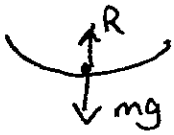
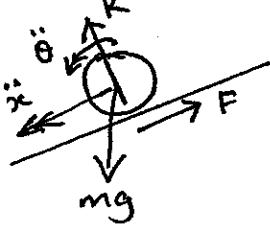
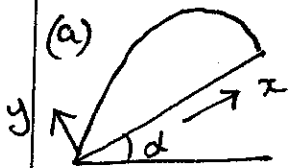


January 2006
6682 Mechanics M6
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) $r = a \sin 2\theta \Rightarrow \dot{r} = 2a \cos 2\theta \cdot \omega$ $v^2 = \dot{r}^2 + (r\dot{\theta})^2 = 4a^2 \omega^2 \cos^2 2\theta + a^2 \omega^2 \sin^2 2\theta$ $\theta = \frac{\pi}{8} \quad v^2 = 4a^2 \omega^2 \cdot \frac{1}{2} + a^2 \omega^2 \cdot \frac{1}{2}$ $\Rightarrow v = \frac{\sqrt{5}}{2} a \omega$</p> <p>(b) $A_{\text{c}T} = r\ddot{\theta} + 2\dot{r}\dot{\theta}; \quad \ddot{\theta} = 0 \Rightarrow A_{\text{c}T} = 4a \omega^2 \cos 2\theta$ $\theta = \frac{\pi}{4} : \cos 2\theta = 0 \quad A_{\text{c}T} = \underline{0} \quad (*)$</p>	<p>M1 A1 M1 M1 A1 (5) M1 M1 A1 (3) (8)</p>
2.	<p>(a) $m\ddot{s} = -mg \sin \psi \Rightarrow \ddot{s} = -\frac{g}{2a} s \quad (*)$</p> <p>(b) $\int \dot{s} d\dot{s} = -\frac{g}{2a} \int s ds$ $\frac{1}{2} \dot{s}^2 = -\frac{g}{4a} s^2 \quad (+C)$ (b=0) $s = \frac{3}{2} a, \quad \dot{s} = 0 \Rightarrow C = \frac{9ga}{16}$ $s = 0 \quad \dot{s}^2 = \frac{9ga}{8} \Rightarrow \dot{s} = \frac{3}{2} \sqrt{\frac{ga}{2}} \quad (*)$</p> <p>(c)  $R - mg = \frac{mv^2}{\rho}$ $\rho = \frac{ds}{d\psi} = 2a \cos \psi$ Sub $\psi = 0, v = \frac{3}{2} \sqrt{\frac{ga}{2}} : R = mg + \frac{m}{2a} \cdot \frac{9ga}{8}$ $= \underline{\frac{25mg}{16}}$</p>	<p>M1 A1 (2) M1 M1 A1 (3) M1 A1 B1 M1 A1 (5) (10)</p>

Question Number	Scheme	Marks
3.	<p>MI of hoop about P = $ma^2 + ma^2 = 2ma^2$</p> <p>Moment of mom^m about P:</p> $m v \cdot \frac{2}{3} a + ma^2 \cdot \omega = 2ma^2 \cdot \omega'$ <p>Rolling before $\Rightarrow v = a\omega$</p> <p>Solve $\rightarrow \omega' = \underline{\underline{\frac{5}{6}\omega}}$</p>	<p>M1 A1</p> <p>M1 A3, 2, 1, 0</p> <p>B1</p> <p>M1 A1</p> <p>(9)</p>
4.	 <p>R(\uparrow) $R = mg \cos \alpha$</p> <p>Sliding $\Rightarrow F = \mu R$</p> $F a = \frac{2}{5} m a^2 \ddot{\theta} (= \mu m g a \cos \alpha)$ <p>R(\downarrow) $m \ddot{x} = m g \sin \alpha - F$</p> $\ddot{x} = (g \sin \alpha - \mu g \cos \alpha) t, +V$ $\ddot{\theta} = \frac{5 \mu g \cos \alpha}{2a} t$ <p>Slips until $\ddot{x} = a \ddot{\theta}$, i.e. when</p> $(g \sin \alpha - \mu g \cos \alpha) t + V = \frac{5}{2} \mu g \cos \alpha t$ $\Rightarrow t = \frac{2V}{g(7\mu \cos \alpha - 2 \sin \alpha)} \quad (*)$	<p>B1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1, A1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>(15)</p>

Qu5.



$$\begin{aligned} \dot{x} &= V \cos \theta - g \sin \alpha t \\ &= V \cos \theta - \frac{3}{5} g t \end{aligned}$$

$$\begin{aligned} y &= V \sin \theta t - \frac{1}{2} g \cos \alpha t^2 \\ &= V \sin \theta t - \frac{2}{5} g t^2 \end{aligned}$$

$$y = 0, \dot{x} = 0 \quad \therefore \text{from } \dot{x} \quad t = \frac{5V \cos \theta}{3g}$$

$$\begin{aligned} \text{Hence (y. y): } V \sin \theta &= \frac{2}{5} g \cdot \frac{5V \cos \theta}{3g} \\ \Rightarrow \tan \theta &= \frac{2}{3} \quad (*) \end{aligned}$$

M1

A1

M1

A1

M1 A1

M1

A1 (3)

(b)

$$x = V \cos \theta t - \frac{3}{10} g t^2$$

$$\dot{x} = 0, t = 2 \Rightarrow 2 = \frac{5V \cos \theta}{3g} \Rightarrow V \cos \theta = \frac{6g}{5}$$

$$\begin{aligned} \Rightarrow x &= \frac{6g}{5} \times 2 - \frac{3}{10} g \times 4 \\ &= 1.2g \approx \underline{11.8 \text{ m}} \end{aligned}$$

M1 A1

M1

M1

A1 (5)

(c)

$$\begin{aligned} V &= \frac{6g}{5 \cos \theta} = \frac{6g}{5 \cos(\arctan 2/3)} \\ &\approx \underline{14.1 \text{ m s}^{-1}} \end{aligned}$$

M1, M1

A1 (3)

(16)

Qu. 6 (a)

$$\text{Trans. accel}^n = 0 \Rightarrow r^2 \dot{\theta} = k$$

$$t=0, r=1, r\dot{\theta} = 3 \Rightarrow k=3$$

$$\text{Tens}^n \text{ in string} = 4(r-1)$$

$$\text{Hence } 2(\ddot{r} - r\dot{\theta}^2) = -4(r-1)$$

$$r\dot{\theta}^2 = r \cdot \frac{9}{r^4} = \frac{9}{r^3}$$

$$\Rightarrow \ddot{r} = \frac{9}{r^3} - 2r + 2 \quad (*)$$

M1

M1 A1

B1

M1 A1

M1

A1 (8)

(b)

$$\int \dot{r} dr = \int \frac{9}{r^3} - 2r + 2 dr$$

$$\frac{1}{2} \dot{r}^2 = -\frac{9}{2r^2} - r^2 + 2r + C$$

$$t=0, r=1, \dot{r}=0 \Rightarrow C = \frac{7}{2}$$

$$\text{Hence } \dot{r}^2 = 7 - \frac{9}{r^2} - 2r^2 + 4r \quad (*)$$

M1 A1

M1

A1 (4)

$$\dot{r}^2 = \frac{1}{r^2} (-2r^4 + 4r^3 + 7r^2 - 9)$$

$$= \frac{1}{r^2} (r-1)(-2r^3 + 2r^2 + 9r + 9)$$

$$= \frac{1}{r^2} (r-1)(3-r)(3+4r+2r^2)$$

M1

M1 A1

$$3 + 4r + 2r^2: \quad "b^2 - 4ac" = -8$$

Hence no real roots - always > 0

M1

$$\text{Hence } (r-1)(3-r) \geq 0$$

$$\Rightarrow \underline{1 \leq r \leq 3}$$

CSO

A1 (5)

(17)