Paper Reference(s)

6672/01

Edexcel GCE

Pure Mathematics P2
Advanced/Advanced Subsidiary

Monday 20 June 2005 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Lilac)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P2), the paper reference (6672), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has 8 questions.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N21140A

1. Solve

(a) $5^x = 8$, giving your answers to 3 significant figures,

(3)

(b) $\log_2 (x + 1) - \log_2 x = \log_2 7$.

(3)

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2. (a) Write down the first three terms, in ascending powers of x, of the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of x is $(-q)$ and the coefficient of x^2 is $11q$,

(b) find the value of p and the value of q.

(4)

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3. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \qquad 0 \leq x \leq 20.$$

(a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

(b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m s^{-1},

(c) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

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4. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{2}{x-1}, x > 1$.

(4)

(*b*) Find $\mathrm{f}^{-1}(x)$.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*b*) Solve $\mathrm{fg}(x) = \frac{1}{4}$.

(3)

5.

Figure 1

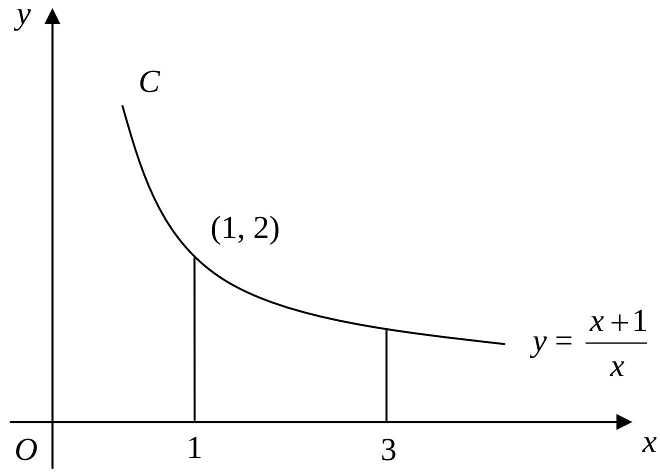


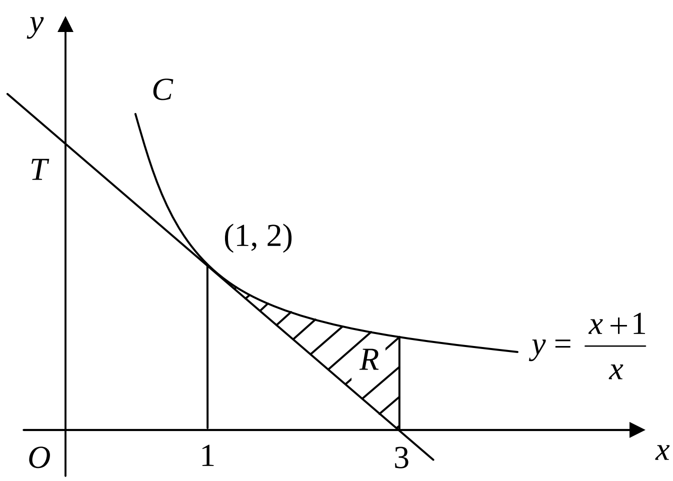
Figure 1 shows part of the curve C with equation $y = \dfrac{x+1}{x}$, $x > 0$.

The finite region enclosed by C, the lines $x = 1$, $x = 3$ and the x-axis is rotated through 360° about the x-axis to generate a solid S.

(*a*) Using integration, find the exact volume of S.

(7)

Figure 2



The tangent T to C at the point (1, 2) meets the x-axis at the point (3, 0). The shaded region R is bounded by C, the line $x = 3$ and T, as shown in Figure 2.

(*b*) Using your answer to part (*a*), find the exact volume generated by R when it is rotated through 360° about the x-axis.

(3)

6. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

7.

Figure 1

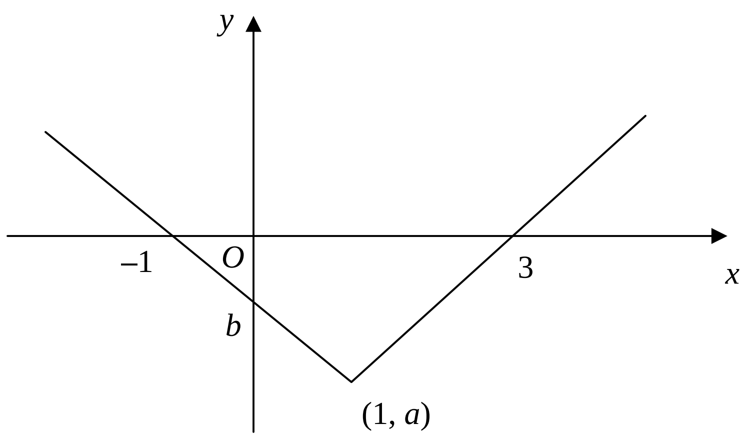


Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = \mathrm{f}(x + 1)$,

(2)

(*b*) $y = \mathrm{f}(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $\mathrm{f}(x) = |x – 1| – 2$, find

(*c*) the value of a and the value of b,

(2)

(*d*) the value of x for which $\mathrm{f}(x) = 5x$.

(4)

8. (*a*) Given that $2 \sin(\theta + 30)° = \cos(\theta + 60)°$, find the exact value of $\tan \theta°$.

(5)

(*b*) (i) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(ii) Hence solve, for $0 \leq x < 2\pi$,

$$\cos 2x = \sin x,$$

giving your answers in terms of π.

(5)

(iii) Show that $\sin 2y \tan y + \cos 2y \equiv 1$, for $0 \leq y < \frac{1}{2}\pi$.

(3)

TOTAL FOR PAPER: 75 MARKS

END