule was well known too and the majority of candidates used this successfully to find the gradient of l. Many went on to find a correct expression for the equation of l (although some used the point B here instead of A) but the final mark in part (a) was often lost as candidates struggled to write their equation in the required form. In part (b) most substituted x =0 into their equation and the examiners followed through their working for the coordinates of C, only a few used $y = 0$ here.

Part (c) caused the usual problems and a variety of approaches (many unsuccessful) were tried. Those who identified OC as the base and 8 as the height usually had little problem in gaining the marks. Some candidates felt uneasy using a height that wasn't a side of their triangle and split the triangle into two then adding the areas, others used a trapezium minus a triangle or a determinant approach. A few attempted to find OB and BC using Pythagoras' theorem in the vain hope of using the $\frac{1}{2}ab\sin C$ formula.

Question 9

Most candidates made a good attempt at expanding the brackets but some struggled with $-4\sqrt{x} \times -4\sqrt{x}$ with answers such as $-4\sqrt{x}$ or $\pm 16\sqrt{x}$ or $\pm 16x^{\frac{1}{4}}$ being quite common. The next challenge was the division by $\sqrt{x}$ and some thought that $\frac{x}{\sqrt{x}} = 1$. Many, but not all, who had difficulties in establishing the first part made use of the given expression and there were plenty of good attempts at differentiating. Inevitably some did not interpret $f'(x)$ correctly and a few attempted to integrate but with a follow through mark here many scored all 3 marks. In part (c) the candidates were expected to evaluate $9^{-\frac{3}{2}}$ or $9^{-\frac{1}{2}}$ correctly and then combine the fractions - two significant challenges but many completed both tasks very efficiently.

Question 10

It seemed clear that many candidates did not appreciate the links between the 3 parts of this question and there was much unnecessary work carried out: differentiating to find turning points and tables of values to help draw, not sketch, the curves.

Part (a) was usually answered well but not all the successful candidates started by taking out the factor of x, rather they tried to use the factor theorem to establish a first factor. Whilst techniques from C2 (or higher units) **may** be used in C1 they are not required and the "best" approach will not use them.

A correct factorisation in (a) should have made the sketch in (b) straightforward. Most drew a cubic curve (but some had a negative cubic not a positive one) and usually their curve either touched or passed through the origin. The most common non-cubic curve was a parabola passing through (0, 0) and (3,0). Part (c) looked complicated but those who spotted that they were sketching f(x - 2) had few problems in securing both marks. Many candidates though embarked upon half a page or more of algebraic manipulation to no avail - this part was only worth 2 marks and a little thought may have helped them realise that such an approach was unlikely to be the correct one.

Question 11

A number of partial attempts at this question may suggest that some were short of time although the final part was quite challenging.

Most secured the mark in part (a) although careless evaluation of $2\times(2)^2$ as 6 spoiled it for some. Apart from the few who did not realise the need to differentiate to find the gradient of the curve, and hence the tangent, part (b) was answered well. Some candidates though thought that the coefficient of x^2 (the leading term) in their derivative gave them the gradient. There was the usual confusion here between tangents and normals with some candidates thinking that $\frac{dy}{dx}$ gave the gradient of the normal not the tangent. In part (c) many knew they needed to use the perpendicular gradient rule but many were not sure what to do. A common error was to find the equation of a straight line (often the normal at P) and then attempt to find the intersection with the curve. Those who did embark on a correct approach usually solved their quadratic equation successfully using the formula, completing the square often led to difficulties with the x^2 term, but a few provided a correct verification.

6664 Core Mathematics C2

Introduction

Although some of the earlier questions on this paper were well answered by the majority, the last four questions often proved particularly challenging, even for good candidates. The unfamiliar style of Q3, Q7 and Q8(b) gave rise to some poor solutions and, in some cases, inappropriate methods that wasted time. A lack of clear strategy in presentation of solutions suggested that many candidates did not seem to look carefully enough at the structure of these questions. It was, however, pleasing to see an improvement in performance on the more familiar questions. Standards of algebra often left much to be desired and, as in recent C2 papers, candidates often had difficulty with trigonometry and logarithms.

There were occasional blank responses to the later questions, suggesting that some candidates may have run short of time, but usually all nine questions were attempted.
As noted in many recent reports, candidates who quote standard formulae before beginning to use them are more likely to safeguard method marks when they make mistakes.

Report on individual questions

Question 1

While most candidates gained method marks for their integration attempt, some did not realise that $\sqrt{x}$ was $x^{\frac{1}{2}}$, or had difficulty in simplifying $3x^{\frac{3}{2}} \div \left(\frac{3}{2}\right)$. More common was the inability to evaluate the definite integral correctly, with $2\times 4^{\frac{3}{2}}$ causing particular problems. Overall, however, it was common to see full marks scored.

Question 2

In part (a), most candidates were aware of the structure of a binomial expansion and were able to gain the method mark. Those who used the $(a+b)^n$ formula were usually able to pick up accuracy marks but many of those who attempted to use the $(1+x)^n$ version made mistakes in simplifying terms, often taking out 2 as a factor rather than 2^7. Coefficients were generally found using ${}^nC_r$, but Pascal's triangle was also frequently seen. The simplified third term was often given as $672kx^2$ instead of $672k^2x^2$, but this mistake was much less common than in similar questions on previous C2 papers.

Part (b) was often completed successfully, but some candidates included powers of x in their 'coefficients'. Compared with recent papers, there seems to be some improvement in the understanding of the difference between 'coefficients' and 'terms'. Accuracy mistakes, including multiplying the wrong coefficient by 6, were common.

Question 3

The style of this question on the remainder theorem was unusual and candidates' performance was generally disappointing. In part (a), finding the value of $\mathrm{f}(k)$ proved surprisingly difficult. Many candidates seemed unable to appreciate that $(3k-2)(k-k)-8$ could be simplified to -8, and $3k-10$ was a popular answer.

Thankfully the majority attempted to use the remainder theorem rather than long division (which was very rarely successful) in part (b), but numerical and algebraic mistakes were very common. Sometimes the expression for the remainder was equated to 0 rather than 4, losing the method mark.

Some candidates had no idea of how to proceed in part (c) and those who made progress were often unable to reach the correct factorised form of the resulting quadratic expression. Some solved a quadratic equation by use of the formula at this stage, never achieving the required factorised form.

Question 4

Part (a) was answered correctly by the majority of candidates, although $\sqrt{2^{2.5}+1}$ was sometimes evaluated as $\sqrt{2.5^2+1}$.
The trapezium rule was often accurately used in part (b), but the common mistake in the value of h $\left(h=\frac{3}{7} \text{ instead of } h=\frac{3}{6}\right)$ was frequently seen. Some candidates had confused bracketing, leaving out the main brackets and multiplying only the first two terms by $0.5h$. The equivalent method of adding the areas of separate trapezia was occasionally seen.

In part (c), some of the candidates' responses clearly indicated a lack of understanding of why the trapezium rule gave an overestimate in this case. To score the final mark, a convincing explanation (with reference to trapezia) was required. Those candidates who supported their reasoning with a simple sketch were usually more successful.

Question 5

There were many excellent solutions to this question. When problems did occur, these were frequently in part (a), where some candidates showed insufficient working to establish the given common ratio and others confused common ratio and common difference, treating the sequence as arithmetic.

Most of those who were confused in part (a) seemed to recover in part (b). In both part (a) and part (b), some candidates used the formula ar^{n-1} and others successfully used the method of repeatedly multiplying or dividing by the common ratio.
Formulae and methods for the sum to 15 terms and the sum to infinity were usually correct in parts (c) and (d). Just a few candidates found the 15$^{\text{th}}$ term instead of the sum in part (c) and just a few resorted to finding all 15 terms and adding.

Question 6

Many candidates had difficulty with this question, with part (c) being particularly badly answered.

In part (a) the method of completing the square was the most popular approach, but poor algebra was often seen, leading to many incorrect answers. Although the correct centre coordinates $(3, -2)$ were often achieved (not always very convincingly), the radius caused rather more problems and answers such as $\sqrt{12}$ appeared frequently. Some candidates inappropriately used the information about the diameter in part (b) to find their answers for part (a), scoring no marks.

There were various possible methods for part (b), the most popular of which were either to show that the mid-point of *PQ* was the centre of the circle or to show that the length of *PQ* was twice the radius of the circle. Provided that either the centre or the radius was correct in part (a), candidates therefore had at least two possible routes to success in part (b), and many scored both marks here. Some, however, thought that it was sufficient to show that both *P* and *Q* were on the circle.

Part (c) could have been done by using the fact that the point *R* was on the circle (angle in a semicircle result), or by consideration of gradients, or by use of Pythagoras' Theorem. A common mistake in the 'gradient' method was to consider the gradient of *PQ*, which was not directly relevant to the required solution.

Many candidates were unable to make any progress in part (c), perhaps omitting it completely, and time was often wasted in pursuing completely wrong methods such as finding an equation of a line perpendicular to *PQ* (presumably thinking of questions involving the tangent to a circle).

Question 7

The style of this question was unfamiliar to many candidates and this produced a generally poor performance, with weaker candidates often scoring no marks at all and many good candidates struggling to achieve more than half marks overall. Much time was wasted on multiple solutions, especially for part (i).

In part (i), where values of $\tan\theta$ and $\sin\theta$ could have been written down directly from the given equation, the most common strategy was to multiply out the brackets. This often led to protracted manipulations involving trigonometric identities and, more often than not, no answers.

Most candidates did a little better in part (ii), starting off correctly by expressing $\tan\theta$ as $\dfrac{\sin\theta}{\cos\theta}$ and often proceeding to divide by $\sin\theta$ and find a value for $\cos\theta$. What very few realised, however, was that $\sin\theta = 0$ was a possibility, giving further solutions 0 and $180°$. Those who tried squaring both sides of the equation were often let down by poor algebraic skills. Just a few candidates resorted to graphical methods, which were rarely successful.

In both parts of the questions, candidates who were able to obtain one solution often showed competence in being able to find a corresponding second solution in the required range. Familiarity with trigonometric identities varied and it was disappointing to see $\sin\theta = 1 - \cos\theta$ so often.

Question 8

In part (a), the majority of candidates showed an understanding of the definition of a logarithm, although the answers - 8 and 8 appeared occasionally instead of $\frac{1}{8}$.

Part (b), however, was often badly done. Not realising that $\log_2 32$ and $\log_2 16$ could be immediately written as 5 and 4 respectively, most candidates launched unnecessarily into laws of logarithms, replacing the numerator by $\log_2 512$ (or sometimes by $\log_2 48$). Although it would have been quite possible to proceed from here to correct answers, confusion often followed at the next stage, with $(\log_2 x)\times(\log_2 x)$ becoming $\log_2 x^2$ and then, perhaps, $2\log_2 x$. Other unfortunate mistakes included writing $\frac{\log 512}{\log x}$ as either $\log_2(512-x)$ or $\frac{512}{x}$. Those candidates who managed to reach $(\log_2 x)^2 = 9$ were usually able to find one correct answer from $\log_2 x = 3$, but the second answer (from $\log_2 x = -3$) appeared much less frequently.

Question 9

Many candidates had difficulty in their attempts to establish the given result for the surface area in part (a) of this question. Solutions often consisted of a confused mass of formulae, lacking explanation of whether expressions represented length, area or volume. Formulae for arc length and sector area usually appeared at some stage, but it was often unclear how they were being used and at which point the substitution $\theta = 1$ was being made. It was, however, encouraging to see well-explained, clearly structured solutions from good candidates.

Having struggled with part (a), some candidates disappointingly gave up. The methods required for the remainder of the question were, of course, more standard and should have been familiar to most candidates.
In part (b), most candidates successfully differentiated the given expression then formed an equation in r using $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$. While many solved $2r - \frac{1800}{r^2} = 0$ successfully, weaker candidates were sometimes let down by their algebraic skills and could not cope correctly with the negative power of *r*. A common slip was to proceed from $r^3 = 900$ to $r = 30$.

In part (c), the majority of candidates correctly considered the sign of the second derivative to establish that the value of *S* was a minimum, although occasionally the second derivative was equated to zero.
Those who proceeded as far as part (d) were usually able to score at least the method mark, except when the value of *r* they substituted was completely inappropriate, such as the value of the second derivative.

6665 Core Mathematics C3

Introduction

This paper proved to be very accessible to many of the candidature and there was little evidence of candidates being short of time. This paper afforded a typical E grade candidate plenty of opportunity to gain marks across many, if not all of the 8 questions.

There were a significant number of candidates who at times failed to give their answers to the degree of accuracy stated in the question. Other candidates gave incorrect answers to Q1(a), Q3(d), Q6(c), Q6(d) and Q8(b) as a result of rounding their intermediate answers to too few significant figures or too few decimal places. There were also a significant number of candidates in question 8 who gave their final answers in degrees when radian answers were required.

In summary, Q1, Q2, Q3, Q4(i)(a), Q4(ii), Q5(d), Q6(c) and Q8(a) were a good source of marks for the average candidate, mainly testing standard ideas and techniques; and Q6(b), Q8(b) and especially Q5 were effective discriminators. A significant proportion of candidates, however, made little progress with Q5(c) and Q5(e) with some candidates writing down exactly the same answers to both of these parts.

Report on individual questions

Question 1

This question was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), the majority of candidates were able to score all three marks. There were a significant number of candidates in this part who incorrectly gave x_3 and x_4 as 2.355 and 2.361 respectively. These incorrect answers were usually achieved by candidates substituting the rounded answer of $x_2 = 2.372$ to find x_3 and substituting their rounded answer of $x_3 = 2.355$ to find x_4. Some candidates are not aware that it is possible to program a basic calculator by using the ANS button to find or even check all four answers. Another common error in this part was for candidates to stop after evaluating x_3.

The majority of candidates who attempted part (b) choose an appropriate interval for x and evaluated y at both ends of that interval. The majority of these candidates chose the interval $(2.3585, 2.3595)$ although incorrect intervals, such as $(2.358, 2.360)$ were seen. There were a few candidates who chose the interval $(2.3585, 2.3594)$. This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 2.359 to 3 decimal places or $\alpha = 2.359$ or even QED.

A minority of candidates attempted part (b) by using a repeated iteration technique. Almost all of these candidates iterated as far as x_6 (or beyond) but most of these did not write down their answers to at least four decimal places. Of those candidates who did, very few candidates managed to give a valid conclusion.

Question 2

In part (a), the majority of candidates started with $\cos^2\theta + \sin^2\theta = 1$ and divided all terms by $\cos^2\theta$ and rearranged the resulting equation to give the correct result. A significant minority of candidates started with the RHS of $\sec^2\theta - 1$ to prove the LHS of $\tan^2\theta$ by using both $\sec^2\theta = \dfrac{1}{\cos^2\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$. There were a few candidates, however, who used more elaborate and less efficient methods to give the correct proof.

In part (b), most candidates used the result in part (a) to form and solve a quadratic equation in $\sec\theta$ and then proceeded to find 120° or both correct angles. Some candidates in addition to correctly solving $\sec\theta = -2$ found extra solutions by attempting to solve $\sec\theta = \frac{2}{3}$, usually by proceeding to write $\cos\theta = \frac{2}{3}$, leading to one or two additional incorrect solutions. A significant minority of candidates, however, struggled or did not attempt to solve $\sec\theta = -2$.

A significant minority of candidates used $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$ to achieve both answers by a longer method but some of these candidates made errors in multiplying both sides of their equation by $\cos^2\theta$.

Question 3

This question was well answered by the overwhelming majority of candidates who demonstrated their confidence in working with exponentials.

Part (a) was almost universally answered correctly, although a few candidates did try to substitute $t = 1$ into the equation for P in order to find the number of rabbits introduced to the island.

In part (b), most candidates were able to use natural logarithms in order to find $t = 12.6$ or $t = 12.63$. Although the expected answer was 13 years, any answer that rounded to 12.6 years was also accepted. Those candidates who continued to round their answer down to 12 or stated "in the 12th year" were not awarded the final accuracy mark as the question required candidates to find the number of years for the number of rabbits to exceed 1000. A few candidates applied a trial and error method in this part and were usually successful in gaining both marks.

In part (c), most candidates correctly stated $\dfrac{dP}{dt}$ as $16e^{\frac{1}{5}t}$. Common errors in this part were candidates giving answers of the form $kte^{\frac{1}{5}t}$ or $16e^{-\frac{4}{5}t}$. A few candidates tried to apply the product rule for differentiation and usually struggled to gain both marks.

In part (d), the vast majority of candidates equated their $\dfrac{dP}{dt}$ found in part (c) to 50 and proceeded to solve for t. A number of candidates failed at this point to use their value for t to find P as required in the question. It was pleasing to see a significant minority of candidates who deduced that $P = 80e^{\frac{1}{5}t} = 5\times16e^{\frac{1}{5}t} = 5\times\dfrac{dP}{dt} = 250$.

Question 4

Part (i)(a) was well answered by the majority of candidates. The most common error was incorrectly differentiating $\cos 3x$ to either $3\sin 3x$ or $-\sin 3x$. A few candidates lost the final accuracy mark for simplification errors such as simplifying $(\cos 3x)(2x)$ to $\cos 6x^2$.

In (i)(b), the quotient rule was generally well applied in most candidates' working. A significant number of candidates, however, struggled to differentiate $\ln(x^2+1)$ correctly. $\frac{1}{x^2+1}$, $\frac{2}{x^2+1}$ or even $\frac{1}{x}$ were common incorrect outcomes. Those candidates who decided to use the product rule in this part were less successful in gaining some or all of the marks.

Again part (ii) was generally well attempted by candidates of all abilities. The most common error was incorrectly differentiating $\sqrt{(4x+1)}$ although a few candidates failed to attempt to differentiate this. A few candidates found the equation of the normal and usually lost the final two marks. Also, a number of candidates failed to write the equation of the tangent in the correct form and so lost the final accuracy mark.

Question 5

In part (a), almost all candidates realised that the transformed curve was the same as the original curve for $x > \frac{1}{2}\ln k$, and only a few failed to reflect the other part of the curve correctly through the *x*-axis. A significant number of candidates in this part struggled to write down the correct coordinates for the *y*-intercept in terms of *k*. The most common incorrect answers were $(0,\ k+1)$ and $(0,\ 1-k)$. A few candidates did not state the coordinates of the *y*-intercept in the simplest form. An answer of $(0,\ |1-k|)$ was accepted on the *y*-axis in this part.

In part (b), many candidates realised that they needed to reflect the given curve through the line $y = x$. Although the majority of these candidates managed to draw the correct shape of the transformed curve, a few seemed unable to visualise the correct position after reflection and incorrectly positioned their curve going through the fourth quadrant. Those candidates who correctly reflected the original curve through the line $y = x$ were often unable to see that the effect of reflecting points *A* and *B* in the line $y = x$ was a reversal of *x*- and *y*-coordinates. The most common incorrect coordinates for the *x* and *y* intercepts in this part were $\left(-\frac{1}{2}\ln k,\ 0\right)$ and $(0,\ -1+k)$ respectively.

In parts (a) and (b), examiners were fairly tolerant with curvature. Those candidates, however, who drew curves going back on themselves to give an upside-down U in the second quadrant in part (a) or a C-shape in the third quadrant in part (b) were not awarded the relevant mark for the shape of their curve.

The majority of candidates struggled with part (c). Some candidates sketched the curve of $y = \mathrm{e}^{2x}$ and proceeded to translate this curve down *k* units in the *y*-direction and in most cases these candidates were able to write down the correct range. A significant number of candidates wrote their range using *x* rather than using either *y* or $\mathrm{f}(x)$. Common incorrect answers for the range were $y \in \mathbb{R}$, $y > 0$, $y > k$ or $y \geq k$

Part (d) was well answered and a majority of candidates were able to score some marks with a large number scoring full marks. The general procedure of changing the subject and switching *x* and *y* was well known. There was the occasional difficulty with taking logarithms of both sides. A common error was an incorrect answer of $\frac{1}{2}\ln(x-k)$. As *x* only appears once in the original function, a few candidates instead chose to use a flowchart method to find the inverse. Nearly all those who did this arrived at the correct answer.

Again, as with part (c), many candidates struggled to give the correct domain for the inverse function in part (e). Those candidates who correctly stated that the domain of the inverse function is the same as the range of the original function failed in many cases to obtain the follow-through mark. This was because they did not change *y* or $\mathrm{f}(x)$ in their range inequality to *x* in their domain inequality.

Question 6

The majority of candidates were able to give a correct proof in part (a). A number of candidates having written $\cos 2A = \cos^2 A - \sin^2 A$ did not make the connection with $\sin^2 A + \cos^2 A = 1$ and were unable to arrive at the given result.

Part (b) proved to be one of the most challenging parts of the paper with many candidates just gaining the first mark for this part by eliminating *y* correctly. A number of candidates spotted the link with part (a) and either substituted $\dfrac{1-\cos 2x}{2}$ for $\sin^2 x$ or $1-\cos 2x$ for $2\sin^2 x$ and usually completed the proof in a few lines. A significant number of candidates manipulated $4\sin^2 x - 2\cos 2x$ to $8\sin^2 x - 2$ and usually failed to progress further. There were some candidates who arrived at the correct result usually after a few attempts or via a tortuous route.

Part (c) was well done. *R* was usually correctly stated by the vast majority of candidates. Some candidates gave α to 1 decimal place instead of the 2 decimal places required in the question. Other candidates incorrectly wrote $\tan\alpha$ as $\frac{4}{3}$. In both cases, such candidates lost the final accuracy mark for this part. There was some confusion between $2x$ and α, leading to some candidates writing $\tan 2x$ as $\frac{3}{4}$ and thereby losing the two marks for finding α.

Many candidates who were successful in part (c) were usually able to make progress with part (d) and used a correct method to find the first angle. A number of candidates struggled to apply a correct method in order to find their second angle. A significant number of candidates lost the final accuracy mark owing to incorrect rounding errors with either one or both of 51.7° or 165.3° seen without a more accurate value given first.

Question 7

Many candidates were able to obtain the correct answer in part (a) with a significant number of candidates making more than one attempt to arrive at the answer given in the question. Those candidates who attempted to combine all three terms at once or those who combined the first two terms and then combined the result with the third term were more successful in this part. Other candidates who started by trying to combine the second and third terms had problems dealing with the negative sign in front of $\dfrac{2}{x+4}$ and usually added $\dfrac{2}{(x+4)}$ to $\dfrac{x-8}{(x-2)(x+4)}$ before combining the result with 1. It was pleasing to see that very few candidates used $(x+4)^2(x-2)$ as their common denominator when combining all three terms.

In part (b), most candidates were able to apply the quotient rule correctly but a number of candidates failed to use brackets properly in the numerator and then found some difficultly in arriving at the given answer.

In part (c), many candidates were able to equate the numerator to the denominator of the given fraction and many of these candidates went onto form a quadratic in e^x which they usually solved. A significant number of candidates either failed to spot the quadratic or expanded $(e^x - 2)^2$ and then took the natural logarithm of each term on both sides of their resulting equation.

In either or both of parts (b) and (c), some candidates wrote e^{x^2} in their working instead of e^{2x}. Such candidates usually lost the final accuracy mark in part (b) and the first accuracy mark in part (c).

Question 8

In part (a), most candidates were able to write down the correct identity for $\sin 2x$.

In part (b), there was a failure by a significant number of candidates who replaced $\operatorname{cosec} x$ with $\dfrac{1}{\sin x}$ to realise the connection between part (a) and part (b) and thus managed only to proceed as far as $1 = 8\sin x \cos x$. Some candidates, however, thought that $8\sin x \cos x$ could be written $\sin 8x$, presumably by continuing the imagined "pattern" with $2\sin x \cos x = \sin 2x$. Nonetheless, the majority of candidates who reached this stage usually used the identity in part (a) to substitute $4\sin 2x$ for $8\sin x \cos x$ and proceeded to give at least one allowable value for x.

A number of candidates lost the final accuracy mark for only giving one instead of two values for x, or for rounding one of their answers in radians incorrectly (usually by writing 1.45 instead of 1.44). Several candidates lost the final accuracy mark for writing their answers in degrees rather than radians. Some candidates, however, worked in degrees and converted their final answers to radians.

6666 Core Mathematics C4

Introduction

The paper proved accessible to candidates and nearly all were able to attempt all 8 questions. Some attempts at Q8 were fragmentary or confused, suggesting a lack of time, but such attempts often followed the use of unnecessarily long methods earlier in the paper, particularly in Q7.

The general standard of presentation was acceptable. These papers are marked online and, if a pencil is used in drawing sketches of graphs, a sufficiently soft pencil (HB) should be used and it should be noted that coloured inks do not come up well and may be invisible.

The standard of algebra was generally acceptable but many candidates showed weaknesses in using brackets and they were often omitted. This can lead to a loss of marks. For example, writing the approximate integral in Q2(b) as $\frac{h}{2}y_0 + y_4 + 2\left(y_1 + y_2 + y_3\right)$ can lead to a failure to multiply $2\left(y_1 + y_2 + y_3\right)$ by $\frac{h}{2}$. There has been a noticeable improvement in the standard of work seen in the topic of implicit differentiation.

Not all candidates were familiar with all the mathematics symbols appropriate to this specification.
In particular, in Q7(b), many candidates stopped at $\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}$, not recognising that finding $\left|\overrightarrow{CB}\right|$ requires finding the magnitude or length of the vector.

Most candidates used their calculators appropriately but candidates need to realise that when, as in Q6, part (b)(i) the question specifies "Hence find $\int_1^2 \left(x-1\right)\sqrt{\left(5-x\right)}\,\mathrm{d}x$", to gain credit, a connection has to be made between this and the previous parts of the question. Obtaining the correct result using the numerical integration facility on a calculator does not do this and does not gain the marks.

Report on individual questions

Question 1

This proved a suitable starting question and the majority of candidates gained 5 or 6 of the available 6 marks. Nearly all could obtain the index as $-\frac{1}{2}$ but there were a minority of candidates who had difficulty in factorising out 4 from the brackets and obtaining the correct multiplying constant of $\frac{1}{2}$. Candidates' knowledge of the binomial expansion itself was good and, even if they had an incorrect index, they could gain the method mark here. An unexpected number of candidates seemed to lose the thread of the question and, having earlier obtained the correct multiplying factor $\frac{1}{2}$ and expanded $\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$ correctly, forgot to multiply their expansion by $\frac{1}{2}$.

Question 2

Most candidates could gain the mark in part (a) although 2.99937, which arises from the incorrect angle mode, was seen occasionally. The main error seen in part (b) was finding the width of the trapezium incorrectly, $\frac{3\pi}{10}$ being commonly seen instead of $\frac{3\pi}{8}$. This resulted from confusing the number of values of the ordinate, 5, with the number of strips, 4. Nearly all candidates gave the answer to the specified accuracy. In part (c), the great majority of candidates recognised that they needed to find $\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x$ and most could integrate correctly. However $\sin x$, $9\sin x$, $3\sin\left(\frac{x}{3}\right)$, $-9\sin\left(\frac{x}{3}\right)$, $-\sin\left(\frac{x}{3}\right)$ and $-3\sin\left(\frac{x}{3}\right)$ were all seen from time to time. Candidates did not seem concerned if their answers to part (b) and part (c) were quite different, possibly not connecting the parts of the question. Despite these difficulties, full marks were common and, generally, the work on these topics was sound.

Question 3

Part (a) was well done with the majority choosing to substitute values of x into an appropriate identity and obtaining the values of A, B and C correctly. The only error commonly seen was failing to solve $5 = \frac{5}{4}A$ for A correctly. Those who formed simultaneous equations in three unknowns tended to be less successful. Any incorrect constants obtained in part (a) were followed through for full marks in part (b)(i). Most candidates obtained logs in part (b)(i). The commonest error was, predictably, giving $\int \frac{4}{2x+1}\mathrm{d}x = 4\ln(2x+1)$, although this error was seen less frequently than in some previous examinations. In indefinite integrals, candidates are expected to give a constant of integration but its omission is not penalised repeatedly throughout the paper. In part (b)(ii) most applied the limits correctly although a minority just ignored the lower limit 0. The application of log rules in simplifying the answer was less successful. Many otherwise completely correct solutions gave $3\ln 3$ as $\ln 9$ and some "simplified" $3\ln 5 - 4\ln 3$ to $\frac{3}{4}\ln\left(\frac{5}{3}\right)$.

Question 4

As noted above work on this topic has shown a marked improvement and the median mark scored by candidates on this question was 8 out of 9. The only errors frequently seen were in differentiating $y\mathrm{e}^{-2x}$ implicitly with respect to x. A few candidates failed to read the question correctly and found the equation of the tangent instead of the normal or failed to give their answer to part (b) in the form requested.

Question 5

Nearly all candidates knew the method for solving part (a), although there were many errors in differentiating trig functions. In particular $\frac{\mathrm{d}}{\mathrm{d}t}\left(2\cos 2t\right)$ was often incorrect. It was clear from both this question and question 2 that, for many, the calculus of trig functions was an area of weakness. Nearly all candidates were able to obtain an exact answer in surd form. In part (b), the majority of candidates were able to eliminate t but, in manipulating trigonometric identities, many errors, particularly with signs, were seen. The answer was given in a variety of forms and all exact equivalent answers to that printed in the mark scheme were accepted. The value of k was often omitted and it is possible that some simply overlooked this. Domain and range remains an unpopular topic and many did not attempt part (c). In this case, inspection of the printed figure gives the lower limit and was intended to give candidates a lead to identifying the upper limit.

Question 6

Throughout this question sign errors were particularly common. In part (a), nearly all recognised that $(5-x)^{\frac{3}{2}}$ formed part of the answer, and this gained the method mark, but $\frac{3}{2}(5-x)^{\frac{3}{2}}$, $-\frac{3}{2}(5-x)^{\frac{3}{2}}$ and $\frac{2}{3}(5-x)^{\frac{3}{2}}$, instead of the correct $-\frac{2}{3}(5-x)^{\frac{3}{2}}$, were all frequently seen. Candidates who made these errors could still gain 3 out of the 4 marks in part (b)(i) if they proceeded correctly. Most candidates integrated by parts the "right way round" and were able to complete the question. Further sign errors were, however, common.

Question 7

This proved the most demanding question on the paper. Nearly all candidates could make some progress with the first three parts but, although there were many, often lengthy attempts, success with part (d) and (e) was uncommon. Part (a) was quite well answered, most finding $\overrightarrow{AB}$ or $\overrightarrow{BA}$ and writing down $\overrightarrow{OA}+\lambda\overrightarrow{AB}$, or an equivalent. An equation does, however need an equals sign and a subject and many lost the final A mark in this part by omitting the "$\mathbf{r} =$" from, say, $\mathbf{r} = 8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k} + \lambda\left(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}\right)$. In part (b), those who realised that a magnitude or length was required were usually successful. In part (c), nearly all candidates knew how to evaluate a scalar product and obtain an equation in $\cos\theta$, and so gain the method marks, but the vectors chosen were not always the right ones and a few candidates gave the obtuse angle. Few made any real progress with parts (d) and (e). As has been stated in previous reports, a clear diagram helps a candidate to appraise the situation and choose a suitable method. In this case, given the earlier parts of the question, vector methods, although possible, are not really appropriate to these parts, which are best solved using elementary trigonometry and Pythagoras' theorem. Those who did attempt vector methods were often very unclear which vectors were perpendicular to each other and, even the minority who were successful, often wasted valuable time which sometimes led to poor attempts at question 8. It was particularly surprising to see quite a large number of solutions attempting to find a vector, $\overrightarrow{CX}$ say, perpendicular to l, which never used the coordinates or the position vector of C.

Question 8

The responses to this question were very variable and many lost marks through errors in manipulation or notation, possibly through mental tiredness. For examples, many made errors in manipulation and could not proceed correctly from the printed $\cos 2\theta = 1 - 2\sin^2\theta$ to $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and the answer $\frac{x}{2} - \frac{1}{4}\sin 2\theta$ was often seen, instead of $\frac{\theta}{2} - \frac{1}{4}\sin 2\theta$.
In part (b), many never found $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or realised that the appropriate form for the volume was $\pi\int y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$.

However the majority did find a correct integral in terms of θ although some were unable to use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify their integral. The incorrect value $k = 8\pi$ was very common, resulting from a failure to square the factor 2 in $\sin 2\theta = 2\sin\theta\cos\theta$. Candidates were expected to demonstrate the correct change of limits. Minimally a reference to the result $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, or an equivalent, was required. Those who had complete solutions usually gained the two method marks in part (c) but earlier errors often led to incorrect answers.

6667 Further Pure Mathematics FP1 (new)

Introduction

Candidates had been well prepared for this paper and the majority made a good attempt at all of the questions. Q5(b), Q6(d) and Q7(c) provided discrimination at the higher levels, as solutions to these questions required interpretation and understanding. The presentation of answers was usually good and most candidates understood that a proof requires each line of reasoning to be explained fully. Some of the matrix work indicated lack of familiarity with vocabulary and uncertainty with transformation work, and proof by induction statements indicated less than full understanding by many candidates.

Report on individual questions

Question 1

Almost all candidates achieved the mark in part (a) for the argand diagram. Also the modulus of a complex number was understood and usually found correctly. Finding the argument of the complex number caused more problems for some candidates as a number of them did not consider which quadrant they needed. Also some candidates used incorrect trigonometry. A few answers were given in degrees and some calculated tan (0.5) instead of arctan (0.5). Part (d), which involved calculating a quotient, was usually answered correctly also. Most successfully multiplied by the conjugate 2 – i and got full marks. Some misread this part and found $\frac{z_1}{z_2}$ which meant that they could only get a maximum of 1 mark for multiplying by their conjugate.

Question 2

In part (a) there were many good solutions, although some candidates increased the algebraic challenge by not extracting all common factors. There were a number of answers which went straight from a cubic to three linear factors with no evidence such as an intermediate quadratic. This question had a printed answer, which candidates were asked to show and all steps of the working are required for full marks.

The evaluation of the sum of the series in part (b) was usually done well. Most used their formula correctly and subtracted the sum of the first twenty terms from the sum of the first forty terms to give their answer. A small minority substituted into the wrong formula and a larger number misquoted their formula for the sum using a fraction ½ instead of 1/12. Very few subtracted the first 19 terms or 21 terms which has been a common error in the past.

Question 3

A substantial minority multiplied out the two brackets which complicated the problem. Most however attempted to solve $x^2 + 4 = 0$, but there were a number of wrong answers, particularly the real answers +2 and –2. The solution of the three term quadratic was usually correct but there were errors in simplification with a substantial number of candidates losing some accuracy in part (a). Indeed the answers –4 + 6i and –4 – 6i were fairly common. In part (b) candidates were asked to find the sum of their roots. Most obtained –8, but this only gained M1A1 if it followed wrong roots. There were candidates who were unfamiliar with the term “sum” and found the product instead.

Question 4

Most candidates were clear about the steps necessary to show that the root of the given equation was between the values 2.2 and 2.3 in part (a). Almost all substituted 2.2 and 2.3 into the left hand side of the equation and gave their numeric answers. A few did not complete the solution by stating that one answer was positive and one negative and that the sign change indicated the presence of a root between 2.2 and 2.3.

The Newton Raphson method in part (b) was well understood and most answered this part of the question correctly. Candidates are advised to show their expression for $f'(x)$ and for $f'(2.2)$. They are also advised to quote the formula and show their substitution. The final answer 2.219 was not acceptable with no working.

There were many good answers to part (c), with most solutions using similar triangles. Those who had learned and quoted a formula often made sign slips. Some used the equation of the line joining (2.2, -0.192) and (2.3, 0.877) and found where it crossed the x axis. This was an acceptable alternative method. A small number of candidates tried interval bisection however, which was not linear interpolation!

Question 5

In part (a) the product of the two matrices was usually executed correctly with few errors. Part (b) caused difficulties for some and there were a number of attempts where pages were covered in matrix work which led nowhere. The common errors included solving $\mathbf{R}^2 = 15\mathbf{R}$ instead of solving $\mathbf{R}^2 = 15\mathbf{I}$. A sizeable minority used the determinant of the matrix, putting it equal to 15 or to 225. They usually did not give a second equation to enable them to find the two unknowns. The successful majority approached the solution by equating the elements of their matrix solution to part (a) to 15 and to 0 as appropriate. Usually they obtained two sets of solutions a = 3 with b = -3 and a = -5 with b = 5. They then discarded the second set of solutions as they had been given the condition a >0, but some candidates failed to discard the second set and lost the final A1 mark.

Question 6

In part (a) candidates either substituted $x = 4t^2$ and $y = 8t$ into the equation $y^2 = 16x$, thereby verifying that the point lies on the parabola, or they compared the coordinates of P with the general point $(at^2, 2at)$ on the general parabola and identified that a = 4.

Most wrote down the correct focus as coordinates in part (b).

Part (c) required candidates to show that the equation of the normal to the parabola at the point P was $y + tx = 8t + 4t^3$. Some seemed to have learned the gradient of a parabola is $1/t$ and began with this result. This does not constitute a complete proof however and the gradient should have been established by differentiation.

Part (d) was found challenging by some and a simple sketch would have helped. Typical problems included making *N* the point where the normal crossed the *y*-axis, and finding the area of the wrong triangle *OPN*. The simplest method for finding the area was to use half base times height for the whole triangle but there were a variety of methods presented which usually led to the correct answer. It was common to divide the triangle into two right-angled triangles, for example. Others used Pythagoras to determine other lengths and used an indirect method, and some used determinant methods to find the area. Those who understood and correctly executed their method frequently made algebraic slips simplifying the final answer, and the final mark was often lost.

Question 7

Those who understood the word singular put their determinant equal to zero and solved the subsequent equation. There were frequently sign errors leading to the solution $a = -\frac{1}{2}$ and other algebraic errors leading to $a = 2$ instead of $a = \frac{1}{2}$. In part (b) most understood the method for finding the inverse matrix, but there were a number of errors and the determinant was often given as 14 instead of 10.

Part (c) could be approached in various ways. The most popular method was to multiply the inverse matrix by the column vector with elements $k - 6$, and $3k + 12$.
The answer obtained was the column vector with elements k and $k + 3$. Candidates then needed to complete their solution by concluding that the point lies on the line
$y = x + 3$.

Another approach involved using the original matrix and multiplying it by the column vector $\begin{pmatrix} x \\ x+3 \end{pmatrix}$ and equating to $\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$, which leads to $x = k$ and $y = k + 3$. Again a conclusion was needed.
Others used the original matrix and multiplied it by the column vector $\begin{pmatrix} x \\ y \end{pmatrix}$, again equating to $\begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}$. This leads straight to the equation $y = x + 3$. It was clear, however, that some candidates were unfamiliar with transformation work using matrices and did not set the transformation matrix first and follow it by a column matrix as required.

Question 8

Most candidates achieved the first four marks in part (a), but there were often slips in the simplification and many of those who first considered $f(k+1) - f(k)$, obtaining a multiple of four stopped at that point. They did not continue a step further to make $f(k+1)$ the subject of their formula and attempt to extract a common factor of 4 to show conclusively that $f(k + 1)$ was divisible by 4.

There were more completely correct answers in part (b), though a number of candidates were unclear that they had to multiply matrices to show this result. Those who multiplied the appropriate matrices sometimes made sign slips.

Candidates do need to learn the basic steps required for induction : true for n=1, **if** (or assuming) true for $n = k$ **then** true for $n=k+1$, therefore by induction true for all integer n. The presentation of their arguments is important in this form of proof and although a majority of candidates gave clear explanations - it was common to see candidates conclude "true for $n = 1$, k and k+1 and so for all n", without a clear indication of an inductive argument.

6668 Further Pure Mathematics FP2 (new)

Introduction

This was the first paper set on this specification and it was evident that the majority of candidates responded positively to it. The paper was found to be accessible with enough material on the paper to enable candidates to demonstrate their abilities and there was no evidence of them being unable to complete the paper owing to time constraints.

There was evidence that some candidates had done little or no revision or had a poor understanding of the topic of Complex Numbers. Statistics show that at least 50% of the candidature scored 3 or fewer marks in question 2 and at least 50% of the candidature scored 4 or fewer marks in question 6. The paucity of good answers by some candidates suggests that teachers may want to especially review this topic when revising FP2 in order to raise their candidates' achievement.

There was also evidence in Q4, Q5(a), Q7 and Q8(b) that some candidates struggled to apply material that they had previously learnt in units Core 3 and Core 4 in order to successfully answer these questions.

In Q4, it was found that there were some incorrect methods that candidates could use to arrive at the correct answer given on the mark scheme. The mark scheme, however, was designed to ensure that only those candidates who applied correct working and algebraic manipulation would be appropriately credited.

In summary, Q1, Q3, Q4, Q7(a), Q7(b) and Q8(a) were a good source of marks for the average candidate, mainly testing standard ideas and techniques; and Q2, Q5, Q6, Q7(c) and Q8(b) were effective discriminators.

Report on individual questions

Question 1

This question was generally well answered with at least 95% of candidates scoring at least 4 of the 6 marks available and the majority of them scoring all 6 marks.

Part (a) was almost universally answered correctly. A few candidates, however, when solving the correct identity, went on to obtain $\frac{1}{2}$ for both of their constants. These candidates made little progress in part (b).

Part (b) was usually well answered with a significant number of candidates making more than one attempt to arrive at the answer given in the question. Some candidates did not write down enough terms in the series summation in order for them to see which ones remained and which ones cancelled out. Sign errors and slips with working with algebraic fractions led to some candidates being unable to obtain the given result.

Question 2

The response to this question was disappointing with a significant number of candidates unable to find the modulus of z^3 and a significant number giving the argument of z^3 as $\frac{\pi}{4}$ instead of $-\frac{\pi}{4}$. Candidates who drew an Argand diagram were usually able to find the correct argument for z^3. Many of these candidates were able to apply de Moivre's theorem correctly in order to find the first of the three roots. A significant number of candidates struggled to find the remaining two roots. Some made no attempt at all, whilst others found the argument for z and then added and subtracted 2π to this result. A number of candidates did not state some or all of their values for z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta < \pi$. It was usual for these candidates to lose the final accuracy mark. There were, however, a number of candidates who were confident in answering this question and so gained all 6 marks available with ease.

Question 3

This question was well answered by candidates and statistics showed that at least 50% of the candidature scored 7 or more marks out the 8 available for this question. Some of these candidates usually lost the final accuracy mark as they either missed out the constant of integration or usually incorrectly manipulated $\frac{y}{\sin x} = 2\sin x + K$ to give $y = 2\sin^2 x + K$.

Most candidates were able to divide all terms in the differential equation by $\sin x$ to achieve an equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$ and most attempted to find the integrating factor. Some candidates wrote down $\mathrm{e}^{\int \frac{\cos x}{\sin x}\mathrm{d}x}$ as their integrating factor instead of $\mathrm{e}^{\int -\frac{\cos x}{\sin x}\mathrm{d}x}$. A few candidates struggled to integrate $-\frac{\cos x}{\sin x}$ correctly, but a significant number simplified $\mathrm{e}^{-\ln\sin x}$ incorrectly to $\sin x$. At this stage, some candidates did not use their integrating factor correctly to achieve an equation of the form $\frac{\mathrm{d}}{\mathrm{d}x}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$, and so lost the remaining four marks. There were also a significant number of candidates who struggled to simplify $\sin 2x \operatorname{cosec} x$ to $2\cos x$.

Question 4

This question was well answered by candidates and statistics showed that around 50% of the candidature gained all 8 marks available for this question.

Most candidates applied the formula $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ but a number of them struggled to write down the correct limits to find the relevant area for their expression. The majority of the errors made by candidates were algebraic; $6a\sin\theta$ was sometimes missed out when $(a + 3\cos\theta)^2$ was expanded. Sometimes a^2 was "taken out" from some candidates' integral. Most candidates knew they needed to substitute $\frac{9(1 + \cos 2\theta)}{2}$ for $9\cos^2\theta$ in order to integrate the expression but sometimes there were errors in dealing with the 9 and the $\frac{1}{2}$. There were a significant minority of candidates who did not know the correct strategy to apply in order to integrate $\cos^2\theta$. These candidates usually lost 7 of the 8 marks available for this question.

Question 5

This question was generally well answered with at least 50% of candidates scoring at least 8 of the 10 marks available and around 25% of them scoring all 10 marks.

In part (a), the majority of candidates usually differentiated y to give $2\sec^2 x \tan x$ and used the product rule to correctly find the second derivative. A noticeable number of candidates struggled to obtain the result given in the question because they did not use the identity $\sec^2 x = \tan^2 x + 1$. Some candidates in this part decided to write $\sec^2 x$ as $(\cos x)^{-2}$ and proceeded to find the first and second derivatives. These candidates were usually less successful. A few candidates wrote $\sec^2 x$ as $2(1 + \cos 2x)^{-1}$ and rarely obtained the result given in the question.

Many candidates were successful in answering part (b). A number of candidates lost some marks in this part by not evaluating $\sec^2\left(\frac{\pi}{4}\right)$ correctly. A significant number of candidates did not realise that they needed to find the third derivative as part of their solution and instead gave a correct Taylor series expansion up to the term in $\left(x - \frac{\pi}{4}\right)^2$. A number of candidates found a Maclaurin expansion or wrote down an expansion of the form $\mathrm{f}\left(\frac{\pi}{4}\right) + x\mathrm{f}'\left(\frac{\pi}{4}\right)x + \frac{\mathrm{f}''\left(\frac{\pi}{4}\right)}{2!}x^2 + \frac{\mathrm{f}'''\left(\frac{\pi}{4}\right)}{3!}x^3 + \ldots$

Question 6

The response to part (a) was disappointing with a significant number of candidates making no progress. At least half of the candidature rearranged $w = \frac{z}{z + \mathrm{i}}$ to make z the subject and took the modulus of both sides of their equation and applied the restriction $|z| = 3$ to give a modulus equation in w only. At this point a significant number of candidates failed to progress further and gained 3 marks. Of those candidates who progressed further, many substituted $u + \mathrm{i}v$ for w, but a number of them struggled to apply Pythagoras correctly by either not grouping the real and imaginary parts or not adding terms and usually wrote down an equation such as $u^2 - v^2 = 3^2\left((u-1)^2 - v^2\right)$. These candidates did not gain any extra credit. Candidates who wrote down $u^2 + v^2 = 3^2\left((u-1)^2 + v^2\right)$ usually progressed further to find the correct centre and radius of the circle, although some candidates did forget to square the 3.

In part (b), the majority of candidates who stated their centre and radius of the transformed circle in part (a) were usually able to able to plot this circle on an Argand diagram in the correct quadrants and usually gained the first mark. Only a minority of candidates realised that they needed to shade in the region outside of their circle. Many, however, shaded in the inside of their circle.

Question 7

This question was reasonably well answered with at least 50% of candidates scoring at least 9 of the 12 marks available and around 25% of them scoring all 12 marks.

In part (a), the majority of candidates were able to sketch the correct shape of the curve, although some candidates missed or incorrectly marked the points where the curve met the coordinate axes. Those candidates who drew a clear sketch in part (a) often improved their chances of finding the correct solution in part (c).

In part (b), the majority of candidates wrote down two equations to solve; the first for the "positive" modulus and the second for the "negative" modulus. Many candidates were able to solve the second equation to find $x = 0$ and $x = 1$ although a few of these candidates omitted $x = 0$. A significant number of candidates failed to solve the quadratic equation $x^2 + x - 2a^2 = 0$ owing to its generalised nature. A noticeable number of them wrote down $x(x + 1) = 2a^2$ leading to incorrect solutions of $x = 2a^2$ and $x = -1 + 2a^2$. Some candidates having found all four correct solutions then proceeded to reject either one or more of them and they were penalised for this. A minority of candidates tried to solve the equation in part (b) by squaring both sides. These candidates were usually less successful and it was rare for them to score more than 3 marks for this part.

Most candidates who obtained all four solutions in part (b) and had drawn a clear sketch in part (a) were able to add the line $y = -x + a^2$ onto their sketch and interpret their results to gain all four marks in part (c). A number of candidates were able to gain some marks in this part either for stating $0 < x < 1$ from their "negative" modulus equation or by stating $x <$ least value and/or $x >$ greatest value found from their "positive" modulus equation.

Question 8

This question discriminated well across candidates of all abilities and statistics show that the median mark for this question was around 8. Part (a) was attempted well by the majority of candidates but a significant number of them made no attempt at part (b). This was because some of these candidates did not make the link with the equation they had written in part (a) and the work they had previously seen in Core 2 on finding and determining the nature of stationary points.

In part (a), most candidates were able to find the complementary function. The majority of these candidates correctly found the particular integral, although a common error at this stage was for some candidates to give a particular integral of the form $\lambda t\,\mathrm{e}^{-t}$. A few candidates did not evaluate the particular integral and these candidates usually gained no credit for the remainder of this part. Many candidates were able to write down a general solution by adding their complementary function to their particular integral and then applied the boundary conditions to find both of their constants. There were a significant number of candidates who incorrectly solved the simultaneous equations and so they usually lost the final accuracy mark. This meant that these candidates were only able to gain a maximum of 4 out of the 7 marks available in part (b), with a function which was more difficult to solve once differentiated.

In part (b), a number of candidates were able to differentiate their answer to part (a) with respect to t and set the result equal to 0. A significant number of candidates at stage struggled to solve the equation $3\mathrm{e}^{-3t} - \mathrm{e}^{-t} = 0$ and usually progressed no further with this part. A number of candidates were able to find the exact value of t and usually stated it as either $\frac{1}{2}\ln 3$ or $-\frac{1}{2}\ln\frac{1}{3}$ and used this value to prove the result given for the maximum value of x, by using exact values, although some candidates at this stage used decimals and were penalised. Many candidates who progressed this far were able to prove that the value of x found was maximum, although a few candidates forgot to include this stage.

6669 Further Pure Mathematics FP3 (new)

Introduction

The early, shorter questions on this paper allowed most candidates to make a good start. The problems really started with Q4, the first integration question. Q5 and Q6 were challenging for many candidates and consequently some of the poor responses seen for Q8 may have been at least partly caused by poor time management. The displayed marks for each part of every question should give candidates an indication of the time expected to be spent on that part. For Q6(b) in particular many candidates did vast quantities of work which led nowhere and must have taken more than the expected 2-3 minutes.

Throughout the paper there were marks lost through numerical slips, particularly when using identities, and careless use of signs. Some candidates clearly make excellent use of their calculators; others should ensure they are fully aware of all the useful operations even basic calculators can perform nowadays. Similarly, some candidates do not seem to be fully aware of the formulae provided in the formula book.

Report on individual questions

Question 1

The majority of candidates correctly replaced the hyperbolic functions with their exponential definitions and subsequently obtained a quadratic equation though it was noticeable how many used the formula instead of factorising. Those who used $\frac{e^{2x}-1}{e^{2x}+1}$ for tanh x often gave up on the complexity of their equation. However a few also used $\frac{2e^{2x}}{e^{2x}+1}$ for sech x which led to a more efficient solution. It was clear that some candidates did not know what a rational number is as sometimes ln 2or $\ln\frac{1}{3}$ was rejected because "a" was not rational. The final answer was often seen as $-\ln 3$ which did not meet the demand of the question.

Question 2

Most candidates knew how to form the vector and the scalar product but it was surprising how many made a fresh start in part (b) rather than using their answer to part (a). There were however some sign errors made and several candidates gave a vector as their answer for the scalar product. Part (c) was the least well answered – a large number tried to use the triple product – and many could not remember the correct fraction ($\frac{1}{6}$) to use in part (d). Once more, in parts (c) and (d), many candidates started again instead of utilising their previous results.

Question 3

In part (a), the vast majority of candidates knew how to set up the characteristic equation but many then multiplied out the resulting cubic instead of using the factor $(7-\lambda)$ which should have been easily seen. Some then divided their cubic by $(7-\lambda)$ as they knew this should be a factor; their result was not always correct. In part (b) there was clear evidence that the standard technique for finding eigenvectors was well understood and most candidates were able to set up the necessary simultaneous equations and thus obtain the eigenvector corresponding to $\lambda = 7$. It was not uncommon to see the equation $7y = 7y$ solved to give $y = 0$. Candidates usually did not notice that using this led to inconsistent equations.

Question 4

Quite a lot of candidates failed to differentiate correctly in part (a), either by forgetting that the chain rule was needed or by leaving the answer wholly or partly in terms of y. Of these only a few managed to complete part (b) correctly by means of the standard arcosh integral. The candidates who had correctly obtained $\frac{\mathrm{d}y}{\mathrm{d}x}$ in part (a) usually recognised the link and progressed efficiently with the solution. Incorrect attempts included trying partial fractions or even multiplying together two integrals as well as a range of arcsin and arctan efforts. Many of those who integrated correctly also completed the solution accurately, the most common error being to stop at $2\ln\left(\frac{3+\sqrt{5}}{2}\right)$ although by no means all candidates attempted to rationalise the denominator of their single logarithm.

Question 5

In part (a) the general quality of solutions was poor with many candidates unclear of the direction in which their solution was going. This should have been a straightforward integral for candidates at this level and a result should have been obtained by recognition, or failing that, by a simple substitution. The condition $0 \leqslant x \leqslant 5$ was not understood by some to be needed to ensure that the denominator was real and instead they thought that the condition provided for limits on the integral. Without a correct result from part (a) candidates were unlikely to make significant progress with part (b) although it was surprising how many candidates performed the integral requested in (a) in the course of their work for part (b) but without realising it and returning to amend their previous work! Many candidates who had a correct result from their integration by parts could not proceed further as they did not realise they could convert their integrand $(n-1)x^{n-2}\sqrt{(25-x^2)}$ to $\frac{(n-1)x^{n-2}(25-x^2)}{\sqrt{(25-x^2)}}$ and hence obtain an equation connecting I_n and I_{n-2}. For some candidates part (c) was the only part of the question on which they gained marks; others possibly needed to read this part more carefully as use of the given result in part (b) could allow this part to be done even if the rest of the question was untouched. Some candidates however, could not evaluate I_0 even though the corresponding integral is given in the formula book.

Question 6

Most candidates could tackle part (a). However it was disappointing to see how many could not cope with the signs correctly when expanding $-a^2(mx+c)^2$. Part (b) saw far fewer correct solutions as many failed to realise that they were looking for a repeated root for the equation obtained in part (a). Those who set "$b^2 - 4ac$" $= 0$ almost invariably proceeded to the desired result. Very few candidates made any progress in part (c). Many simply turned to the next question; others attempted to find the equation of the tangent to the hyperbola at the point $(1,4)$ *on* the curve by use of differentials. As the point was not on the curve this method was inappropriate. The minority who managed to obtain the correct equations connecting c and m usually managed to proceed to the correct tangent equations.

Question 7

Part (a) was a straightforward exercise for the majority of candidates. There were some careless errors made when solving the equations and a few candidates could not contemplate a zero solution for λ and so gave up at that point. In part (b) most candidates knew they needed to form the vector product of the direction vectors of the two lines and could use this vector together with the equation **r.n** = **a.n** to form the required Cartesian equation of the plane. A few candidates were confused about which vectors should be used to obtain **n** and to represent **a**. Alternative methods using three suitable points in the equation $ax+by+cz+d=0$ or using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ were also seen. In general, candidates who used one of these methods knew exactly what they needed to do and the only errors were due to algebraic slips. The formula for the distance between two skew lines was also widely known although some candidates used $|\mathbf{b}|\times|\mathbf{d}|$ in the denominator instead of $|\mathbf{b}\times\mathbf{d}|$. Some candidates found the perpendicular distance from a point to a plane using the formula provided in the formula book. The method was successful provided any point on the second line was taken as the point (α,β,γ).

Question 8

In general the responses to this question were poor with many candidates lacking the ability to apply the given substitutions. The formula for the area of a surface of revolution together with correct expressions for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ and $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ were used to obtain a correct integral with respect to θ. Although most could then attempt part of the work required for the substitution $c=\cos\theta$ only a few could deal with the limits correctly. Some made no attempt to change the limits; others obtained the correct values for the new limits but used them the wrong way round thinking that the larger value always has to go at the top of the integral sign. Consequently, very few legitimately obtained the printed result with the correct values of $k=10$ and $\alpha=1$. In part (b), some could substitute correctly and use one or both of the necessary identities but the multiples and limits often caused problems. It was not uncommon to see an otherwise correct solution spoilt at the final stage when the limits were substituted and a final answer obtained. The correct answer of 117 was rare.

6674 Further Pure Mathematics FP1 (legacy)

Introduction

The paper proved accessible and the questions did seem to be in increasing order of difficulty. There seemed to be sufficient material for both the very able to demonstrate their understanding and to allow the weaker candidates to access most questions. The better candidates showed sufficient working to make their method clear.

Most candidates attempted all eight questions and there did not seem to be any time difficulties.

Report on individual questions

Question 1

This was an accessible question. Candidates generally recognised that if $x = i$ is a root then $x = -i$ is also a root, but some candidates confused roots and factors referring to the 'root $(x - i)$'. This meant that some candidates omitted $x= -i$ from their list of solutions to the equation. A few candidates misquoted the quadratic formula using b rather than –b.

Question 2

Most candidates were able to find the complimentary function correctly, although many mixed up the variables with many writing it as $y = f(x, t)$. The majority of candidates were able to find the particular integral but some were unable to set it up properly, either having no constant, or using t in place of a constant or not realising the index was $-4t$. Many candidates again mixed up their variables in stating the general solution and did not give their final answer as $x = f(t)$.

Question 3

The vast majority made a good start, expanded $r(r+1)(r+2)$ correctly and then used the correct formulae to substitute for the sums. Those who then extracted the common factors rather than expanding all the terms had an easier, and quicker, route through the question. Too many candidates did not make clear their method of getting from the cubic to the fully factorised form. Examiners needed to see the quadratic factor providing the link between these two expressions.

Almost all the candidates were able to use part (a) in order to find the answer to (b), with only a few using 21 instead of 20.

Question 4

This was generally well done. In part (a) most rearranged to get $(5+2pi)/(1+i)$. Those who then multiplied top and bottom by $(1+i)$ were able to find a and b very quickly. Quite a few multiplied top and bottom by $(a+ib)$ and usually were unable to progress further, despite lengthy working. A few equated real and imaginary parts and used simultaneous equations. Part (b) was usually well done, but some candidates put Im/Re= arctan4 rather than 4 or inverted to give Re/Im =4, a few made errors in solving the linear equation. Part (c) was generally well answered. Most candidates were able to represent their points on a correct argand diagram though a number failed to label their points.

Question 5

Most candidates were able to evaluate f(0.8) and f(0.9) correctly but not all stated the link between their results and the conclusion for part (a). Many candidates did not differentiate ln 3x correctly with many getting 1/3x (with 3/x a slightly less popular error). Most candidates knew what to do in part (c) the most common errors were premature approximation often leading to 0.887 and carelessness over signs, particularly 0.019. Those who drew a diagram were notably more accurate.

Question 6

Only a very few failed to find the correct integrating factor. Some candidates could not integrate $\sin^2 x$. Many candidates ignored the instruction to give their answer as y = and some did not include a constant, or did not divide their constant by sin x. Nearly all the candidates knew what to do in part (b); most found C correctly although many slips in solving the equation with $c = 3\pi/4$ not being uncommon. The final answer was given for the candidates and many candidates allowed terms to vanish or appear in order to obtain the given answer. It is hoped, at this level, that candidates will go back and check their working if they have to resort to such tactics, having said that, many fully correct solutions were seen here.

Question 7

In part (a) the majority of candidates were able to draw the correct, fully labelled sketch. The most common error was in failing to state the coordinates of the axes interceptions. Many candidates failed to state the equations of the two asymptotes with y= 0 being particularly rare. Part (b) was often well done with most candidates finding the two surds and then able to sort out the correct intervals.

Question 8

Most candidates were able to find $r \sin\theta$ and went on to gain full marks for this part. Some used $r \cos \theta$ and others were not able to solve their trigonometric equation. In part (b) most were able to evaluate the integral of $\frac{1}{2} r^2$ although some could not integrate $\cos^2 \theta$ and others used incorrect limits. Most, but by no means all, were able to find the Cartesian coordinates of P correctly. Only the best candidates recognised that they needed to find the area of the trapezium, more commonly partitioning it into a rectangle and triangle, and then subtract from this the answer to their integral. Many candidates only found the triangle or the rectangle rather than both.

6675 Further Pure Mathematics FP2 (legacy)

Introduction

This paper proved suitable for testing a wide range of abilities. Q1, Q2, Q3, Q4 and Q6 proved very accessible and completely correct solutions to these questions were common. Q5 and Q7 were more discriminating and Q8, designedly, provided a challenge for even the ablest candidates. Overall the paper was proved very similar in difficulty to previous years and in this, the last summer examination in which this specification was taken, there was no evidence that the general level of attainment had changed significantly.

The majority of candidates used calculators appropriately and the general standard of presentation was satisfactory. Very few candidates wrote answers outside the spaces provided.

Report on individual questions

Question 1

This proved a good starting question and fully correct solutions were common. The most frequently seen error was to lose a factor of 2 when differentiating $\left(\text{arsinh}\, 2x\right)^2$ and obtaining $\dfrac{2\,\text{arsinh}\, 2x}{\sqrt{\left(4x^2+1\right)}}$ instead of $\dfrac{4\,\text{arsinh}\, 2x}{\sqrt{\left(4x^2+1\right)}}$. Those who differentiated correctly almost always completed the question correctly. A few candidates rewrote the equation as $\sin y^{\frac{1}{2}} = 2x$ and differentiated implicitly with respect to *x*. This is, of course, a correct and acceptable method but it did give more opportunities for error than the more direct method and slips were often made.

Question 2

This was a very straightforward question and the median mark was full marks. In part (a), a few candidates thought that *b* was 8, rather than $\sqrt{8}$, and some lost a mark for not realising that, as $a > 2\sqrt{2}$, $a = -4$ is not a possible answer. In part (b) most split the shape into two or four triangles although some, realising that the shape is a square, found one side and squared it. A surprisingly common error was to multiply together the two diagonals of the quadrilateral, which gave twice the correct area.

Question 3

In this question, almost all candidates knew how to proceed but sign errors in integration were common. In part (a), the limits gave some difficulty and the result that $\left[n\left(1-x\right)^{n-1}\cosh x\right]_0^1$ is $-n$ and not *n* was not always clearly established. In part (b), most used the reduction formula to obtain I_4 in terms of I_2 and, hence, in terms of I_0. However, in the course of manipulation, an incorrect constant was often obtained, -4 instead of -28 being particularly common. Most could evaluate I_0 but a few left this as an expression in e^x.

Question 4

Most candidates handled this question well and this was another question in which the median mark was full marks. In part (a), a few lost the final mark by giving the answer as $\ln 8$ instead of as $3\ln 2$, as is required by the wording of the question. Those who used the second derivative test in part (b) usually completed this part successfully. In this case, other methods are more difficult to carry out and often led to some loss of marks.

Question 5

This question gave many candidates difficulty. Almost all knew the correct formula to apply and obtained the correct integral $36\pi\int t^3\left(t^2+1\right)^{\frac{1}{2}}\,\mathrm{d}t$ but many could get no further and those who did find a possible substitution often found one that led to an unnecessary amount of work. The simple substitution $u^2=t^2+1$, a method which is in the C4 specification, was rarely found. Some very long methods were seen. A popular method was to use $t=\sinh u$ and, with the hyperbolic identity $\sinh^2 u=\cosh^2 u-1$, this gives a reasonable solution. However some, on further manipulation, reached $\int\left(\sinh^5 u+\sinh^3 u\right)\mathrm{d}u$, and then proved a reduction formula for $J_n=\int_0^{\operatorname{arsinh}1}\sinh^n u\,\mathrm{d}u$. A few candidates completed this method correctly, which was a very impressive achievement, but such methods must cause time pressures later in the paper.

Question 6

This was for the majority a straightforward question. A substantial number of candidates made the error of substituting $\mathrm{d}\theta=\sinh\theta\,\mathrm{d}u$ instead of $\mathrm{d}\theta=\dfrac{\mathrm{d}u}{\sinh\theta}$. Candidates who substituted correctly often failed to make further progress because of weaknesses in elementary algebra. Many, on reaching $\displaystyle\int\frac{u+1}{\left(u^2-1\right)\left(u-1\right)^2}\,\mathrm{d}u$, were unable to simplify this to $\displaystyle\int\frac{1}{\left(u-1\right)^3}\,\mathrm{d}u$, an integral that can be written down by inspection. However, the great majority were able to handle the limits correctly, evaluating $\cosh\left(\ln 4\right)$ and $\cosh\left(\ln 2\right)$ correctly and using them.

Question 7

In part (a), most candidates could obtain $\tan\psi=\cot x$ but then even the strongest candidates often had difficulty giving a convincing reason why this led to $\psi=\dfrac{\pi}{2}-x$. The examiners accepted the relation $\cot x=\tan\left(\dfrac{\pi}{2}-x\right)$ as a sufficient reason for making this deduction. There are many possible approaches to part (b). Possibly the neatest is to note that as $\psi=\dfrac{\pi}{2}-x$, $\dfrac{\mathrm{d}x}{\mathrm{d}\psi}=-1$. Then,

as $\dfrac{\mathrm{d}s}{\mathrm{d}\psi}=\dfrac{\mathrm{d}s}{\mathrm{d}x}\times\dfrac{\mathrm{d}x}{\mathrm{d}\psi}$ and $\dfrac{\mathrm{d}x}{\mathrm{d}s}=\cos\psi$, $\dfrac{\mathrm{d}s}{\mathrm{d}\psi}=-\sec\psi$ and $s=-\int\sec\psi\,\mathrm{d}\psi$. This also enables part (c) to be completed quickly as $\rho=\dfrac{\mathrm{d}s}{\mathrm{d}\psi}=-\sec\psi$ can just be written down.

Question 8

Part (a) was well done and almost all candidates gained full marks. In part (b) most candidates were able to eliminate one of the variables, usually x. Those who recognised that the resulting expression was a quadratic in y usually went on to solve it correctly to obtain the printed answer. Surprisingly, as the topic is not in the specification, some very neat solutions using the sum of the roots of a quadratic were also seen.

In part (c), only the best candidates achieved a full solution. To make progress beyond writing down an expression for PQ, it is necessary to tidy up the expression for the length so that it can be differentiated, but few candidates had the technique to do this. For example, $PQ^2 = \frac{16a^2\left(p^2+1\right)^3}{p^4}$ can be differentiated and then factorised relatively easily. Some very elegant and unexpected solutions were seen from a few candidates who used the parameters $\left(aq^2, 2aq\right)$ for Q and simplified the expression for PQ^2 to $a^2\left(p-q\right)^2\left(\left(p+q\right)^2+4\right)$ before substituting $p+q=-\frac{2}{p}$ and $p-q=2\left(p+\frac{1}{p}\right)$.

6676 Further Pure Mathematics FP3 (legacy)

Introduction

The paper as a whole proved quite accessible for many candidates. Most candidates were able to complete Q1 quickly and accurately and Q2 involved standard work on vectors. Performances from weaker candidates were sometimes 'patchy', showing good understanding of some topics but lack of confidence in other parts of the specification. Most managed to show methods and working clearly, although presentation of proofs often left much to be desired in some cases.

Work of a very high standard was seen from many candidates and the discriminators worked well.

Report on individual questions

Question 1

This standard question was very well answered, proving an easy starter for the vast majority of candidates. There were some cases where wrong values were used and in particular using the value of x as 0.2 and 0.3 when they should have been 0.1 and 0.2 respectively. There were a few cases of premature rounding for the second approximation which because of the requirement of 3 significant figures for the third approximation, was not penalised further.

Question 2

This question was answered well with the allocation of follow through marks making it a good source of marks for many candidates.

In part (a), most candidates obtained the vector correctly using the cross product. However a significant number made a sign error on the **j** component.

In part (b), again most formed the dot product correctly and were allowed a follow through accuracy mark from part (a). A surprising number of candidates thought the result was a vector (5**j**) or made errors in pairing up the components for the scalar. 5 + 5 = 10 was not uncommon.

In part (c) most realized the link with part (a) although some found the area of the wrong triangle or attempted $\frac{1}{2}|a||b|\sin\theta$ meeting with less success.

In part (d) a significant number of candidates thought that the volume was $\frac{1}{3}|a.(bxc)|$ rather than $\frac{1}{6}|a.(bxc)|$.

Question 3

Candidates performed well on this question, particularly in part (a).

In part (a), most opted to obtain the characteristic equation which they processed correctly to verify the given eigenvalue and to obtain the other two eigenvalues. A significant number chose to verify that 7 was an eigenvalue independently.

In part (b) most candidates could set up the appropriate equations and could proceed to obtain a correct eigenvector. However, there were a significant number of instances of algebraic errors as well as cases where candidates thought that 7y = 7y meant that y = 0.

Question 4

The majority of candidates performed very well on this question.

In part (a), implicit differentiation and the use of the product rule were usually sound, enabling most candidates to obtain the third derivative correctly. The most common error was to obtain $2y\frac{\mathrm{d}y}{\mathrm{d}x}$ rather than $2y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ in that part of the product.

Most candidates knew how to proceed in part (b) and generally, the only mistakes were as a result of incorrect differentiation in part (a) although the first term was sometimes given as zero rather than 1.

Question 5

The majority of candidates used De Moivre's Theorem and the binomial theorem to efficiently and accurately obtain the printed identity. Some other methods were seen that also involved De Moivre but were less successful.

In part (b), candidates used the hence and used the correct identity for $\cos 2\theta$ to make progress in solving the equation. It was common to see division through by $\cos^2\theta$ and hence a solution lost. However, many could proceed to factorise and solve the remaining quadratic in $\cos^2\theta$ but all too often, the $\pm$ was omitted again resulting in lost solutions.

Question 6

Good candidates had no difficulty with either part (a) or part (b) and many solutions to these parts were efficiently and clearly presented.

Part (a) saw the greater success and many candidates could accurately perform the required matrix multiplication and complete the proof correctly and thoroughly.

In (b) the more successful candidates chose the method of showing f(k + 1) – f(k) to be a multiple of 16. Those who chose to deal with f(k + 1) directly were less successful and in many cases were required to use induction again to complete their argument, for example to show that $3 + 5^{k+1}$ is divisible by 4.

Question 7

Most candidates could use the fact that the lines intersected to correctly find the value of α.

In (b) there were some cases where candidates did not know how to start but those who did, correctly found a normal vector and proceeded to use a point in the plane to obtain the Cartesian form.

In (c), success was varied. Those who remembered the formula often completed successfully but there were many cases where the formula had not been remembered correctly, usually with an incorrect expression in the denominator. Alternative methods were rare and those that were seen were almost invariably unsuccessful.

Question 8

Most candidates knew the correct method for establishing the cartesian equation of the locus in part (a), although a few forgot to square the 2. Those who included i in the modulus calculation usually failed to notice that the equation they produced was not that of a circle.

In part (b), there were very few fully correct responses. Many candidates could at least progress to $|3w-12| = 2|4iw-12|$ but were then unsure how to proceed to obtain the required form. The step from $|12-4iw|$ to $4|w+3i|$ was beyond all but the best.

6677 Mechanics M1

Introduction

The paper seemed to be of a suitable length for the vast majority although because the last question was on vectors it wasn't always clear whether weaker candidates were running out of time or running out of ideas. The paper did prove to be more demanding than some summer papers from previous years. In many cases candidates seemed to have great problems applying mechanics principles in slightly different scenarios, showing a lack of real understanding. Many candidates do not realise that a magnitude must be a positive number and many do not understand the difference between speed and velocity. The first question proved to be a problem for some candidates and Q3, Q6(c), Q7(c) and Q8(d) caused some problems also. The best source of marks were Q4 and Q5. Overall, candidates who used large and clearly labelled diagrams and who employed clear and concise methods were the most successful.

In calculations the numerical value of g which should be used is 9.8, as advised on the front of the question paper. Final answers should then be given to 2 (or 3) significant figures – more accurate answers will be penalised.

If candidates run out of space in which to give their answer than they are advised to use a supplementary sheet – if a centre is reluctant to supply extra paper then it is crucial for the candidate to say whereabouts in the script the extra working is going to be done.

Report on individual questions

Question 1

This was a more difficult question 1 than usual, in that neither u nor a could be found directly from the given information and it was necessary to set up a pair of simultaneous equations. Many were able to write down an equation for the motion from P to Q but then struggled to find another one. By far the most common error was to say that the average velocity over an interval was equal to the actual velocity at one end of it. Those candidates who produced two correct equations invariably produced the correct answers. A few candidates found the acceleration but then forgot to find the value of u.

Question 2

In part (a) the majority of candidates were able to find the correct angle. For those that didn't the most common error was to find the complementary angle. In the second part, provided that it was realised that the sum of the **j** components was equal to zero, full marks were usually achieved. A significant number of candidates equated the sum of the **i** components to zero.

Question 3

Impulses continue to cause problems and a correct solution to part (a) was rarely seen. Most candidates know that impulse = change in momentum but few can cope with the signs correctly and the impulse in the first part almost always had the wrong sign. The second part produced more success and if the impulse-momentum principle was used again, this part was independent of part (a) and so full marks could be scored. Some tried to use the conservation of momentum principle in part (b), but this relied on using their possibly incorrect answer to part (a).

Question 4

This question was well done by the majority of candidates. Most made valid attempts at resolving parallel and perpendicular to the plane. The most common error was where candidates obtained the sin/cos of the complementary angle. Others used sin(4/5) or cos(3/5). Many successful candidates used the actual angle 53.1 rather than working with fractions for the trig. ratios. Some thought that the friction force was 1/3. A few managed to obtain the “correct” answer fortuitously by using $R = 0.5a$.

Question 5

Most candidates scored three marks for $F = P\cos 50°$ and for $F = 0.2R$. However, errors were often made in the vertical resolution, with some ignoring P completely, giving $R = 15g$, while others included a component of P but made a sign error. A small minority of candidates was unable to eliminate R legitimately between their equations, while a significant number lost the final A1 for giving the answer as 36.9 (or 36.93).

Question 6

Part (a) was well done by the majority of candidates and a good number went on to use the answer correctly in part (b). If mistakes were made they were the usual sign errors or more seriously, in terms of marks lost, missing terms.

The third part was poorly done. There was confusion over the direction of the forces and the concept of thrust. A few candidates halved the thrust and used 50N in each equation. Some used the values of the acceleration and tension from previous parts.

Question 7

Most candidates chose to take moments about Q in part (a). Common errors were incorrect distances, missing g's or lack of a distance in the moment of T_P. Some used the answer for part (a) and resolved vertically to obtain the answer for part (b).

The third part proved to be difficult for many candidates and answers with inequalities in x were offered. Some candidates used $x = 0.1$ or 1.39 to calculate the boundaries. A few managed to get the correct boundaries but did not express their answers correctly.

In the final part a significant minority lost marks by using $T_P = 3\ T_q$ to obtain their answer.

Question 8

Most candidates were able to gain the first six marks and most seemed to know that, in part (c), they needed to perform a subtraction on $\mathbf{r}_H$ and $\mathbf{r}_K$ although some were unsure which way round to do it. Another common error was to equate the position vectors and then fudge the answer. This received no credit.

In part (d) many candidates assumed that the hikers would meet and equated just one pair of components to produce $t = 20$. If they then used just one hiker to find $24\mathbf{i} + 82\mathbf{j}$ they scored only 2 out of 5, if they used **both** hikers, they scored full marks. There were a number of other ways of obtaining $t = 20$, some spurious, but provided that the candidate verified that both hikers were at the point with position vector $24\mathbf{i} + 82\mathbf{j}$ at $t = 20$, they could score all of the marks.

6678 Mechanics M2

Introduction

This paper proved to be accessible to the majority of candidates. Much of the work seen was of a high standard - clearly and concisely set out and accompanied by clear diagrams. A major problem noted by the examiners was the poor technique in manipulating algebra demonstrated by many candidates – there were too many examples of responses receiving poor marks because of simple errors in transforming equations. Candidates need to be reminded that their working needs to be particularly thorough when they are working towards a given answer – too many succumb to the temptation to fudge the result.

It was pleasing to see relatively few accuracy errors in solutions involving the use of an approximate value for *g*.

Most candidates offered solutions to all eight questions, and they were able to complete their solutions in the space provided in the booklet.

As in previous papers, the candidates found the work–energy principle (Q7) difficult to apply. The thrust (Q4) also proved to be very challenging.

Report on individual questions

Question 1

Candidates found this very accessible with the majority obtaining the correct velocity. Unfortunately many did not proceed to find the speed, which was a careless loss of two marks. Common errors included sign errors in the original equation, or in rearranging the equation, and errors in manipulating the fractions. Some candidates made the mistake of trying to work with the magnitudes of impulse and momentum.

Question 2

This question provided the opportunity for candidates to show that they could both differentiate the velocity function to find the acceleration and integrate it to find the displacement. In general both were done successfully, although as usual there were candidates who incorrectly attempted to solve the problem using constant acceleration formulae.

Although the majority of candidates used differentiation in part (a), there was also a large number who treated it by completing the square, and they were often successful in this approach. A number of candidates produced a table of discrete time values and corresponding speeds of the particle. Unfortunately they rarely scored full marks for their effort as the supporting statement about the symmetry of a quadratic function was usually missing. The most common error among candidates using differentiation was to stop when they had found the time and not go on to find the speed. In part (b) it would have been reassuring to have seen more candidates - even the successful ones - giving a more rigorous treatment of the constant of integration. Algebraic errors in solving the equation $4T^2 - \frac{1}{3}T^3 = 0$ were surprisingly common.

Question 3

Most candidates dealt with this question very well; a great proportion of candidates being able to score full marks for both parts. Part (a) was invariably correct, but a small number of candidates added a mass/weight to the resistance given in the question.

In part (b) the usual approach was to find the force of 330 N separately and then to substitute into $P=Fv$. Some candidates used the power from (a) as a force so scored nothing. Some wrote a correct equation for the total resistance to motion but forgot to include the 120 when they completed the calculation; a few included the 120 twice in some way - either adding or subtracting from 330.

A significant minority lost the final A mark through over-accuracy following the use of an approximate value for g or incorrect rounding.

Question 4

In part (a) candidates who realised the thrust in the strut acted in the direction *CD* were generally successful in finding its magnitude. The overwhelming majority applied the most simple method of taking moments about *A*, although much longer alternative methods were often seen. A significant number, however, appeared to be confused by the use of the word thrust and many took this to be a vertical force acting at *D*.

For part (b) many candidates were able to find the horizontal and vertical components of the reaction at *A* and then the correct magnitude of this force. Most went on to find a direction but a significant number were unable to describe this direction properly. It was surprising to see relatively few diagrams and yet a diagram would have shown direction clearly.

The confusion over the direction of the thrust led to many errors in the vertical component of the reaction. Some candidates falsely assumed that the reaction at A acted vertically/horizontally. The candidates who took the force at *D* to be vertical were unable to complete this part as there was no horizontal force present.

Question 5

This question was generally well answered by most candidates. It was pleasing to see many correct evaluations of the centre of mass of the triangle and many completely correct solutions to part (a). A few candidates attempted breaking up the lamina into rectangles and triangles rather than subtracting the moment of the triangle from the moment of the rectangle and so made the question much more difficult. Of those who calculated the areas of the rectangle and triangle correctly some failed to subtract these and added them instead. Most seemed to be happy with the use of large numbers for areas and only a few reduced these to a ratio. A very small number of candidates tried to replace the lamina by a framework of rods.

In part (b) a significant number of candidates failed to recognise that the lamina was symmetrical and wasted time in finding the distance to the centre of mass from *AB* using the same method, rather than using symmetry to write it down.

Many candidates lost the last two marks by finding the angle between *AD* and the vertical instead of the angle required.

Question 6

Part (a) was answered well by many candidates. The structure of the question, explicitly requiring the horizontal and vertical components as a starting point, led to many more successful answers than might otherwise have been the case. Most achieved the method marks successfully for both components but several had a sign error for the vertical component or some sin/cos confusion. As the answer was given, most candidates managed to recover sufficiently from initial errors to gain full marks. There were a few cases when a candidate had an incorrect expression but then wrote down the correct result! Some lost the final mark because another step was needed to show the given result.

In part (b) many candidates managed to use the given formula, allowing them to find u correctly, even if they had made an error in part (a). However, a significant number of candidates made errors in manipulating the given expression arriving at an incorrect value for u. A few candidates did not see that parts (a) and (b) were connected and started again with horizontal and vertical components. Others did not realise they could find u by substituting into the given equation. It was common for candidates to lose the last three marks. In finding u some thought they had already found the speed as it passed over the fence and proceeded no further.

A few candidates chose to use the more simple method of work-energy for the final part. These candidates usually scored full marks. The most common approach was to try to combine horizontal and vertical components of the final velocity. Some found only one component of the speed (usually the vertical one) and gave this as their answer. Others used u unresolved in either the vertical or horizontal direction and then combined to find v incorrectly. There were a significant number of responses making inappropriate use of $v^2 = u^2 + 2as$ to gain 9.13 incorrectly.

A few candidates scored no marks in part (b) because they made no attempt to find u.

Question 7

In this question a significant minority tried to find the acceleration in part (a), despite being instructed to use work and energy. For those candidates using the required method, the most common error was to include the work done against gravity with the work done against the friction but then also include the change in gravitational potential energy, thus double counting. Although many answered successfully, there were some who rather "fudged" the arrival at the given answer. A common error was to assume that the vertical height gained was 7, and this lead inevitably to the correct answer without the necessary algebraic treatment of the situation. The most common approach to part (b) was to revert to the use of $F = ma$, rather than sticking to the work-energy principle, for which they had already done most of the work in part (a). Many who attempted a work-energy approach forgot to include the work done against friction in this part, even though they had used it in part (a). Very few candidates used energy and point X as the initial and final position to find the answer.

Question 8

Parts (a) and (b) were tackled with confidence by most candidates although a few were not sufficiently careful with signs and/or had long-winded algebraic manipulation to achieve the given result. CLM and the impact law were generally used correctly.

Part (c) proved to be more challenging and differentiated between the stronger candidates and those with confused concepts. No mention of e in the question caused some to ignore it or, more commonly, to assume the previous value. There were various arguments used to justify their final statement but, encouragingly, the range for e seemed to be understood.

6679 Mechanics M3

Introduction

There were some very straightforward questions on this paper, giving weaker candidates a good opportunity to show what they knew, but also some questions which challenged the most able. Candidates' attention should be drawn to the advice on the front of the question booklets – "You should show sufficient working to make your methods clear to the examiner" and "Answers without working may not gain full credit". Some candidates are developing a tendency to ignore the former of these. For example, failure to show substitution of (the correct) limits when integrating can cost several marks if the candidate then makes a small slip when using the calculator. If the answer is wrong the examiner cannot award for the numbers that have been substituted. Mechanics is a branch of applied mathematics; it follows that some knowledge of the pure mathematics that is being applied will be needed. Candidates had difficulties with this in Q4 and Q6. To their credit when they had difficulties, their good exam preparation and technique ensured that this did not adversely affect their performance in other questions.

Report on individual questions

Question 1

This was a straightforward opening question but many candidates spoiled their solutions with careless errors. Most managed to use Pythagoras to calculate the length of the extended string. However some appeared to have misinterpreted the information given in the question and took the natural length as 12 m instead of 8 m. Hooke's law was well known but problems arose as candidates were confused between the full string and half strings, with some thinking the tensions in these were different. Significant numbers gave the mass as their final answer instead of the weight as demanded. Part (b) gave rise to fewer problems. The formula for the elastic potential energy was known by all but a small number of candidates and many who had an incorrect extension in part (a) either completely recovered to gain full marks or gained 2 of the 3 marks by follow through.

Question 2

This question produced many completely correct solutions. Some candidates however ignored the information provided about the masses of the two parts of the route marker and calculated their own "masses" by using volumes instead, usually assuming both sections to be solids. Almost all could produce a valid, if not correct, moments equation and so gained some marks. The most frequently seen error in part (b) was to have the required fraction upside down resulting in $h = \frac{r}{24}$. Some candidates lost the final mark here through giving r in terms of h instead of answering the question asked.

Question 3

Some candidates found this question to be very straightforward and gave very neat concise solutions to both parts. However, that was not so in the majority of cases. Many produced horizontal and vertical equations with correct components of the reaction. However, not all knew what to do with these equations and no further work was shown. Frequently candidates equated the horizontal component of the reaction to $mr\omega^2$ (where *r* was the given radius of the bowl). Most of these candidates then found $\tan\theta = \frac{3}{2}$ and linked back to the reaction and *d* (in terms of *r*) via $\sin\theta$ and $\cos\theta$. As they thought that *r* was the horizontal radius (rather than the radius of the bowl) they also used $\tan\theta = \frac{r}{d}$ to arrive at what appeared to be a correct result. Few marks could be awarded, however, for work which was based on such a serious initial error. Some candidates simply wrote down $F = mr\omega^2 = \dfrac{3mg}{2}$, which is a correct equation, although it was often not clear that the candidates really understood this. Many could not produce any equations to enable *d* to be found or even revealed their lack of understanding by proceeding to treat $\dfrac{3mg}{2}$ as a component of the required reaction.

Question 4

Solutions to this question were spoilt by poor integration; $\int x^{-4}\,dx$ was seen as x^{-5}, x^{-3}, $-3x^{-3}$ and even $\ln\left(x^4\right)$. Many marks were lost by candidates who were determined to arrive at the given answer of 21π even though their working could not support this result. A large number of good candidates lost the final mark in part (b) as they completely forgot that *coordinates* had been requested and so a *y*-coordinate was needed as well as the *x*-coordinate. Other errors in part (b) included finding the centre of mass of a lamina, ignoring the volume found in (a), and various further integration mistakes. There was some confusion over the significance of the lower limit for *x* being $\frac{1}{4}$; some candidates seemed to think it was necessary to subtract $\frac{1}{4}$ from their result.

Question 5

Part (a) of this question was a fairly standard vertical circular motion question and many candidates could produce two equations by considering change in kinetic and potential energies and resolving along the radius. Some tried to resolve vertically with little success. The need to find a difference in potential energy between two points created problems for some candidates who seemed to think the particle either started at the top of the circle or from the horizontal level of *A*. The presence of a printed answer did enable some candidates to retrace their steps and correct the signs in their working. This was not a problem as long as the result was consistently correct. Part (b) produced a different set of problems. Some candidates thought that the string broke at the point where $T = 0$ and so found an incorrect value for the initial velocity of the projectile. The projectile problem could be solved by using vertical motion under gravity or by energy considerations. Many who opted for energy forgot that the particle was still moving (horizontally) at the highest point of its path; some of those who opted for vertical motion under gravity forgot that the initial velocity they had found needed to be resolved vertically before use in the appropriate equation. Most who had worked to this point in the question remembered to finish off by finding the distance below the horizontal through *A*.

Question 6

In all three parts this question challenged all but the most able candidates. The question drew on topics from C2, C4 and M2. Power is an M2 topic and can be used in that or any subsequent mechanics unit. It was the difficulty of using and applying several different areas of mathematics that seemed to cause problems. At times the core repertoire of questions will be extended and candidates did struggle particularly with part (c) which was slightly unusual.

Success in part (a) needed the use of power = driving force $\times$ speed; many candidates had forgotten this fact and others quoted power = net force $\times$ speed, being confused between the driving force and the resultant force. In many responses there was a difficulty to interpret the proportionality statement in the question correctly. Only a minority realised that using $a = 0$ with $v = 20$ would enable them to obtain the constant of proportionality. It was pleasing to see some reasonable attempts at setting up an equation of motion, though without the constant of proportionality candidates were unable to make much progress. Some candidates hoped they could use the given answer to obtain a value for the constant. Those who know what to do with a question of this type are almost always better advised to work *towards* the printed answer rather than start from it. A few did not include the mass, thinking this was testing section 1 of the specification rather than section 3.

It was hoped that the pure mathematics in part (b) would have been accessible to candidates at this level. Many candidates capably gained the first two marks (for separating the variables), but sometimes these became the only marks gained on this question. A proportion of candidates had difficulty with applying the principles involved in separating variables. The following integration was poorly completed. Zeros were frequently lost in otherwise correct work. The most able candidates realised that counting zeros once was enough and used standard form thereafter. In part (c), candidates needed to work with the printed answer from part (a), after changing the derivative form of the acceleration. It was common to see candidates attempting to use their work in part (b).

Question 7

Part (a) was usually handled confidently with Hooke's Law in the equilibrium position applied to each spring and the resulting tensions equated. There were some unfortunate careless errors such as "total extension $= 5 - 2 - 1 = 3$" which appeared far too often. Mistakes in the extensions led to difficulties in the later parts of the question but some able candidates didn't think to check their extensions when their confidently applied method in (b) failed to work as expected. The work seen in part (b) suggested that many candidates are not aware that to prove SHM they need an equation of motion that reduces to the form $\ddot{x} = -\omega^2 x$; some even tried to give a written explanation of the motion, with no equations provided at all. Even if the question had not told them that the equilibrium position was the centre of the oscillation, candidates should have known this and measured their displacement from this point; many chose some other point instead. There were also numerous sign errors in the equation of motion, often "corrected" but not always in a valid manner.

Lack of success in part (b) meant many candidates had no suitable value for ω to use in part (c). Some were content to invent a number and continue, thereby making the method and follow through marks available, others gave up, though this may have been due to lack of time. Most who worked on this part realised that the amplitude could be obtained using $v_{\max} = a\omega$ and hence the time required using $x = a\sin\omega t$. Solutions using $x = a\cos\omega t$ were seen occasionally but the extra work needed to obtain the necessary time was often omitted. The instruction to find the *proportion* of the time was usually ignored completely resulting in the loss of three marks as the period was not calculated since the necessity for it was not apparent.

6680 Mechanics M4

Introduction

Well prepared candidates found this paper very accessible, although parts of some questions challenged even the most able candidates. A wide range of quality of responses was seen, with some candidates clearly picking out selected topics, and others offering concise and clearly argued solutions to all six questions. A few candidates scored very highly, but many still have work to do in order to be fully confident with the underlying mechanics in this specification. As has been commented on before, too few candidates support their work by drawing clear diagrams, which could help to clarify their understanding of the question and assist them in explaining their solution. Candidates need to remember that where an answer is given they need to be particularly rigorous in their working in order to demonstrate that they have indeed derived the required result. Similarly, candidates need to be more careful over the accuracy of their solutions: there were many instances of candidates approximating too much or too soon in their working, leading to inaccuracy in the final answer.

Report on individual questions

Question 1

Most candidates started by applying CLM parallel to the plane to obtain an equation connecting the speed of approach and the speed of rebound, which enabled them to form an expression for the fraction of kinetic energy lost. Many candidates, who also considered the components of the speeds perpendicular to the surface and used the impact law to find the value of e, did eventually reach a correct conclusion, but in most instances this method took longer because they were doing more work than necessary. Some candidates found the amount of kinetic energy lost and did not go on to complete the question. There was also evidence of some confusion amongst candidates who worked only with the components of velocity perpendicular to the plane and did not express their change in kinetic energy as a fraction of the *total* initial kinetic energy – candidates should be discouraged from thinking of energy as resolvable, as this has clearly led to some misconceptions.

Question 2

This proved to be a particularly accessible question, with many candidates offering completely correct solutions. The most common errors involved incorrect signs in the initial differential equation, or problems in substituting the limits and simplifying the answer. Some candidates integrated $\dfrac{v}{-g-kv^2}$ to obtain $-\dfrac{1}{2k}\ln\left(-g-kv^2\right)$ and went on to work with the logarithms of negative quantities with no indication at all of any recognition that this should have concerned them.

A few candidates started with a differential equation in v and t. Although it is possible to complete the solution by this route, very few of these attempts got as far as a correct first integral, and almost all stopped work before achieving a differential equation involving x.

Question 3

Clear, correctly orientated diagrams were particularly helpful in this question. Many candidates understood that in order for Q to pass as close as possible to P the course set by Q needed to be perpendicular to the relative velocity. It proved to be more difficult to determine the actual bearing, with many choosing the direction opposite to the required direction. When the working was correct, the final mark was often lost through not expressing the bearing to the nearest whole degree.

Candidates with a good diagram usually used a correct method to determine the shortest distance between P and Q in part (b), but this work was often affected by rounding errors due to premature rounding in calculating the angle in the triangle.

Several correct solutions to (c) were seen, but some candidates were not able to find the time taken to the point of closest approach because they did not realise that they had sufficient information to find the relative velocity.

Some completely correct vector solutions were seen, although candidates using this method were generally less successful.

Question 4

Although this was a standard problem, many candidates lost marks in part (a) because they fudged the result for the lighter particle rather than calculate the distance of the particle below B. Some candidates offered no attempt to part (a) and simply went on to use the given result. In parts (b) and (c) the majority of candidates were confident in using the given expression for the potential energy to identify the position of equilibrium of the system and to conclude that this was a point of stable equilibrium. The problems occurred in trying to explain why there was only one solution for $\cos\theta$ when their quadratic equation had two roots. The given condition on L was often interpreted as an upper bound rather than a lower bound leading to some spurious reasons for rejecting one of the roots. Some responses demonstrated little understanding of the physical properties of the model, and of strings in particular.

Question 5

In part (a) the majority of candidates demonstrated their dislike of vectors by calculating the $\mathbf{i}$ and $\mathbf{j}$ components of the velocity of B in two separate equations. Many candidates lost marks because they only considered the $\mathbf{j}$ component. In their attempts to derive the given result without the use of vectors some candidates created some unconvincing notation and arguments.

Most candidates in part (b) understood that impulse causes a change in momentum, but there were many sign errors in finding the impulse, with large numbers of candidates arriving at the negative of the required result when considering the impulse on A. Equally, some candidates started by finding the impulse on B and did not take the final step to answer the question. When candidates had found an impulse acting in the right direction they were usually able to draw the correct conclusion about the line of centres of the two spheres, although here too some of the presentation was rather muddled and the examiner was often left to interpret the candidate's use of the word 'it' to apply to whatever the candidate had in mind.

Part (c) challenged many candidates, and was a good indicator of the level of understanding of what was happening in this collision. Only the stronger candidates appreciated the need to resolve their velocities parallel to the line of centres, and for these candidates the initial velocity of A proved the most difficult to deal with. This was largely because the usual approach was to draw a diagram and use a trigonometry, rather than using the scalar product of the vectors.

Question 6

In part (a) many candidates claimed to have derived the given equation, but very few actually did so correctly having considered the equation of motion of the particle *P*. Those candidates who started with a clear diagram were far more likely to realise that they needed to consider both the distance moved by *P* and the extension in the spring. It was very common for candidates to score only one mark here, for finding the tension in the spring correctly.

Almost all candidates in part (b) were confident in attempting to solve the second order differential equation, although several did not choose a correct form for the complementary function, and a few struggled with the particular integral. It was common to see the correct solution for x.

In part (c) it was reassuring to find many candidates knowing that the maximum value of $(1-\cos nt)$ is 2, although 1 was a popular alternative answer. Candidates who did not use the basic properties of the trig function were often able to find the maximum extension by using calculus, but here too there were some difficulties in identifying the value(s) of t for which $\sin nt = 0$.

The response to part (d) confirmed that many candidates had little or no idea of the correct derivation of the equation in (a). Most candidates believed that the speed of *P* and the rate of change of the extension in the spring were equal. Correct responses were seen, but usually only from the stronger candidates.

6681 Mechanics M5

Introduction

Overall the paper proved to be accessible and marginally more straightforward than last summer's examination, with candidates of all abilities given the opportunity to demonstrate what they could do. Moreover, the vast majority were able to complete it in the time allowed. The questions which seemed to cause most difficulty were Q4 and Q6 i.e. those involving parts of the specification dealing with rotation. Despite comments in previous reports, some candidates continue to ignore the examiners' advice (and indeed the question!) when dealing with the period of a compound pendulum – see the comment on Q6. The best sources of marks were Q2 and Q5.

In calculations the numerical value of g which should be used is 9.8, as advised on the front of the question paper. Final answers should then be given to 2 (or 3) significant figures – more accurate answers will be penalised.

If candidates run out of space in which to give their answer than they are advised to use a supplementary sheet – if a centre is reluctant to supply extra paper then it is crucial for the candidate to say whereabouts in the script the extra working is going to be done.

Report on individual questions

Question 1

Most candidates scored at least 4 out of 7 on this question by using the work-energy principle to find the speed of P but some did not then go on to find a velocity, using the fact that the velocity is parallel to AB. Candidates whose dot product resulted in a vector gained only the first mark for **AB**. There were a number of other approaches which involved finding the acceleration, either as a scalar or as a vector, followed by the use of an appropriate constant acceleration formula.

Question 2

This question was a good source of marks for many candidates. There were two common errors: finding the values of the arbitrary constants before adding in the particular integral to obtain the complete general solution; calculating the value of the constant for the particular integral as $\frac{1}{8}\mathbf{j}$ but then forgetting to put the e^{2t} back in. Some candidates unnecessarily doubled their workload by considering separate differential equations for the **i** and **j** components.

Question 3

The majority of candidates recognise that they must start variable mass questions by considering the change in momentum over a small time interval (in this question the change in momentum was zero!). They must put the total mass at the beginning of the interval as m and that of the rocket at the end of the interval as $m + \delta m$ (in the case of rocket questions, δm is negative). Candidates who simply try to memorise the solution to one version of this question and then try to adapt it are almost invariably unsuccessful.

Although a significant number struggled with the first part, there was more success with part (b). The easiest method was to use the chain rule to split up dv/dt, but some candidates successfully integrated the result from part (a) by separating the variables and then differentiating the result with respect to t.

Question 4

The simplest way to split the lamina into strips for part (a) was to use the hint in the question and have the strips parallel to AB. Those who used strips parallel to OA made the question more complicated and more difficult but were, nevertheless, often successful.

The commonest errors in the second part were either to get the position of the centre of mass wrong or to forget to multiply J by $2a$ when applying the rotational impulse-momentum principle.

Question 5

Many candidates scored well on the first part of this question, although there were many sign errors. It was common for even good candidates to throw away 2 marks by not finding the magnitude of $\mathbf{F}_3$.

In part (b), most candidates used vector products to find the moments and then put the sum of these equal to zero but some did not know how to proceed when they found that their equations were not independent. A surprising number forgot to put "**r** =" at the beginning of their line equation.

The final part of the question was usually well answered but again some forgot to find the magnitude of the couple.

Question 6

The weakness of candidates when answering questions about rotational mechanics was shown by the large number who only attempted part (a) of this question. This sometimes involved blatant "fudging".

In part (b), the attempt at the energy equation was often reasonable, but using Newton's second law along the rod often contained dimensional errors, such as multiplying the component of the weight by a distance.

In the final part it was necessary to obtain an expression for the angular acceleration either by taking moments or by differentiating the energy equation from part (b). Without this starting point, it was not possible to use the approximation $\theta \approx \sin\theta$ and so few marks could be obtained. There are still a number of candidates who learn a formula for the period of a compound pendulum and this should be strongly discouraged as the questions are usually designed to test understanding and the ability to work from 1st principles.

6683 Statistics S1

Introduction

There was a lower standard of responses than in previous S1 papers and, although candidates showed which techniques to select and also how to use them, they found some of this paper challenging.

Even though the specification emphasises the general rule of answers being given to 3 significant figures and the importance of fractions or exact decimals, a number of candidates do not follow these instructions. For example, many rounded to 0.8 or 0.81 in Question 1 and used 0.83 instead of $\frac{5}{6}$ or $0.8\dot{3}$ in Q2. Good practice such as setting out the formulae used in the probability questions and a table for the calculation of the mean and standard deviation in Q4 usually led to higher marks.

In Q3 and Q4 where the idea of proportionality was needed were badly done. A major weakness showed up in the interpretation of the correct class limits resulting in a minimum loss of 2 marks in Q3 and 6 marks in Q4.

Calculation of the standard deviation continues to defeat a considerable number of candidates in spite of the availability of calculators to check the answer.

Comments tended to be verbose and interpretation weak. Comments are expected to be in the context of the question and that, at this level, the use of statistical terms and reference to the quantities calculated in the question are required.

Report on individual questions

Question 1

The vast majority scored full marks in part (a). The most common reason for losing marks for the correlation coefficient was for rounding to less than 3 significant figures without having stated the more accurate answer first. A large proportion of candidates still believe that stating ‘it’s a high level of correlation’ will be enough to gain the mark for interpretation. A fully contextual comment is required here, using the named variables of pressure and temperature and not just the letters p and t.

Question 2

Part (a) and part (b) were generally very well done with few candidates not knowing the correct structure of the tree diagram. A number did not fully label the tree diagram thus potentially losing the mark for the probabilities. Some candidates do not help themselves or the examiner by drawing very small diagrams. In part (b) it was pleasing to see very few candidates resorting to decimals and those who did seem to have got the message that exact equivalents are required using recurring decimals where appropriate. In part (c) many candidates demonstrated a lack of understanding of conditional probability. They could not transfer the context of the question into a formula and many still use P(*A/B*) with no explanation as to what *A* and *B* represent. Of those who did manage to write P(F’/L) many failed to see the significance of part (b)(ii).

Question 3

Although there were more correct solutions than in previous papers for this type of question the process required to answer this question was not applied successfully by a large number of candidates. The most common error in part (a) was to give an answer of 0.8. In tandem with this was an answer to part (b) of 7.5 where candidates recognised that the answer to part (a) times the answer to part (b) should be 6. Many candidates divided 9 by 3 in part (b) but failed to multiply by 2. Other candidates however produced two correct answers but nothing else. The variety of approaches may suggest some logical thinking rather than a taught approach to this type of problem.

Question 4

Very few candidates got full marks for this question, being unable to perform the calculations for grouped data, although the mean caused the least problems. Those candidates with good presentation particularly those who tabulated their workings tended to fare better. In spite of the well defined groups many candidates subtracted or added 0.5 to the endpoints or adjusted the midpoints to be 0.5 less than the true value with the majority getting part (a) incorrect as a result. As usual all possible errors were seen for the calculation of $\sum fx^2$ i.e. $(\sum fx)^2$, $\sum (fx)^2$, $\sum f^2x$ and $\sum x^2$. Use of 17.1 for the mean in the calculation of the standard deviation led to the loss the accuracy mark. Candidates are once again reminded not to use rounded answers in subsequent calculations even though they usually gain full marks for the early answer. The comment in part (c) was often forgotten perhaps indicating that candidates are able to work out the figures but do not know what they mean, although many did appreciate in part (d) that there is no skew in a normal distribution. As opposed to question 1, correlation was often mentioned instead of skewness although again this is becoming less common.

Question 5

There were some good responses to this question, but some candidates calculated the slope as 59.99/120.1, although 3/5 marks were obtained if they went on to produce the equation as $w = 6.8 + 0.50l$ provided a minimum of 2 significant figures were used. Candidates should be able to identify the independent and dependent variables from a contextual question. The accuracy mark for the calculation of the intercept was lost if they used the rounded value of 1.8 for the slope in the calculation for the intercept. Many candidates did not believe 60mm to be quite far enough away from the data range to be called extrapolation showing that they did not go back and read the question carefully enough and consider the range of values given.

Question 6

This entire question was usually very well done if part (b) was correct. Some candidates did not identify that the sum of probabilities should equal one and had problems trying to find the values of a and b resorting to guessing. Even if candidates could form two correct equations, some lacked the ability to solve these relatively simple equations. A number of candidates who had no success with part (b) gave up at this point but others managed to get part (d) and part (f) correct using the values given in the question. In part (e) many knew that they had to take 1.6^2 from their figure, and not having got the figures for a and b correct in part (b) they adjusted their number to come to 4.2, so that $4.2 - 1.6^2$ came to 1.64. On occasion it was difficult to distinguish between $\sum$ and E in the candidate's handwriting.

Question 7

Generally this question was not well answered by a large number of candidates. The terms and properties relating to probability do not seem to be fully understood, especially by weaker candidates. Part (a) was done surprising badly, with often the rest of the question fully correct. Part (c) was often correct when all else was wrong, demonstrating that candidates can use the conditional probability formula even if they do not understand it. Too few candidates write down the formula they are trying to use, which in part (d) was helpful in ascertaining if they were trying to use the correct method.

Question 8

More able candidates made a good start to this question. Part (a) was well done and part (b) usually gained the method mark. Part (c) proved to be much more of a challenge despite it being very similar to questions set in previous papers on this topic. Many candidates gained 4 marks, and there were a number who could see what was required but could not quite answer fully, submitting solutions which had most of the components, but not in the right sequence. Many adjusted the sign, either losing it during the calculation, or right at the end when -50 did not appear to be correct. Many candidates did not use 0.8416, settling for 0.84. A few used a probability rather than a z value but this was less than in previous years. Some candidates drew diagrams to help their thought processes. In part (d) candidates lost marks as they were not confident in interpreting what their figures meant. Many candidates did not use correct statistical language and thus lost the marks, others commented on the standard deviation for Y being lower than that for X without considering the magnitude of the difference.

6684 Statistics S2

Introduction

Candidates would appear to have had just enough time to complete this paper. There were few questions where no attempt had been made to produce an answer.

The level of work was very mixed. There were a number of candidates who had little idea about significance testing and quite a few who had problems with selecting the correct distributions.

The standard of presentation was very weak – it was often difficult to follow the work. The presentation of working on a page was often disorganised, this was particularly the case on Q7 where candidates made many attempts to answer the question

Report on individual questions

Question 1

The majority of candidates achieved full marks on this question with the most common errors caused by difficulties in identifying, interpreting and/or working with the inequalities. In part (b) whilst a few candidates wrote down $P(X \leq 13)$ they were unable to find this probability correctly. The most common error was to use $1 - P(X \leq 12)$

Question 2

Candidates seemed better prepared for this type of question than in previous years. Marks were often lost for not using λ or μ in the hypotheses and for not putting the conclusion into context. A significant minority of candidates found $P(X=1)$ instead of $P(X \leq 1)$ but only a few candidates chose the critical region route.

Question 3

This question was either answered very well with some text book solutions, although it seemed that only a minority of candidates earned all five marks, or badly with some strange descriptions. A reasonable number of candidates responded with comments that were very close to those in the mark scheme: evidence possibly of deliberate preparation and learning whilst others had internalised the concepts and provided responses in their own words. Whilst these responses might not have matched the 'official' answers, they nevertheless captured the essence of the concepts and were fully acceptable. There was confusion with the definition of statistics and parameters and part (b) was often attempted badly with candidates not knowing the definition of a probability distribution. On the whole this was one of the worst answered questions in the paper.

In part (a) candidates gave various definitions sometimes all muddled up. Not many candidates gave clear definitions but a common error was candidates writing "***any** function*" or "*no* ***other*** *quantities*".

In part (b) again the candidates had mixed success. A significant minority scored marks by knowing that a sampling distribution involved all possible values of the statistic and their associated probabilities.

In part (c) many could identify (ii) correctly and a variety of reasons were seen. This part seemed to be done well even by candidates who could not answer part (a).

It was interesting to see that a relatively large proportion of candidates who earned both marks for part (c), were unable to achieve either of the two marks in part (a). There was a connection between parts (a) and (c) that many candidates failed to recognise. If those candidates who wrote "(ii) is not a statistic because it has unknown parameters" had then reflected on their responses to parts (a) and (c), they could then have gone back to modify their answer to (a) in order to earn more marks.

Question 4

This was a very well answered question. Candidates were able to use binomial tables and gave the answer to the required number of decimal places. As in previous years there were some candidates who confused the critical region with the probability of the test statistic being in that region but this error has decreased. Candidates were able to describe the acceptance of the hypothesis in context although sometimes it would be better if they just repeated the wording from the question which would help them avoid some of the mistakes seen. There were still a few candidates who did not give a reason in context at all.

In part (a) many candidates failed to read this question carefully assuming it was identical to similar ones set previously. Most candidates correctly identified B(20,0.3) to earn the method mark and many had 0.0355 written down to earn the first A mark, although in light of their subsequent work, this may often have been accidental. A majority of candidates did not gain the second A mark as they failed to respond to the instruction "state the probability of rejection in each case". In the more serious cases, candidates had shown no probabilities from the tables, doing all their work mentally, only writing their general strategy: "$\mathrm{P}(X \le c) < 0.05$". Whilst many candidates were able to write down the critical region using the correct notation there are still some candidates who are losing marks they should have earned, by writing $\mathrm{P}(X \le 2)$ for the critical region $X \le 2$

Part (b) was usually correct.

Part (c) provided yet more evidence of candidates who had failed to read the question: "in the light of your critical region". Some candidates chose not to mention the critical region and a number of those candidates who identified that 11 was in the critical region did not refer to the manager's question.

Question 5

Part (a) was answered well with the majority of candidates gaining full marks.

Part (b) was also a good source of marks for a large majority of the candidates. Common errors included using 23.9…for variance and 19.5 instead of 20.5. A sizeable minority of candidates used 21.5 after applying the continuity correction. A few candidates had correct working up to the very end when they failed to find the correct probability by not subtracting the tables' probability from 1.

Question 6

Many candidates attained full or nearly full marks for this question.

In part (a) many candidates were unable to correctly state $\mathrm{P}(A > 3) = \frac{2}{5}$.

In part (b) some candidates multiplied their answer to (a) by 3 rather than finding (a) cubed.

Part (c) was well done and apart from those who made errors in the differentiation candidates gained full marks for this part.

The most common error in question 6 was drawing the sketch graph incorrectly in part (d). A straight line was often seen, either sloping from 0 to 5 or parallel to the x-axis.

In part (e) a few candidates attempted to calculate the mode rather than reading it straight from the graph. Not only did this waste time the result was usually incorrect.

In part (f) candidates confused f(x) with F(x) and/or used the formula for E(Y) incorrectly. Errors in the simple integration were often seen.

In part (g) few candidates chose to use P(X > 3) = 1- F(3) and instead used the method involving integration where too often the incorrect limits were used.

Question 7

A minority of candidates achieved a high rate of success on this question.

In part (a) most candidates were able to write E(X) = 2 without difficulty.

A variety of methods were seen in part (b). The method of the mark scheme was seen, perhaps only from a minority of candidates. Many candidates preferred to use calculus: $\int \mathrm{f}(x)\mathrm{d}x = 1$. However, the use of calculus requires more subtlety and sensitivity than was available to many of the candidates. Answers of a = ½ and b = 2 seemed to be not uncommon, resulting from the incorrect methods: $\int_0^2 ax\ \mathrm{d}x = 1$ and $\int_2^4 (b - ax)\ \mathrm{d}x = 1$.

There were candidates who obtained the correct answers using calculus, but it often took considerable working, in contrast to the expected method.

There were some candidates who obtained full marks to part (c) with solutions that were confident, fluent and accurate. Furthermore, many of these responses were also efficient: four or five lines of working provided a solution that was not just correct but contained all the required details. However, a wide variety of alternative responses were also seen. Some were indeed correct, but inefficient. Other candidates used an incorrect strategy. Some candidates only worked with the domain $2 \le x \le 4$. Others worked with both domains, but wanted to keep the domains separate, resulting in two separate versions of $\mathrm{Var}(X)$:

$$\mathrm{Var}(X) = \int_0^2 x^2 \frac{1}{4} x\ \mathrm{d}x - \left(\int_0^2 x \frac{1}{4} x\ \mathrm{d}x \right)^2 \text{ and } \mathrm{Var}(X) = \int_2^4 x^2 \left(x - \frac{1}{4} x \right) \mathrm{d}x - \left(\int_2^4 x \left(x - \frac{1}{4} x \right) \mathrm{d}x \right)^2$$

Some candidates then calculated the average of these two versions of the variance.

Many candidates also found E(X) from scratch in this part rather than using the answer they had in part (a). Not only did this waste time, but whilst they often had it correct in part (a) they gained an incorrect value by integration in this part which they then went on to use.

It must be noted that where the answer to a question is given, marks cannot be gained by restating this without sufficient working. Some attempts were made to describe the answer as proven even though no real working had been done.

A reasonable number of correct solutions to part (d) were seen. Some candidates went so far as to specify fully the cumulative distribution function before using the correct part to find the lower quartile. Even though this extra work was not required, strictly speaking, it did provide these candidates with a ready method for part (e).

It would appear that whilst most candidates attempted part (e), their responses consisted of a simple statement, usually "greater than 0.5" together with an irrelevant reason. A tiny minority of candidates responded in the manner intended. A few provided a clear diagram to illustrate this same argument. However, the majority of successful candidates preferred to evaluate the probability. This was not straightforward, except for those who had already obtained a full and correct version of the cumulative distribution function. Part (e) seemed to challenge all but the most able candidates.

Question 8

Parts (a) and (b) were completed successfully by most candidates. The most common errors seen were using the wrong Poisson parameter or identifying the incorrect probability in part (b).

Part (c) proved to be a good discriminator with only those with good mathematical skills able to attain all the marks for this part of the question. Few candidates used the method given on the mark scheme and chose to use natural logarithms instead. Whilst this is an accepted method this knowledge is not expected at S2 and full marks were gained by the most able candidates using the given method.

In part (d) a few candidates seemed confused by this with some using 1.7 or 2/15 as a probability rather than the 0.8 given in the question, and far too many seemed unable to use 60p and £1.50 correctly when calculating the profit.

6691 Statistics S3

Introduction

The paper was similar in standard to previous years. Q2(b), Q3(a) and Q4 were answered very well by most candidates but Q7(b) in particular proved more challenging.

Report on individual questions

Question 1

Most candidates realised that they would need to sample every 500$^{\text{th}}$ name on the list in part (a) but a number did not explain how to select the first member of their sample at random from the first 500 names. In part (b) it was clear that many candidates had learnt some standard reasons from a textbook and nearly everyone scored something here. A few candidates confused systematic and stratified sampling in (ii) but for the most part this question was answered quite well.

Question 2

It is often the case when we set questions like part (a) that many candidates simply calculate a 95% confidence interval for the mean and the same happened here. They then repeated the technique in part (b), usually with the correct z value, and there were many fully correct confidence intervals found here. Some candidates were not prepared to make a decision in part (c) and hedged their comments, possibly because 19.5 was "in" the answer to part (a) but not in the confidence interval in (b). The examiners required a clear rejection of the grower's claim, based on the fact that 19.5 was outside the 98% confidence interval and a good number of candidates gained both marks for these two simple statements.

Question 3

Part (a) was a fairly standard application of Spearman's rank correlation and it was answered very well with most ranking BMI from low to high. In part (b) there were fewer candidates losing marks for failing to give their hypotheses in terms of ρ and there were many good answers here too although sometimes it was difficult to interpret their conclusion: "there is positive correlation between BMI and finishing position" may be true but is not as clear as saying that "there is evidence to support the doctor's belief" or "there is evidence that a low BMI leads to a greater level of fitness". Many missed the point in part (c) and simply mentioned that Spearman's rank correlation was "easier" or that there were no tied ranks.

Question 4

This was a very straightforward question and most gained full marks. Sometimes their notation was far from clear. It would be nice to see all candidates writing $X \sim \mathrm{N}(55, 3^2)$ and then stating $\bar{X} \sim \mathrm{N}\left(55, \frac{3^2}{8}\right)$ before calculating $\mathrm{P}\left(\bar{X} > 57\right)$ but it was clear from their calculations that they knew what they were doing even if they did not always communicate their intentions clearly.

Question 5

Parts (a) and (b) were answered very well and most scored full marks on these two parts but part (c) proved more challenging. Many insisted on including the mean of 1.05 in their hypotheses even though this was incompatible with their correct treatment of the degrees of freedom. The pooling of the last two groups was usually carried out and the calculation of the test statistic was often correct. There was some confusion over the calculation of the degrees of freedom though: many subtracted 2 but others only 1 and some were not sure whether to subtract from the number of classes before or after the pooling. A number failed to score the final mark because their conclusion was not given in context: comments such as "there is evidence to support the manager's claim" or "there is evidence that the number of goals scored in football matches does follow a Poisson distribution" are fine; "the data follows a Poisson distribution" is not.

Question 6

Part (a) was usually answered very well with most stating their conclusion in context but a few losing a mark for simply using μ_1 and μ_2 in their hypotheses without giving any indication which population was which. In part (b) the Central Limit Theorem was a popular wrong answer (it is not an assumption but a theorem that is invoked because of the large samples) but many did mention independence and some that the sample variances were assumed equal to the population variances but it was rare to see a candidate earn both marks here.

Question 7

Most answered part (a) well with the calculations being clearly laid out. Part (b) caused many problems. Many only considered one "tail" so they effectively used $\mathrm{P}\left(\bar{X}-\mu<0.05\right)=0.90$, others realised that the calculations from a confidence interval were involved and equations involving $\frac{0.2}{\sqrt{n}}\times z$ appeared but they were not always correct. Some did muddle through to the correct answer but there were few clearly set out solutions.

Question 8

Part (a) was answered very well with most scoring both marks. The majority secured all the marks in part (b) too but some forgot to square the 3 and the 4 and others forgot to add the variances. Most were able to write down E(B) and often Var(B) too but many did not clearly form a new distribution $D = B - A$ and proceed to write down the distribution of D and state the problem as P(D > 0). Those who did were usually able to complete the question successfully but those who didn't often floundered being unsure how to standardize their expression.

6686 Statistics S4

Introduction

Overall the paper worked well enabling nearly all candidates to demonstrate what they knew but also allowing the stronger candidates to show their true potential. Most candidates found Q1, Q2, Q4 and Q5 accessible with many scoring highly here. Q3 and Q6 proved to be good discriminators and only the better candidates made significant progress through these. Generally candidates were able to carry out calculations but showed a lack of understanding when they had to use or interpret the answer to their calculations.

Report on individual questions

Question 1

This proved to be a good starter question and most candidates gave good solutions. A minority of candidates did not state a conclusion in context.

Question 2

In part (a) few candidates realise that it is the “differences” which needs to be normally distributed and not the distributions themselves. In part (b) many candidates were able to write in some form that the data was collected in pairs. In part (c) the most common error was an incorrect standard deviation. The majority of candidates were able to apply the method correctly.

In part (d) it was pleasing to see that many candidates were able to write in context what the type two error would be although a few could only regurgitate the definition from a book showing that they had no real understanding of what it is.

Question 3

Many candidates were able to write down the definitions in one form or another in part (a) and part (b). Only a few did not read the question and write it in terms of H_0 and/or H_1.

Part (c) was not well done. Many candidates recognised the correct distribution but were unable to gain the correct critical regions. In part (d)(i) whilst many candidates were able to find the P(Type II error) using their answer to part (c) a sizeable number simply worked out the significance level used. Nearly all candidates knew how to calculate the power of the test correctly.

In part (e) the better candidates had learnt this and gained both marks.

Question 4

Part (a) was answered well with many candidates gaining full marks. In part (b) although a pooled estimate was worked out correctly by many candidates they then failed to use the square root of it in their calculations of the confidence interval.

In part (c) candidates knew that to find the confidence interval/pooled estimate that the variances needed to be equal but few commented on the fact that this has been established in part (a).

Question 5

The most able candidates gained full marks for this question. The most common error was in part (a) when they used $\sqrt{25.2}$ rather than $\sqrt{\frac{25.2}{15}}$. In part (b) many candidates gained the first 3 marks but were then unable to correctly calculate the interval for σ^2. In part (c) many candidates knew that they needed to use the highest values from parts (a) and (b), but then either did not square root the "62.5" or used $\sqrt{\frac{\text{"62.5"}}{15}}$ when finding z.

Question 6

This question proved to be the most challenging for many candidates. In part (a) many candidates tried to prove it was equal to $\frac{k}{2}$ and few made the concluding statement that it was unbiased.

In part (b) few candidates were able to find $a + b = 2$ and hence made little progress. Those who did find this were able to gain full marks.

In part (c) a mix of both of the given methods on the mark scheme were used. If they chose the first method the majority of candidates did not prove that it was a minimum. If they chose the second they rarely completed the square correctly choosing to leave out the $\frac{k^2}{6}$.

6689 Decision Mathematics D1

Introduction

This paper proved accessible to the candidates. The questions differentiated well, with each giving rise to a good spread of marks. All questions contained marks available to the E grade candidate and there also seemed to be sufficient material to challenge the A grade candidates also.

The questions were presented in probable order of difficulty and this ordering proved correct, for the candidature as a whole.

Almost all the candidates tackled all the questions suggesting that the removal of material into D2 has reduced the pressure caused by content and time on this paper.

Candidates are reminded that they should not use methods of presentation that depend on colour, but are advised to complete diagrams in (dark) pencil. This remains a particular problem in the questions on matchings (Q3 on this paper)

Candidates are also reminded that this is a 'methods' paper. They need to make their method clear: 'spotting' the correct answer, with no working, rarely gains credit.

Some candidates are using methods of presentation that are too time-consuming. The space provided in the answer booklet and the marks allotted to each section should assist candidates in determining the amount of working they need to show.

Report on individual questions

Question 1

This was a good starter and was well answered. Some candidates headed the columns correctly but then did not **list** the arcs correctly; they should be encouraged to list each arc as it is selected. As expected the commonest error was to look for the smallest entry in the latest column rather than in all numbered columns.

Question 2

This was often a good source of marks for candidates. Most were able to calculate the lower bound getting 3.8 and therefore 4 bins. Most candidates applied first fit correctly, although some candidates did not offer each item to each bin in turn, starting with bin 1 each time, so often the 9 was the first item to be misplaced. Some candidates wasted time by replacing each letter by a number, or explaining in lengthy detail the steps they took to place each item. Not all candidates realised that they should fill as many bins as possible when using full bin, most found one full bin, but only the best found two.

Question 3

This was a good source of marks for well-prepared candidates. Most are making the alternating paths they used clear. Candidates are reminded that they are instructed to **list** their alternating paths, rather than draw them, which makes them clearer for the examiners to give full credit. The change status step must be made clear, either by writing ‘change status’ or by re-listing the alternating path with the connective symbols switched. Terms such as ‘add’ ‘remove’ ‘switch’ are not accepted as evidence of the change status step. Many candidates did not list the complete improved matching at the end of (a) often omitting S=1 or C=3. Candidates are reminded that colour is not visible and so colour should not be used to indicate the change status step or used to ‘pick out’ the improved matching on a bipartite graph showing more than the minimum necessary arcs. In part (b) candidates needed to be specific and name the people and the tasks used in their explanation. The most efficient answer involved C, 3 and 6 only. Alternating paths **must** go from an unmatched node to an unmatched node; in part (c) this meant that the path must go from E to 6, a number of candidates started at S.

Question 4

This was generally well done. A disappointingly large number of candidates only chose one pivot per iteration, rather than choosing one pivot per sublist, and some candidates used lengthy methods of presentation that isolated each sublist in turn, making it difficult to see if they were choosing more than one pivot per iteration. The examiners would advise candidates to refrain from showing this unnecessary detail and simply indicate the pivots selected at each iteration. Some candidates did not select a pivot where the sublist was of order two, with the two items being in the correct order, and some did not consistently pick ‘middle left’ or ‘middle right’ when the sublist was of even order. Candidates are reminded that when the items are being transferred to the next line, the order of the items should be preserved, so if item Y is to the left of item X in the current line, neither of them being a pivot, then Y should be to the left of X in the next line. The best candidates allowed each item to become a pivot before declaring the sort complete. Some candidates did not check that their final list was in alphabetical order. In part (b) some candidates tried to apply the algorithm to the original unsorted list given at the start of (a) and others did not discard the pivot at each stage, but generally the binary search was very well done. A few candidates selected J as the first pivot, the specification makes it clear that candidates must take the ‘middle right’ where necessary.

Question 5

Almost all candidates found three pairings of the correct odd nodes, but few found all three correct totals. The answers 85, 83 and 112 were very common. Some weaker candidates listed all six couples, without pairing them, gaining no marks. After identifying the shortest arcs from their parings most candidates were able to calculate the length of the shortest route, but some wasted time listing the shortest route. Examiners were pleased that more candidates then previously were able to make progress in part (b). A significant number failed to indicate, when choosing DE, that it was the shortest route between two odd nodes. Some incorrectly chose EC at 39 as the shortest, since it was in their current route. Some candidates lost a mark by not working out 660 even if they had got the other two marks in this part.

Question 6

Many candidates found part (a) straightforward and gained full marks, but candidates are reminded that it is the **order** of the working values that is key for examiners to determine if the algorithm has been applied correctly. Many candidates missed the shortcut to C, giving the third (and smallest) value of 21, coming from D and it appeared that some candidates were working 'geographically' left to right through the network rather in order of their arithmetic nearness to A. This misconception often led to incorrect, additional or incorrectly ordered working values particularly at C, D, G and I. Many candidates did not read part (b) of the questions carefully and stated the shortest distance from A to I rather than A to G.

Question 7

There were some very good, and very poor, solutions seen to this question. Almost all candidates were able to write down the correct inequality in part (a) with only a very few getting the wrong coefficients or replacing the inequality with an equals sign. Part (b) proved challenging for many candidates. Candidates struggled in particular to interpret $y \leq 4x$. The usual error was to confuse 'small' with 'large' but many failed to refer to, or reversed, the inequality. The most able described the inequality in terms of percentages; where this was seen it was almost always correct. Most candidates drew $5x + 7y = 350$ and $y = 20$ correctly. Most candidates used a ruler and most plotted the axes interceptions accurately. Unsurprisingly $y = 4x$ caused the most difficulty, often replaced by $x = 4y$. Most candidates used shading sensibly although some shaded so scruffily that they obscured their line. Most candidates labelled R correctly; most candidates did not label their lines. Most candidates were able to write down the correct objective function. Part (e) was often poorly done with many candidates failing to make their method clear; if using the objective line method candidates MUST draw an objective line, and of a sensible length, so that its accuracy can be checked; if using point testing then the points and their values must be stated. As always those who use the objective line method are more successful than those who use point testing. When point testing, all vertices in the feasible region must be tested. Many candidates assumed that the point (36, 20) was a vertex; it was pleasing to see a small number of scripts where this was tested and found to be outside the feasible region. Others found the precise point but then did not seek integer solutions to complete their answer.

Question 8

This was a challenging question for many, but those candidates who were well-prepared made good progress and some good solutions were seen. Most, but not all, candidates were able to complete part (a) correctly. Most identified the critical activities but many included I as a critical activity. Examiners were pleased that most candidates showed their calculations when determining the length of the total floats, as directed, and most calculated the floats correctly. Some omitted, or made only a token attempt at part (d); others drew a scheduling diagram, but far fewer than in previous years. Many candidates made a good attempt at the cascade diagram and most handled the critical activities correctly. Activities K and M were often omitted, and the floats on F and G were often incorrectly drawn. Only the best candidates completed part (e) correctly. Many omitted C for day 31 and many listed 4 or 5 activities for day 16.

6690 Decision Mathematics D2

Introduction

The paper proved accessible to most candidates, who demonstrated good knowledge and understanding, and most had sufficient time to complete the paper, blank pages were rare.

The general quality of presentation was good but there were many arithmetic errors, especially in Q1, Q2, Q3, Q6 and Q7. Poorly written numbers caused problems for a number of candidates, misreading their own writing. Candidates continue to use clear and efficient styles of presentation and good use of the tables given in the answer book.

The questions were arranged in order of perceived difficulty and this proved a correct ordering. There seemed sufficient material for the weaker candidates to make progress and also sufficient material for the more able.

Candidates are reminded that they should not use methods of presentation that depend on colour, but are advised to complete diagrams in (dark) pencil.

Report on individual questions

Question 1

The great majority of candidates found this an easy first question. Most candidates completed parts (a) and (b) correctly. In part (c) a number of candidates used 1, instead of 2, as the 'minimum uncovered' element in the second iteration.

Question 2

This proved a good discriminator with able candidates producing concise, accurate solutions. Most candidates made a good attempt at the difference between the classical and practical problems, but some muddled the two and others referred to arcs/edges rather than vertices/nodes. Most went on to correctly use the nearest neighbour algorithm but some used it to find a path from A to C and then doubled it. A few candidates found the minimum spanning tree and doubled it. Most candidates selected the lower upper bound. In part (d) a significant number of candidates did not find the correct residual minimum spanning tree, although most did then add the two shortest arcs from B. Most candidates were able to select the higher lower bound.

Question 3

This also proved a good discriminator. Part (a) was often a good source of marks for most. In part (b) most candidates deleted column 3, although a significant number deleted column 1. The matrix transposition and/or sign changes were frequently omitted in part (c). Most candidates were able to set up three probability expressions with relatively few algebraic errors, but the graphs were often poorly drawn, some without a ruler or linear scale. A significant number of candidates were not able to correctly select the optimal point.

Question 4

Most candidates were able to evaluate at least one cut correctly, but only the best were able to calculate both accurately. Many candidates were able to gain full marks in part (b), with many simply listing the flow-augmenting route, its flow and the maximum flow. A significant number complicated the question by not starting from the given flow, provided in (a).

Question 5

This question caused problems for most candidates. Very few were able to read off the correct values, or to write down the correct objective function, expressions with two equal signs being the most common error here.

Question 6

Most candidates started this question well and found the correct stepping stone route; few stated the correct exiting cell, with ZC being a very common incorrect answer. Many wasted time unnecessarily calculating shadow costs and improvement indices to confirm ZA as the entering cell. As in other papers improved solutions were marred by leaving zero in the exit cell. Those who found the correct (5 term) solution generally went on to find the correct shadow costs and improvement indices and the second improved solution, although many did not state the entering and exiting cells. Some candidates wasted time listing the stepping stone route rather than drawing it. Ample blank tables are provided in the answer book, but some candidates cram three or four sets of information into one table, often putting improved solution, improvement indices, new stepping stone route and costs all on one diagram.

Question 7

This was a challenging question, requiring candidates to think carefully, the examiners were pleased by the way in which many of the candidates tackled this question. Most candidates were able to make good progress with part (a), with only a small minority unable to attempt anything or just the first four rows. Most worked correctly through the subsequent stages without much difficulty; although some omitted stage 2 state 50 and/or 0 and some did unnecessary work in stage 3. Some poor arithmetic was seen with 125+60=195 and 190+60=240 being fairly frequently seen. Those who completed the table were then able to answer the subsequent questions, although some were unable to state the correct maximum income. In part (b) most were able to define ‘scheme’ correctly, but the definitions of state and action challenged many.

Question 8

This proved to be the most challenging question on the paper, with many attempts marking little progress, but very few blanks seen. . Many candidates did not answer the question as stated and did not give their answer from Laura’s point of view and did not give their constraints as inequalities. Very few added 6 to all terms to make the entries positive. Most did attempt to define their variables, but some tried to structure the problem in terms of nine unknowns, confusing Game Theory with Transportation or Allocation. Many stated an objective but then either did not state if they were maximising or minimising or selected the wrong one. The majority of candidates used rows instead of columns to set up the constraints and those that sis correctly use columns often made mistakes with the inequality signs. Very many candidates unnecessarily made two attempts at the question, firstly in terms of V and p’s and then in terms of x.

Grade Boundary Statistics

The tables below give the lowest raw marks for the award of the stated uniform marks (UMS).

Module	**80**	**70**	**60**	**50**	**40**
6663 Core Mathematics C1	55	47	39	31	23
6664 Core Mathematics C2	53	45	38	31	24
6665 Core Mathematics C3	67	59	52	45	38
6666 Core Mathematics C4	58	52	46	40	35
6667 Further Pure Mathematics FP1	63	55	47	39	31
6668 Further Pure Mathematics FP2	58	50	42	34	26
6669 Further Pure Mathematics FP3	50	44	38	32	26
6674 Further Pure Mathematics FP1	64	56	49	42	35
6675 Further Pure Mathematics FP2	54	48	42	36	31
6676 Further Pure Mathematics FP3	63	57	51	45	39
6677 Mechanics M1	55	46	38	30	22
6678 Mechanics M2	62	54	46	39	32
6679 Mechanics M3	48	42	36	30	25
6680 Mechanics M4	57	49	42	35	28
6681 Mechanics M5	57	49	42	35	28
6683 Statistics S1	60	52	44	36	29
6684 Statistics S2	62	54	46	38	30
6691 Statistics S3	62	54	47	40	33
6686 Statistics S4	62	53	45	37	29
6689 Decision Maths D1	63	56	50	44	38
6690 Decision Maths D2	57	49	41	34	27

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