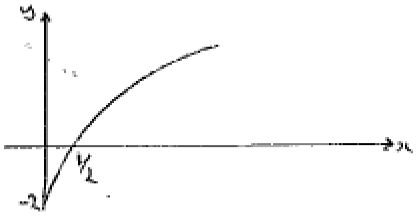
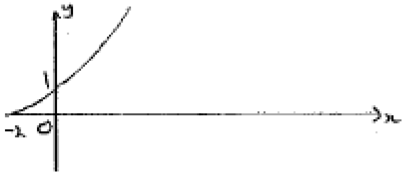
January 2005

6672 Pure Mathematics P2

Mark Scheme

Question Number	Scheme		Marks
1a)	$\frac{(x-3)(x+2)}{x(x-3)}$; $= \frac{(x+2)}{x}$ or $1+\frac{2}{x}$	B1 numerator, B1 denominator ; B1 either form of answer	B1,B1,B1 (3)
1b)	$\frac{(x+2)}{x} = x+1 \Rightarrow x^2 = 2$	M1 for equating f(x) to $x + 1$ and forming quadratic. A1 candidate's correct quadratic	M1 A1√
	$x = \pm\sqrt{2}$		A1 (3)
2a)		Reflected in x - axis $0<x<1$ Cusp + coords Clear curve going correct way Ignore curve $x < 0$	M1 A1 (2)
		General shaped and -2 (1/2, 0) Ignore curve $x < 0$	B1 B1 (2)
		Rough reflection in $y = x$ (0,1) or 1 on y - axis (-2, 0) or -2 on x - axis and no curve $x < -2$	B1 B1 B1 (3)

Question Number	Scheme		Marks
3a)	$u_2 = (-1)(2) + d = -2 + d$		B1
	$u_3 = (-1)^2(-2 + d) + d = -2 + 2d$	Attempting to find u_3 in terms of d	M1
	$u_4 = (-1)^3(-2 + 2d) + d = 2 - d$	u_3 and u_4 correct	A1
	$u_5 = (-1)^4(2 - d) + d = 2$ * cso	fully correct	A1*
			(4)
b)	$u_{10} = u_2 = d - 2$ o.e.	their u_2 must contain d	B1√
			(1)
c)	$-2 + 2d = 3(-2 + d) \Rightarrow d = 4$	M1 equating their u_3 to their $3u_2$ must contain d	M1 A1
			(2)
4a)	(0,4), or $x = 0$ and $y = 4$		B1
			(1)
b)	$V = \pi \int x^2 dy$	attempt use of, must have pi	M1
	$x^2 = y - 4$ or $x = \sqrt{y - 4}$		B1
	$V = (\pi)\int (y-4)dy$	attempt to integrate	M1
	$= (\pi)\left[\frac{y^2}{2} - 4y\right]$	correct integration ignore pi	A1
	using limits in a changed form to give	8,4 either way but must subtract	M1
	$\pi[(32-32)-(8-16)] = 8\pi$. (c.a.o)		A1
			(6)

Question Number	Scheme		Marks	
5a)	$\log 3^x = \log 5$	taking logs	M1	
	$x = \frac{\log 5}{\log 3}$ or $x \log 3 = \log 5$		A1	
	$= 1.46$ cao		A1	(3)
b)	$2 = \log_2 \frac{2x+1}{x}$		M1	
	$\frac{2x+1}{x} = 4$ or equivalent;	4	B1	
	$2x + 1 = 4x$	multiplying by x to get a linear equation	M1	
	$x = \frac{1}{2}$		A1	(4)
c)	$\sec x = 1/\cos x$		B1	
	$\sin x = \cos x \Rightarrow \tan x = 1 \quad x = 45$	use of $\tan x$	M1, A1	(3)

Question Number	Scheme		Marks	
6a)	I = $3x + 2e^x$		B1	
	Using limits correctly to give 1 + 2e. (c.a.o.)	must subst 0 and 1 and subtract	M1 A1	(3)
b)	$A = (0, 5)$;	$y = 5$	B1	
	$\frac{dy}{dx} = 2e^x$		B1	
	Equation of tangent: $y = 2x + 5$; c = –2.5	attempting to find eq. of tangent and subst in $y = 0$, must be linear equation	M1; A1	(4)
c)	$y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \quad \Rightarrow 4y - 2 = 5x - xy$	putting $y =$ and att. to rearrange to find x .	M1; A1	
	$g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent	must be in terms of x	A1	(3)
d)	gf(0) = g(5); =3	att to put 0 into f and then their answer into g	M1; A1	(2)

Question Number	Scheme	Marks
7a)	Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	M1
	$DE = 4 \sin \theta$ * (c.s.o.)	A1*
		(2)
b)	$P = 2\ DE + 2EF$ or equivalent. With attempt at EF	M1
	$= 8\sin \theta + 4\cos \theta$ * (c.s.o.)	A1*
		(2)
c)	$8\sin \theta + 4\cos \theta \ = R \sin (\theta + \alpha)$	
	$= R \sin \theta \cos \alpha + \text{R} \cos \theta \sin \alpha$	
	Method for R, method for α — need to use tan for 2nd M	M1 M1
	[$R \cos \alpha = 8$, R $\sin \alpha = 4$ $\quad \tan \alpha = 0.5$, R = $\sqrt{(8^2 + 4^2}$]	
	$R = 4\sqrt{5}$ or 8.94, $\alpha = 0.464$ (allow 26.6), — awrt 0.464	A1 A1
		(4)
d)	Using candidate's $R \sin (\theta + \alpha) = 8.5$ to give $(\theta + \alpha) = \sin^{-1} \frac{8.5}{R}$	M1
	Solving to give $\theta = \sin^{-1} \frac{8.5}{R} - \alpha$, $\theta = 0.791$ (allow 45.3)	M1 A1
	Considering second angle: $\theta + \alpha = \pi$ (or 180) – $\sin^{-1} \frac{8.5}{R}$;	M1
	$\theta = 1.42$ (allow 81.6)	A1
		(5)

Question Number	Scheme		Marks
8a)	$f'(x) = -\frac{1}{2x^2}; + \frac{1}{x}$	M1 for evidence of differentiation. Final A –no extras	M1A1;A1
	$f'(x) = 0 \Rightarrow \frac{-1+2x}{2x^2} = 0; \Rightarrow x = 0.5$	(or subst x = 0.5)	M1A1 * cso (5)
b)	$y = 1 - 1 + \ln\left(\frac{1}{4}\right); = -2\ln 2$	Subst 0.5 or their value for *x* in	M1;A1 (2)
c)	f(4.905) = < 0 (–0.000955), f(4.915) = > 0 (+ 0.000874)	evaluate	M1
	Change of sign indicates root between and correct values to 1 sf)		A1 (2)
d)	$\frac{1}{2x} - 1 + \ln\left(\frac{x}{2}\right) = 0; \quad \Rightarrow 1 - \frac{1}{2x} = \ln\left(\frac{x}{2}\right)$		
	$\Rightarrow \frac{x}{2} = e^{\left(1-\frac{1}{2x}\right)}; \Rightarrow x = 2e^{\left(1-\frac{1}{2x}\right)}$ * (c.s.o.)	M1 for use of e to the power on both sides	M1;A1 (2)
e)	$x_1 = 4.9192$		B1
	$x_2 = 4.9111, \ x_3 = 4.9103,$	both, only lose one if not 4dp	B1 (2)